COMP3411/9414/9814: Artificial Intelligence

Week 6: Perceptrons

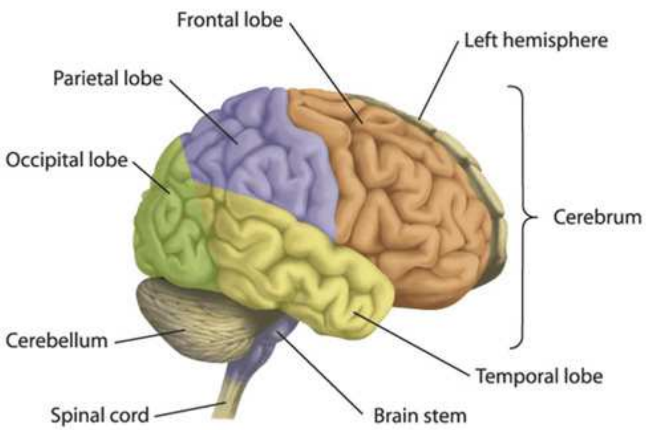
Russell & Norvig: 18.6, 18.7

Outline

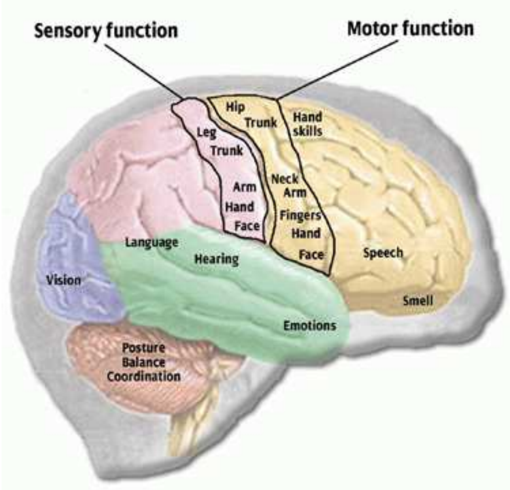
- Neurons – Biological and Artificial
- Perceptron Learning
- Linear Separability
- Multi-Layer Networks

Sub-Symbolic Processing

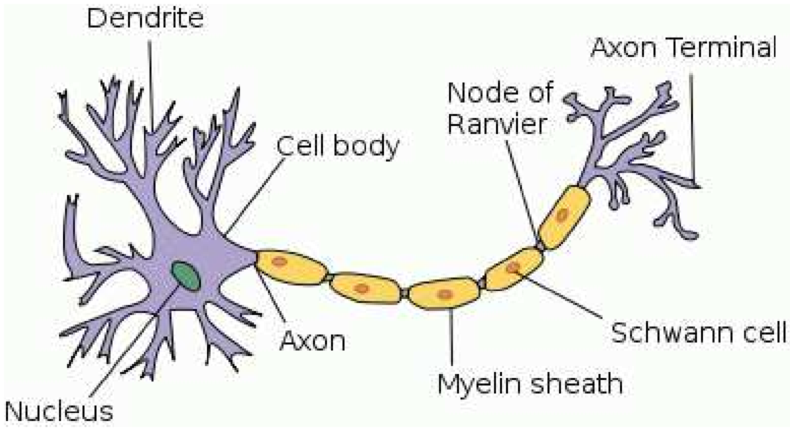
Brain Regions



Brain Functions



Structure of a Typical Neuron



Biological Neurons

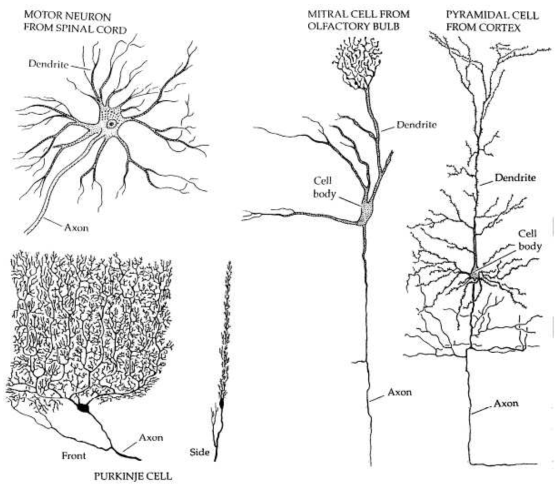
The brain is made up of neurons (nerve cells) which have

