COMP 4161

NICTA Advanced Course

Advanced Topics in Software Verification

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Slide 1

Binary Search (`java.util.Arrays`)

```
1:     public static int binarySearch(int[] a, int key) {
2:         int low = 0;
3:         int high = a.length - 1;
4:
5:         while (low <= high) {
6:             int mid = (low + high) / 2;
7:             int midVal = a[mid];
8:
9:             if (midVal < key)
10:                 low = mid + 1
11:              else if (midVal > key)
12:                 high = mid - 1;
13:              else
14:                 return mid; // key found
15:          }
16:          return -(low + 1);  // key not found.
17:     }
```

```
6:          int mid = (low + high) / 2;
```

http://googleresearch.blogspot.com/2006/06/extra-extra-read-all-about-it-nearly.html

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Organisatorials

When	Wed	9:00 – 10:30
	Fri	9:00 – 10:30
Where	Mon:	MatSc G10
	Fri:	MatSc G11

http://www.cse.unsw.edu.au/~cs4161/

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About us

Members of the seL4 verification team

- ➜ Functional correctness of a C microkernel
  Isabelle/HOL model ↔ Haskell model ↔ C code
- ➜ 10 000 KLOC / 300 000 lines of proof script (!)
- ➜ 25 person years / $6 million

Read all about it: `http://ertos.nicta.com.au/publications/`

Slide 4

What you will learn

- ➜ how to use a theorem prover
- ➜ background, how it works
- ➜ how to prove and specify
- ➜ how to reason about programs

Health Warning

Theorem Proving is addictive

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Content — Using Theorem Provers

- ➜ Intro & motivation, getting started (today)
- ➜ Foundations & Principles
  - Lambda Calculus
  - Higher Order Logic, natural deduction
  - Term rewriting
- ➜ Proof & Specification Techniques
  - Datatypes, recursion, induction
  - Inductively defined sets, rule induction
  - Calculational reasoning, mathematics style proofs
  - Hoare logic, proofs about programs

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Credits

some material (in using-theorem-provers part) shamelessly stolen from

Tobias Nipkow, Larry Paulson, Markus Wenzel

David Basin, Burkhardt Wolff

Don't blame them, errors are mine

Slide 7

What is a proof?

to prove (Marriam-Webster)

- ➜ from Latin probare (test, approve, prove)
- ➜ to learn or find out by experience (archaic)
- ➜ to establish the existence, truth, or validity of
  (by evidence or logic)
  prove a theorem, the charges were never proved in court

pops up everywhere

- ➜ politics (weapons of mass destruction)
- ➜ courts (beyond reasonable doubt)
- ➜ religion (god exists)
- ➜ science (cold fusion works)

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What is a mathematical proof?

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In mathematics, a proof is a demonstration that, given certain axioms, some statement of interest is necessarily true. (Wikipedia)

Example: $\sqrt{2}$ is not rational.

Proof: assume there is $r \in \mathbb{Q}$ such that $r^2 = 2$.

Hence there are mutually prime p and q with $r = \frac{p}{q}$.

Thus $2q^2 = p^2$, i.e. p^2 is divisible by 2.

2 is prime, hence it also divides p, i.e. $p = 2s$.

Substituting this into $2q^2 = p^2$ and dividing by 2 gives $q^2 = 2s^2$. Hence, q is also divisible by 2. Contradiction. Qed.

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Nice, but..

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- ➜ still not rigorous enough for some
  - what are the rules?
  - what are the axioms?
  - how big can the steps be?
  - what is obvious or trivial?
- ➜ informal language, easy to get wrong
- ➜ easy to miss something, easy to cheat

Theorem. A cat has nine tails.
Proof. No cat has eight tails. Since one cat has one more tail than no cat, it must have nine tails.

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What is a formal proof?

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A derivation in a formal calculus

Example: $A \wedge B \longrightarrow B \wedge A$ derivable in the following system

Rules: $\frac{X \in S}{S \vdash X}$ (assumption) $\quad \frac{S \cup \{X\} \vdash Y}{S \vdash X \longrightarrow Y}$ (impl)

$\frac{S \vdash X \quad S \vdash Y}{S \vdash X \wedge Y}$ (conjI) $\quad \frac{S \cup \{X, Y\} \vdash Z}{S \cup \{X \wedge Y\} \vdash Z}$ (conjE)

Proof:

1.	$\{A, B\} \vdash B$	(by assumption)
2.	$\{A, B\} \vdash A$	(by assumption)
3.	$\{A, B\} \vdash B \wedge A$	(by conjI with 1 and 2)
4.	$\{A \wedge B\} \vdash B \wedge A$	(by conjE with 3)
5.	$\{\} \vdash A \wedge B \longrightarrow B \wedge A$	(by impl with 4)

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What is a theorem prover?

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Implementation of a formal logic on a computer.

- ➜ fully automated (propositional logic)
- ➜ automated, but not necessarily terminating (first order logic)
- ➜ with automation, but mainly interactive (higher order logic)

- ➜ based on rules and axioms
- ➜ can deliver proofs

There are other (algorithmic) verification tools:

- ➜ model checking, static analysis, ...
- ➜ usually do not deliver proofs

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Why theorem proving?

