NICTA

COMP 4161

NICTA Advanced Course

**Advanced Topics in Software Verification**

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# type classes & locales

# Last Time

- ➜ more C verification
- ➜ preventing undefined execution
- ➜ finite machine words
- ➜ concrete C data types
- ➜ C memory model and pointers

# Content

Rough timeline

- ➜ Intro & motivation, getting started [1]
- ➜ Foundations & Principles
  - Lambda Calculus, natural deduction [2,3,4[a]]
  - Higher Order Logic [5,6[b],7]
  - Term rewriting [8,9,10[c]]
- ➜ Proof & Specification Techniques
  - Isar [11,12[d]]
  - Inductively defined sets, rule induction [13[e],15]
  - Datatypes, recursion, induction [16,17[f],18,19]
  - Calculational reasoning, mathematics style proofs [20]
  - Hoare logic, proofs about programs [21[g],22,23]

[a] a1 out; [b] a1 due; [c] a2 out; [d] a2 due; [e] session break; [f] a3 out; [g] a3 due

# Type Classes

**Common pattern in Mathematics:**

- ➜ Define abstract structures (semigroup, group, ring, field, etc)
- ➜ Study and derive properties in these structures
- ➜ Instantiate to concrete structure: (nats with + and * from a ring)
- ➜ Can use all abstract laws for concrete structure

**Type classes in functional languages:**

- ➜ Declare a set of functions with signatures (e.g. plus, zero)
- ➜ give them a name (e.g. c)
- ➜ Have syntax 'a :: c for: type 'a supports the operations of c
- ➜ Can write abstract polymorphic functions that use plus and zero
- ➜ Can instantiate specific types like nat to c

**Isabelle supports both.**

# Type Class Example

**Example:**

**class** semigroup =

**fixes** mult :: 'a $\Rightarrow$ 'a $\Rightarrow$ 'a (**infix** $\cdot$ 70)

**assumes** assoc: $(x \cdot y) \cdot z = x \cdot (y \cdot z)$

**Declares:**

- ➜ a name (semigroup)
- ➜ a set of operations (fixes mult)
- ➜ a set of properties/axioms (assumes assoc)

# Type Class Use

**Can constrain type variables 'a with a class:**

**definition** sq :: ('a :: semigroup) $\Rightarrow$ 'a **where** sq x $\equiv$ x · x

More than one constraint allowed. Sets of class constraints are called **sort**.

**Can reason abstractly:**

**lemma** "sq x · sq x = x · x · x · x"

**Can instantiate:**

**instantiation** nat :: semigroup

**begin**

**definition** "(x::nat) · y = x * y"

**instance** $< proof >$

**end**

# Demo: Type Classes

# Type constructors

Basic type instantiation is a special case.

**In general:**
Type constructors can be seen as functions from classes to classes.

**Example:**
product type prod :: (semigroup, semigroup) semigroup
(or: pairs of semigroup elements again form a semigroup)

Declarations such as *(semigroup, semigroup) semigroup* are called **arities**.

**Fully integrated with automatic type inference.**

# Subclasses

Type classes can be extended:

**class** rmonoid = semigroup +

**fixes** one :: 'a

**assumes** $x \cdot \text{one} = x$

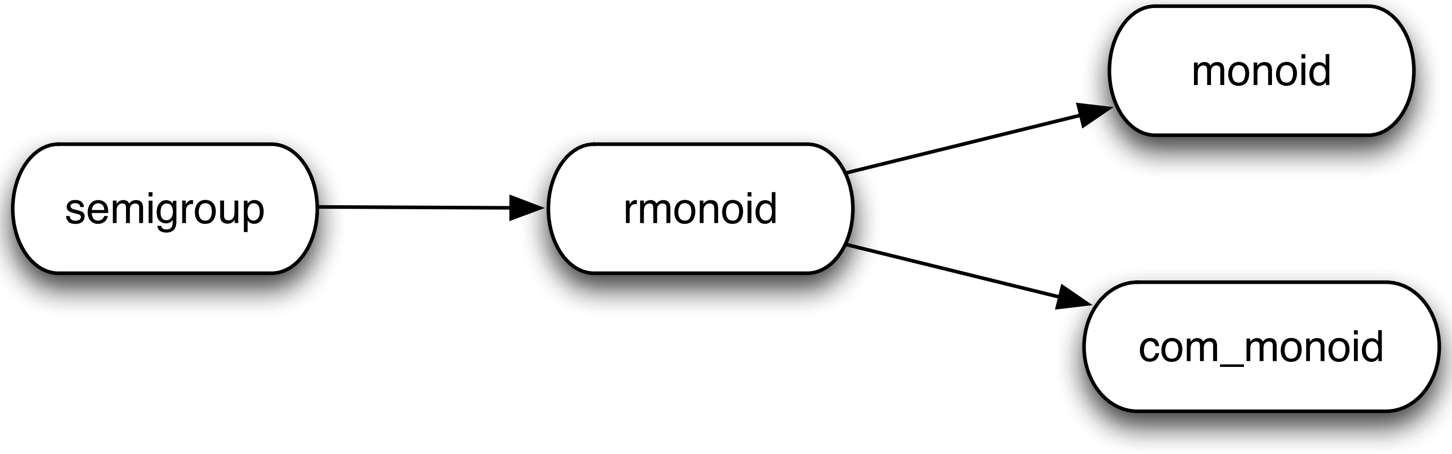
rmonoid is a **subclass** of semigroup

Has all operations & assumptions of semigroup + additional ones.

Can build hierarchies of abstract structures.