- a cell body (soma)
- dendrites (inputs)
- an axon (outputs)
- synapses (connections between cells)

Synapses can be exitatory or inhibitory and may change over time.

When the inputs reach some threshhold an action potential (electrical pulse) is sent along the axon to the outputs.

Variety of Neuron Types



The Big Picture

- human brain has 100 billion neurons with an average of 10,000 synapses each
- latency is about 3-6 milliseconds
- therefore, at most a few hundred "steps" in any mental computation, but massively parallel

Artificial Neural Networks

(Artificial) Neural Networks are made up of nodes which have

- inputs edges, each with some weight
- outputs edges (with weights)
- an activation level (a function of the inputs)

Weights can be positive or negative and may change over time (learning).

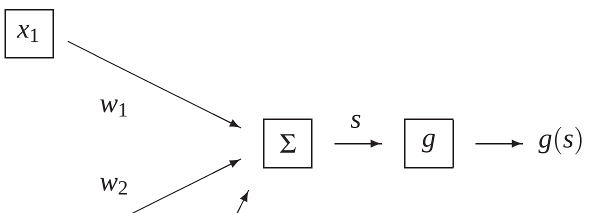
The input function is the weighted sum of the activation levels of inputs.

The activation level is a non-linear transfer function g of this input:

$$\text{activation}_i = g(s_i) = g(\sum_j w_{ij} x_j)$$

Some nodes are inputs (sensing), some are outputs (action)

McCulloch & Pitts Model of a Single Neuron



w_0=-th

$\boxed{1}$ $s = w_1x_1 + w_2x_2 - \text{th}$
$\quad = w_1x_1 + w_2x_2 + w_0$

x_1, x_2 are inputs

w_1, w_2 are synaptic weights

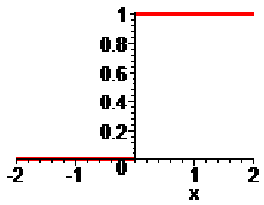
th is a threshold

w_0 is a **bias** weight

g is transfer function

Transfer function

Originally, a (discontinuous) step function was used for the transfer function:



$$g(s) = \begin{cases} 1, & \text{if} \quad s \geq 0 \\ 0, & \text{if} \quad s < 0 \end{cases}$$

(Later, other transfer functions were introduced, which are continuous and smooth)

Linear Separability

Q: what kind of functions can a perceptron compute?

A: linearly separable functions

Examples include:

AND	$w_1 = w_2 = \ \ 1.0,$	$w_0 = -1.5$
OR	$w_1 = w_2 = \ \ 1.0,$	$w_0 = -0.5$
NOR	$w_1 = w_2 = -1.0,$	$w_0 = \ \ 0.5$

Q: How can we train it to learn a new function?

Perceptron Learning Rule

Adjust the weights as each input is presented.

recall: $s = w_1x_1 + w_2x_2 + w_0$

if $g(s) = 0$ but should be 1,

$$\begin{aligned} w_k &\leftarrow w_k + \eta\, x_k \\ w_0 &\leftarrow w_0 + \eta \\ \text{so} \quad s &\leftarrow s + \eta\,(1 + \sum_k x_k^2) \end{aligned}$$

