

# COMP3411/9814: Artificial Intelligence

## Extension 3. Motion Planning

# Motion Planning Approaches

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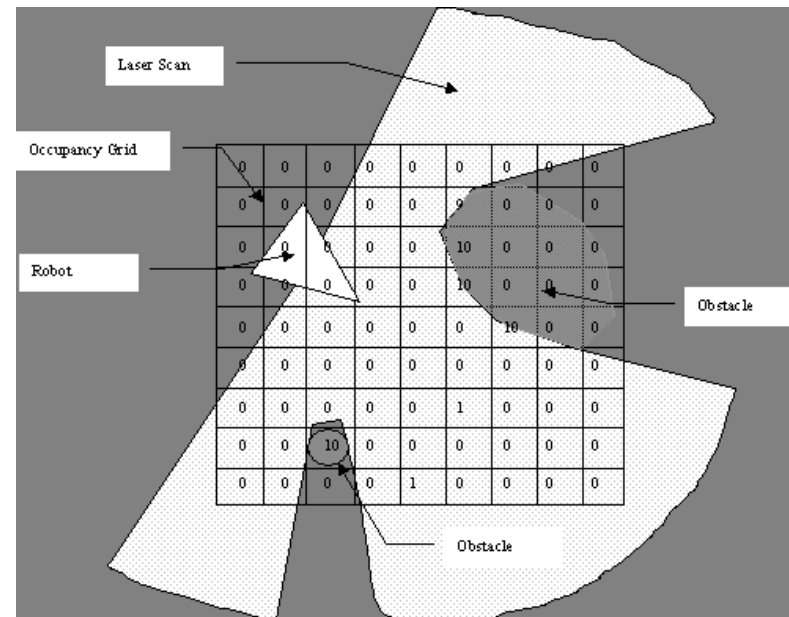
- Partially Observable Environments (on-board sensors only)
  - ▶ Occupancy Grid
  - ▶ Potential Field
  - ▶ Vector Field Histogram
  
- Fully Observable Environments (overhead cameras)
  - ▶ Delaunay Triangulation
  - ▶ Parameterized Cubic Splines

# Robots

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# Occupancy Grid



- divide environment into a Cartesian grid
- for each square in the grid, maintain an estimate of the probability of an obstacle in that square

# Potential Field

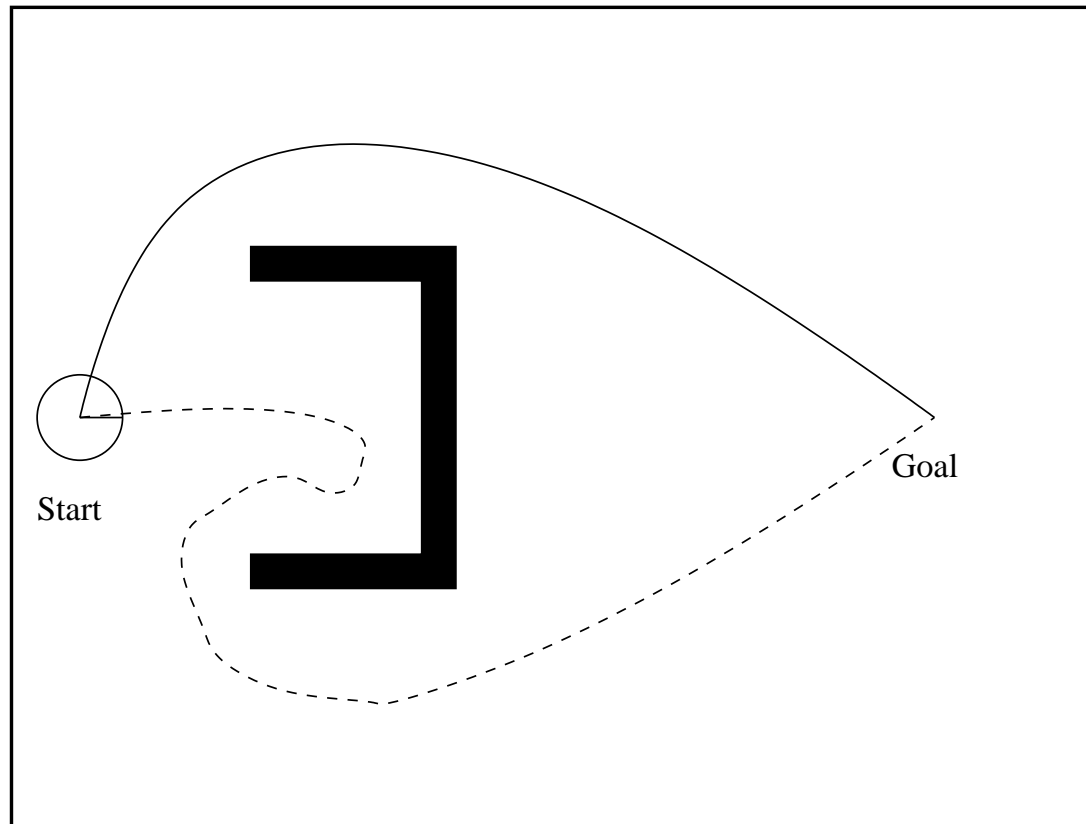
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Using an occupancy grid for its map, an agent can plan a path using a Potential Field.

