

Advanced Algorithms COMP4121 2020

Practice Problems 1

Here are a few problems of the kind you will be asked to solve on the final exam.

1. Consider the deterministic Linear Time Algorithm for Order Statistic.
 - (a) Assume we split the numbers in groups of seven elements. Derive the asymptotic run time of the algorithm, following closely what we did in case when we split the input numbers into groups of five.

Hint: You have to figure out what the recursion should look like and for what K you can derive $T(n) < K \cdot C \cdot n$. In case of splitting into groups of five $K = 11$ worked. If you split into groups of seven you will need a different K . Try setting K a variable and see what you get if you substitute this into the right side of the recurrence; it should be easy to figure out what K works.

- (b) Assume we split the numbers in groups of three elements. Explain why the proof breaks down.

Hint: Show that no K works.

2. Write explicitly the pseudo-code of the Quick Sort algorithm which runs in time $n \log n$ even if all elements in the array are equal.

Hint: make sure elements equal to the pivot get into both sides of the partition evenly, using either randomisation or keeping track where the previous element of the pivot went.

3. Modify the SkipList data structure so that finding the k^{th} smallest element can also be done in expected time $O(\log n)$.

4. Consider Karger's MinCut algorithm. How many repetitions of the 4-CONTRACT(G) would you need to make to guarantee that MinCut is found with a probability of $1 - \frac{1}{n^2}$ where n is the number of vertices of G . How does the asymptotic run time of the whole algorithm change?
5. The present day "publish or perish" madness in academia involves counting number of papers researchers have published, as well as the number of citations their papers got.
 - (a) One might argue that not all citations are equally valuable: a citation in a paper that is itself often cited is more valuable than a citation in a paper that no one cites. Design a PageRank style algorithm which would rank papers according to their "importance", and then use such an algorithm to rank researchers by their "importance".
 - (b) Assume now that you do not have information on the citations in each published paper, but instead you have for every researcher a list of other researchers who have cited him and how many times they cited him. Design again a PageRank style algorithm which would rank researchers by their importance.
6. (This is a somewhat tricky one!) Prove that if in an irreducible Markov Chain one state is aperiodic, then all states must be aperiodic. Thus, equivalently, assume you have a directed strongly connected graph G which has one vertex v which satisfies that there is NO number K such that the length of every loop containing v is divisible by K . Show that in this case all vertices have the same property.

Hint: Let u be an aperiodic vertex and v an arbitrary vertex. Since the graph is strongly connected, there is a path p_1 from v to u and a path p_2 from u back to v . Show that for every integer K you can find a path containing v whose length is not divisible by K . You might want to make several trips starting at v , traversing only p_1 and then p_2 back to v and just one trip traversing p_1 then going through the loop containing u whose length is not divisible by K and then traversing path p_2 back to v .