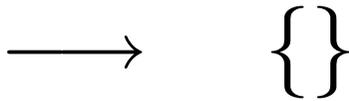




COMP 4161
NICTA Advanced Course

Advanced Topics in Software Verification

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Slide 1



Last Time

- Conditional rewriting
- Rewriting with assumptions
- Case splitting
- Congruence rules
- Permutative rewriting, AC rules

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Content

- Intro & motivation, getting started with Isabelle
- Foundations & Principles
 - Lambda Calculus
 - Higher Order Logic, natural deduction
 - **Term rewriting**
- **Proof & Specification Techniques**
 - **Inductively defined sets, rule induction**
 - Datatypes, recursion, induction
 - Calculational reasoning, mathematics style proofs
 - Hoare logic, proofs about programs

Slide 2



Back to Confluence

Last time: confluence in general is undecidable.

But: confluence for terminating systems is decidable!

Problem: overlapping lhs of rules.

Definition:

Let $l_1 \rightarrow r_1$ and $l_2 \rightarrow r_2$ be two rules with disjoint variables.

They form a **critical pair** if a non-variable subterm of l_1 unifies with l_2 .

Example:

Rules: (1) $f x \rightarrow a$ (2) $g y \rightarrow b$ (3) $f (g z) \rightarrow b$

Critical pairs:

$$\begin{array}{l} (1)+(3) \quad \{x \mapsto g z\} \quad a \xrightarrow{(1)} f g t \xrightarrow{(3)} b \\ (3)+(2) \quad \{z \mapsto y\} \quad b \xrightarrow{(3)} f g t \xrightarrow{(2)} b \end{array}$$

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Completion

(1) $f x \rightarrow a$ (2) $g y \rightarrow b$ (3) $f (g z) \rightarrow b$
is not confluent

But it can be made confluent by adding rules!

How: join all critical pairs

Example:

(1)+(3) $\{x \mapsto g z\}$ $a \xleftarrow{(1)} f g t \xrightarrow{(3)} b$

shows that $a = b$ (because $a \xleftarrow{*} b$), so we add $a \rightarrow b$ as a rule

This is the main idea of the Knuth-Bendix completion algorithm.

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Orthogonal Rewriting Systems

Definitions:

A rule $l \rightarrow r$ is **left-linear** if no variable occurs twice in l .

A **rewrite system** is **left-linear** if all rules are.

A system is **orthogonal** if it is left-linear and has no critical pairs.

Orthogonal rewrite systems are confluent

Application: functional programming languages

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DEMO: WALDMEISTER

Slide 6



THAT WAS TERM REWRITING

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MORE ISAR

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Last Time on Isar

- basic syntax
- proof and qed
- assume and show
- from and have
- the three modes of Isar

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Backward and Forward



Backward reasoning: ... have " $A \wedge B$ " proof

- **proof** picks an **intro** rule automatically
- conclusion of rule must unify with $A \wedge B$

Forward reasoning: ...

assume AB: " $A \wedge B$ "

from AB **have** "... " **proof**

- now **proof** picks an **elim** rule automatically
- triggered by **from**
- first assumption of rule must unify with AB

General case: from $A_1 \dots A_n$ have R proof

- first n assumptions of rule must unify with $A_1 \dots A_n$
- conclusion of rule must unify with R

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Fix and Obtain



fix $v_1 \dots v_n$

Introduces new arbitrary but fixed variables
(~ parameters, \wedge)

obtain $v_1 \dots v_n$ **where** <prop> <proof>

Introduces new variables together with property

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DEMO

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Fancy Abbreviations

this = the previous fact proved or assumed

then = **from this**

thus = **then show**

hence = **then have**

with $A_1 \dots A_n$ = **from** $A_1 \dots A_n$ **this**

?thesis = the last enclosing goal statement

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Moreover and Ultimately



have $X_1: P_1 \dots$	have $P_1 \dots$
have $X_2: P_2 \dots$	moreover have $P_2 \dots$
\vdots	\vdots
have $X_n: P_n \dots$	moreover have $P_n \dots$
from $X_1 \dots X_n$ show \dots	ultimately show \dots

wastes lots of brain power
on names $X_1 \dots X_n$

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General Case Distinctions



show *formula*

proof -

have $P_1 \vee P_2 \vee P_3$ <proof>

moreover { **assume** $P_1 \dots$ **have** ?thesis <proof> }

moreover { **assume** $P_2 \dots$ **have** ?thesis <proof> }

moreover { **assume** $P_3 \dots$ **have** ?thesis <proof> }

ultimately show ?thesis **by blast**

qed

{ ... } is a proof block similar to **proof ... qed**

{ **assume** $P_1 \dots$ **have** P <proof> }

stands for $P_1 \implies P$

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Mixing proof styles



```
from ...
have ...
  apply - make incoming facts assumptions
  apply (...)
  :
  apply (...)
done
```

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Sets in Isabelle



Type 'a set: sets over type 'a

- $\{\}, \{e_1, \dots, e_n\}, \{x. P x\}$
- $e \in A, A \subseteq B$
- $A \cup B, A \cap B, A - B, \neg A$
- $\bigcup x \in A. B x, \bigcap x \in A. B x, \bigcap A, \bigcup A$
- $\{i..j\}$
- insert :: $\alpha \Rightarrow \alpha \text{ set} \Rightarrow \alpha \text{ set}$
- $f:A \equiv \{y. \exists x \in A. y = f x\}$
- ...

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BUILDING UP SPECIFICATION TECHNIQUES: SETS



Proofs about Sets



Natural deduction proofs:

- equality: $[A \subseteq B; B \subseteq A] \Longrightarrow A = B$
- subset: $(\bigwedge x. x \in A \Longrightarrow x \in B) \Longrightarrow A \subseteq B$
- ... (see Tutorial)

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Bounded Quantifiers



→ $\forall x \in A. P x \equiv \forall x. x \in A \longrightarrow P x$

→ $\exists x \in A. P x \equiv \exists x. x \in A \wedge P x$

→ **ball**: $(\bigwedge x. x \in A \implies P x) \implies \forall x \in A. P x$

→ **bspec**: $[\forall x \in A. P x; x \in A] \implies P x$

→ **bexI**: $[P x; x \in A] \implies \exists x \in A. P x$

→ **bexE**: $[\exists x \in A. P x; \bigwedge x. [x \in A; P x] \implies Q] \implies Q$

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DEMO: SETS

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