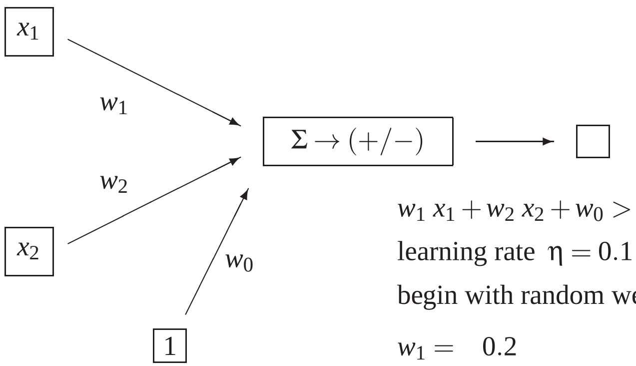
if $g(s) = 1$ but should be 0,

$$\begin{aligned} w_k &\leftarrow w_k - \eta\, x_k \\ w_0 &\leftarrow w_0 - \eta \\ \text{so} \quad s &\leftarrow s - \eta\,(1 + \sum_k x_k^2) \end{aligned}$$

otherwise, weights are unchanged. ($\eta > 0$ is called the **learning rate**)

Theorem: This will eventually learn to classify the data correctly, as long as they are **linearly separable**.

Perceptron Learning Example



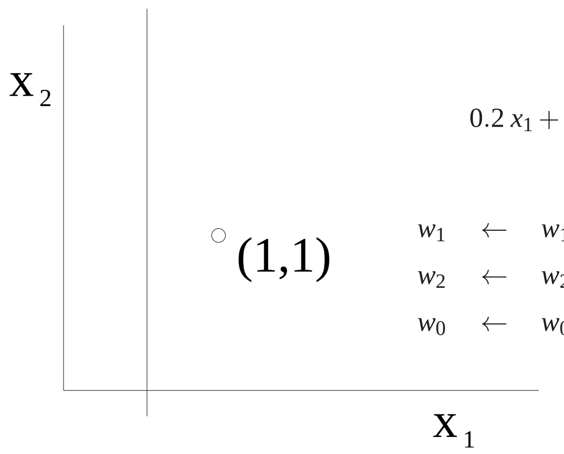
$w_1\, x_1 + w_2\, x_2 + w_0 > 0$

learning rate $\eta = 0.1$

begin with random weights

$$\begin{aligned} w_1 &= \ \ 0.2 \\ w_2 &= \ \ 0.0 \\ w_0 &= -0.1 \end{aligned}$$

Training Step 1



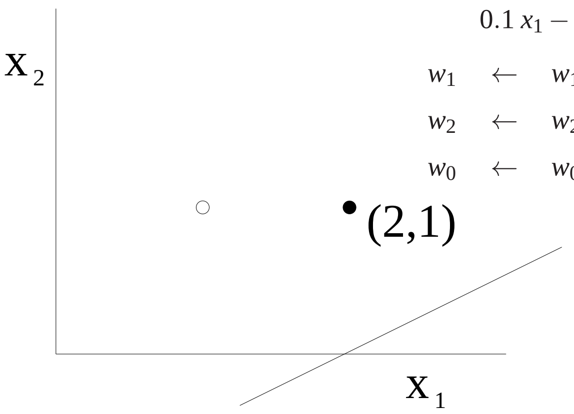
$0.2\, x_1 + 0.0\, x_2 - 0.1 > 0$

$$\begin{aligned} w_1 &\leftarrow w_1 - \eta\, x_1 &= \ \ 0.1 \\ w_2 &\leftarrow w_2 - \eta\, x_2 &= -0.1 \\ w_0 &\leftarrow w_0 - \eta &= -0.2 \end{aligned}$$