# More Subclasses

**Example structure:**

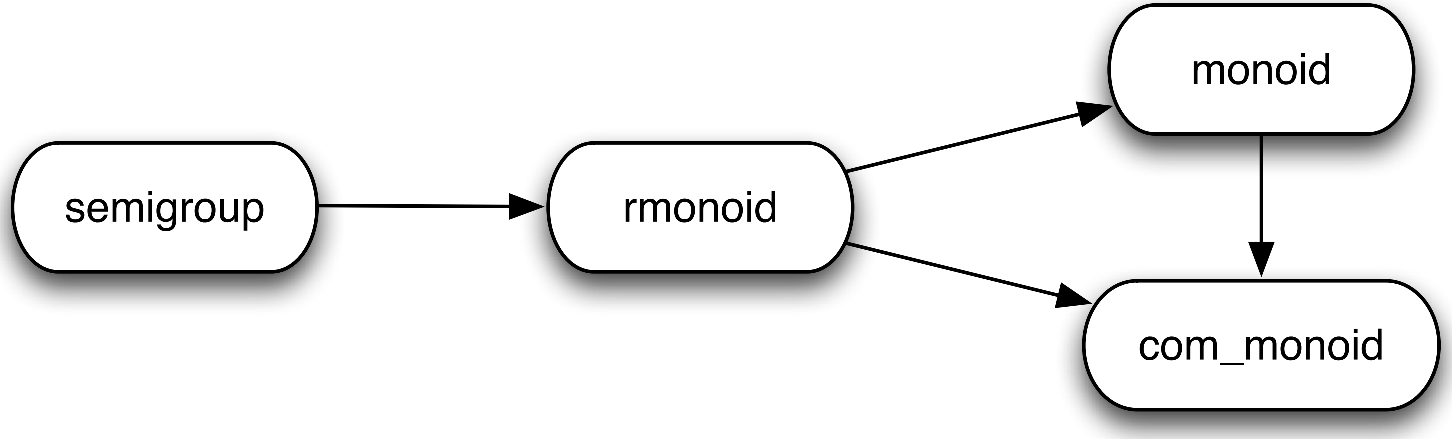



**Can prove:** every com_monoid is also a monoid.

Can tell Isabelle that connection:

**subclass** (in com_monoid) monoid $< proof >$

**Result:**



# Limitations

**Operations (fixes) are implemented by overloading**

➜ each type constructor can implement each operation only once

**Type inference must remain automatic, with unique most general types**

➜ type classes can mention only one type variable

➜ type constructor arities must be co-regular:

$K :: (c_1, ..., c_n)c$ and $K :: (c'_1, ..., c'_n)c'$ and $c \subseteq c' \implies \forall i.\ c_i \subseteq c'_i$

# DEMO: SUBCLASSES

# Isar Is Based On Contexts

**theorem** $\bigwedge x.\ A \Longrightarrow C$

**proof** -

  **fix** $x$

  **assume** $Ass$: $A$

  $\vdots$

  **from** $Ass$ **show** $C$ . . .

$x$ and $Ass$ are visible

inside this context

**qed**

# Beyond Isar Contexts

Locales are extended contexts, look similar to type classes

- ➜ Locales are **named**
- ➜ Fixed variables may have **syntax**
- ➜ It is possible to **add** and **export** theorems
- ➜ It is possible to **instantiate** locales
- ➜ Locale expression: **combine** and **modify** locales
- ➜ No limitation on type variables
- ➜ Term level, not type level: no automatic inference

# Context Elements

Locales consist of **context elements**.

| | |
|---|---|
| **fixes** | Parameter, with syntax |
| **assumes** | Assumption |
| **defines** | Definition |
| **notes** | Record a theorem |

# Declaring Locales

Declaring **locale** (named context) $loc$:

**locale** $loc$ =

$loc1$ + Import

**fixes** . . . Context elements

**assumes** . . .

# Declaring Locales

Theorems may be stated relative to a named locale.

**lemma (in** $loc$**)** $P$ [simp]: $proposition$

$proof$

or

**context** $loc$ **begin**

**lemma** $P$ [simp]: $proposition$

$proof$

**end**

- ➜ Adds theorem $P$ to context $loc$.
- ➜ Theorem $P$ is in the simpset in context $loc$.
- ➜ Exported theorem $loc.P$ visible in the entire theory.

# DEMO: LOCALES 1

# Parameters Must Be Consistent!

➜ Parameters in **fixes** are distinct.

➜ Free variables in **defines** occur in preceding **fixes**.

➜ Defined parameters cannot occur in preceding **assumes** nor **defines**.

# Locale Expressions

Locale name: $n$

Rename: $n:\ e\ q_1 \dots q_n$

Change names of parameters in $e$,

Give new locale the name prefix $n$ (optional)

Merge: $e_1 + e_2$

Context elements of $e_1$, then $e_2$.

# DEMO: LOCALES 2

# Normal Form of Locale Expressions

Locale expressions are converted to flattened lists of locale names.

- ➜ With full parameter lists
- ➜ **Duplicates removed**

Allows for **multiple inheritance**!

# Instantiation

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Move from **abstract** to **concrete**.

**interpretation** label: loc ”parameter 1” . . . ”parameter n”

- ➜ Instantiates locale **loc** with provided parameters.
- ➜ Imports all theorems of **loc** into current context.
  - Instantiates theorems with provided parameters.
  - Interprets attributes of theorems.
  - Prefixes theorem names with **label**
- ➜ version for local Isar proof: **interpret**

# Sublocales

Similar to type classes:

**sublocale** (in sub_loc) parent_loc $< proof >$

makes facts of parent_loc available in sub_loc.

# DEMO: LOCALES 3

# We have seen today ...

- ➜ Type Classes + Instantiation
- ➜ Locale Declarations + Theorems in Locales
- ➜ Locale Expressions + Inheritance
- ➜ Locale Instantiation