- ➜ Analysing systems/programs thoroughly
- ➜ Finding design and specification errors early
- ➜ High assurance (mathematical, machine checked proof)
- ➜ it's not always easy
- ➜ it's fun

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Main theorem proving system for this course

Isabelle

- ➜ used here for applications, learning how to prove

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What is Isabelle?

A generic interactive proof assistant

- ➜ **generic:**
  not specialised to one particular logic
  (two large developments: HOL and ZF, will mainly use HOL)
- ➜ **interactive:**
  more than just yes/no, you can interactively guide the system
- ➜ **proof assistant:**
  helps to explore, find, and maintain proofs

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Why Isabelle?

- ➜ free
- ➜ widely used systems
- ➜ active development
- ➜ high expressiveness and automation
- ➜ reasonably easy to use
- ➜ (and because we know it best ;-))

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If I prove it on the computer, it is correct, right?

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If I prove it on the computer, it is correct, right?

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No, because:

① hardware could be faulty
② operating system could be faulty
③ implementation runtime system could be faulty
④ compiler could be faulty
⑤ implementation could be faulty
⑥ logic could be inconsistent
⑦ theorem could mean something else

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If I prove it on the computer, it is correct, right?

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No, but:

probability for

➜ OS and H/W issues reduced by using different systems
➜ runtime/compiler bugs reduced by using different compilers
➜ faulty implementation reduced by right architecture
➜ inconsistent logic reduced by implementing and analysing it
➜ wrong theorem reduced by expressive/intuitive logics

No guarantees, but assurance immensely higher than manual proof

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If I prove it on the computer, it is correct, right?

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Soundness architectures

careful implementation	PVS
LCF approach, small proof kernel	HOL4 Isabelle
explicit proofs + proof checker	Coq Twelf Isabelle HOL4

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Meta Logic

Meta language:
The language used to talk about another language.

Examples:
English in a Spanish class, English in an English class

Meta logic:
The logic used to formalize another logic

Example:
Mathematics used to formalize derivations in formal logic

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Meta Logic – Example

Syntax:

Formulae: $F ::= V \quad | \quad F \longrightarrow F \quad | \quad F \wedge F \quad | \quad False$

$V ::= [A - Z]$

Derivable: $S \vdash X$ $\quad X$ a formula, S a set of formulae

logic / meta logic

$$\frac{X \in S}{S \vdash X} \qquad \frac{S \cup \{X\} \vdash Y}{S \vdash X \longrightarrow Y}$$

$$\frac{S \vdash X \quad S \vdash Y}{S \vdash X \wedge Y} \qquad \frac{S \cup \{X, Y\} \vdash Z}{S \cup \{X \wedge Y\} \vdash Z}$$

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Isabelle's Meta Logic

$$\bigwedge \qquad \Longrightarrow \qquad \lambda$$

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$\bigwedge$

Syntax: $\bigwedge x.\ F$ $\qquad$ (F another meta level formula)

in ASCII: `!!x. F`

- ➜ universal quantifier on the meta level
- ➜ used to denote parameters
- ➜ example and more later

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$\Longrightarrow$

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Syntax: $A \Longrightarrow B$ (A, B other meta level formulae)

in ASCII: `A ==> B`

Binds to the right:

$$A \Longrightarrow B \Longrightarrow C \quad = \quad A \Longrightarrow (B \Longrightarrow C)$$

Abbreviation:

$$[\![A; B]\!] \Longrightarrow C \quad = \quad A \Longrightarrow B \Longrightarrow C$$

- ➜ read: A and B implies C
- ➜ used to write down rules, theorems, and proof states

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Example: a theorem

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mathematics: if $x < 0$ and $y < 0$, then $x + y < 0$

formal logic: $\vdash\ x < 0 \land y < 0 \longrightarrow x + y < 0$

variation: $x < 0; y < 0 \ \vdash\ x + y < 0$

Isabelle: **lemma** "$x < 0 \land y < 0 \longrightarrow x + y < 0$"

variation: **lemma** "$[\![x < 0; y < 0]\!] \Longrightarrow x + y < 0$"

variation: **lemma**
assumes "$x < 0$" and "$y < 0$" shows "$x + y < 0$"

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Example: a rule

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logic: $\dfrac{X \quad Y}{X \land Y}$

variation: $\dfrac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y}$

Isabelle: $[\![X; Y]\!] \Longrightarrow X \land Y$

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Example: a rule with nested implication

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logic: $\dfrac{X \lor Y \quad \begin{array}{c} X \\ \vdots \\ Z \end{array} \quad \begin{array}{c} Y \\ \vdots \\ Z \end{array}}{Z}$

variation: $\dfrac{S \cup \{X\} \vdash Z \quad S \cup \{Y\} \vdash Z}{S \cup \{X \lor Y\} \vdash Z}$

Isabelle: $[\![X \lor Y; X \Longrightarrow Z; Y \Longrightarrow Z]\!] \Longrightarrow Z$

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λ

Syntax: $\lambda x.\ F$ (F another meta level formula)

in ASCII: `%x. F`

➜ lambda abstraction
➜ used for functions in object logics
➜ used to encode bound variables in object logics
➜ more about this in the next lecture

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ENOUGH THEORY!
GETTING STARTED WITH ISABELLE

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