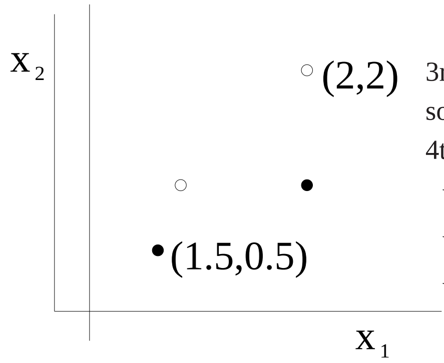
Training Step 2

$0.1\,x_1 - 0.1\,x_2 - 0.2 > 0$

$$\begin{aligned} w_1 &\leftarrow w_1 + \eta\, x_1 &= 0.3 \\ w_2 &\leftarrow w_2 + \eta\, x_2 &= 0.0 \\ w_0 &\leftarrow w_0 + \eta &= -0.1 \end{aligned}$$



Training Step 3



$0.3\,x_1 + 0.0\,x_2 - 0.1 > 0$

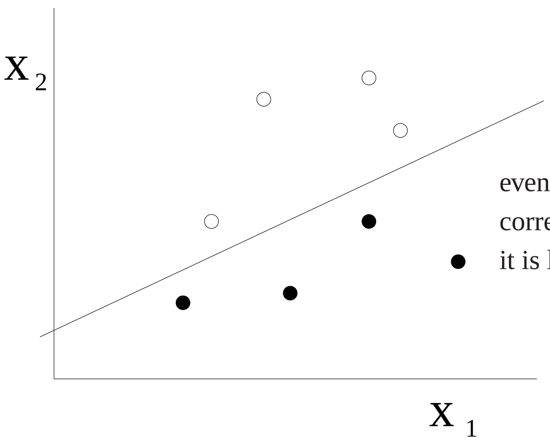
3rd point correctly classified, so no change

4th point:

$$\begin{aligned} w_1 &\leftarrow w_1 - \eta\, x_1 &= 0.1 \\ w_2 &\leftarrow w_2 - \eta\, x_2 &= -0.2 \\ w_0 &\leftarrow w_0 - \eta &= -0.2 \end{aligned}$$

$0.1\,x_1 - 0.2\,x_2 - 0.2 > 0$

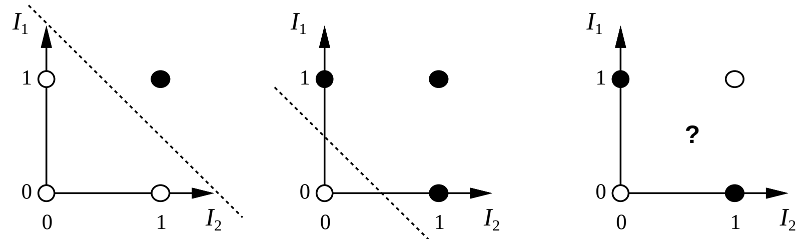
Final Outcome



eventually, all the data will be correctly classified (provided it is linearly separable)

Limitations of Perceptrons

Problem: many useful functions are not linearly separable (e.g. XOR)



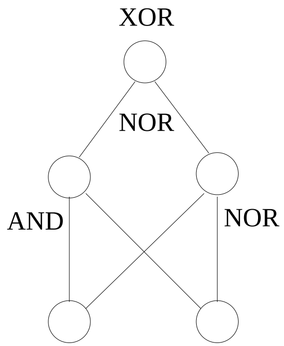
(a) I_1 **and** I_2 **(b)** I_1 **or** I_2 **(c)** I_1 **xor** I_2

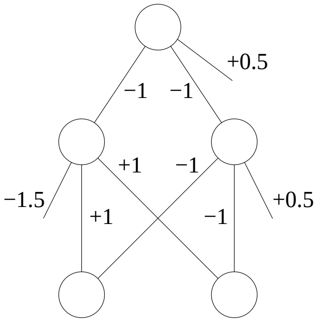
Possible solution:

x_1 XOR x_2 can be written as: (x_1 AND x_2) NOR (x_1 NOR x_2)

Recall that AND, OR and NOR can be implemented by perceptrons.

Multi-Layer Neural Networks





Problem: How can we train it to learn a new function? (credit assignment)