- treat robot's configuration as a point in a potential field that combines attraction to the goal, and repulsion from obstacles
- very rapid computation, but can get stuck in local optima, thus failing to find a path

# Problem - Local Optima

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# Vector Field Histogram

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A more sophisticated approach - less likely to get stuck in local optima - is a Vector Field Histogram

- Cartesian histogram grid, continuously updated
- Polar histogram, based on current position/orientation of robot
- candidate valleys (contiguous sectors with low obstacle density)
- select candidate valleys, based on proximity to target direction

# Delaunay Triangulation

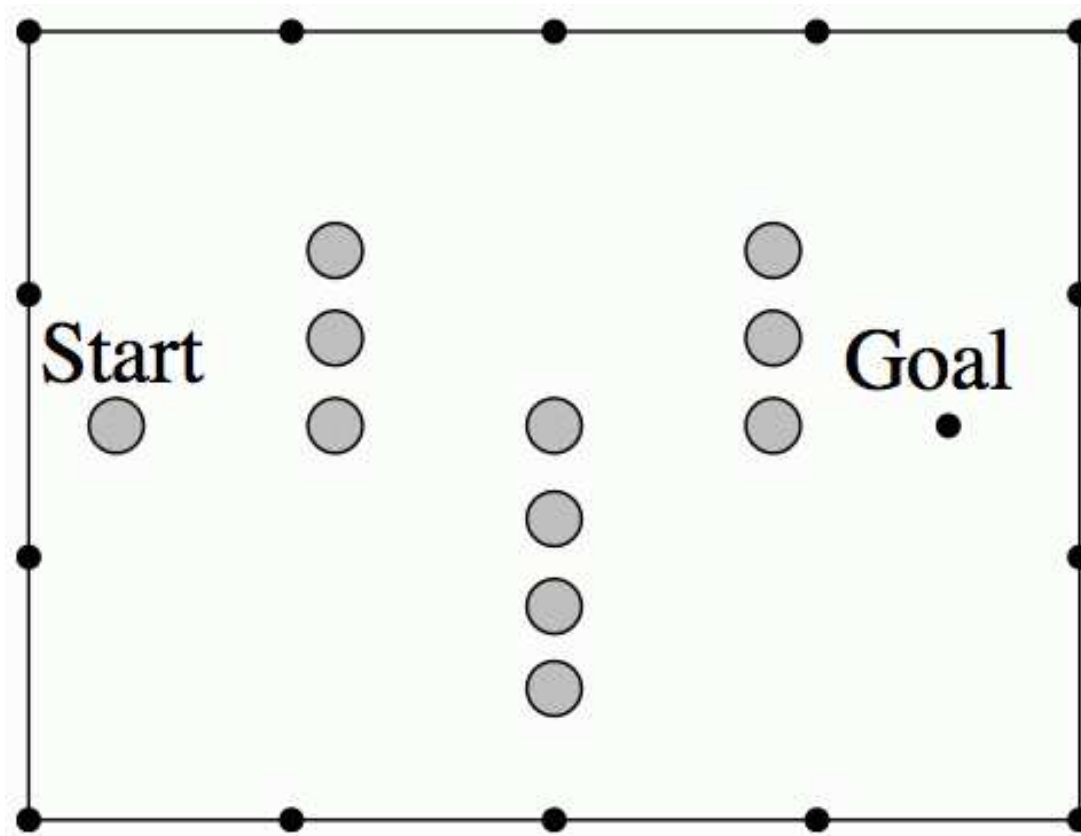
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- applicable in situations where the environment is well mapped by overhead cameras (museums, shopping centres, robocup small league)
- add line segments between (closest points of) obstacles, sorted according to length (shortest segments first)
- do not add any segment that crosses an existing segment



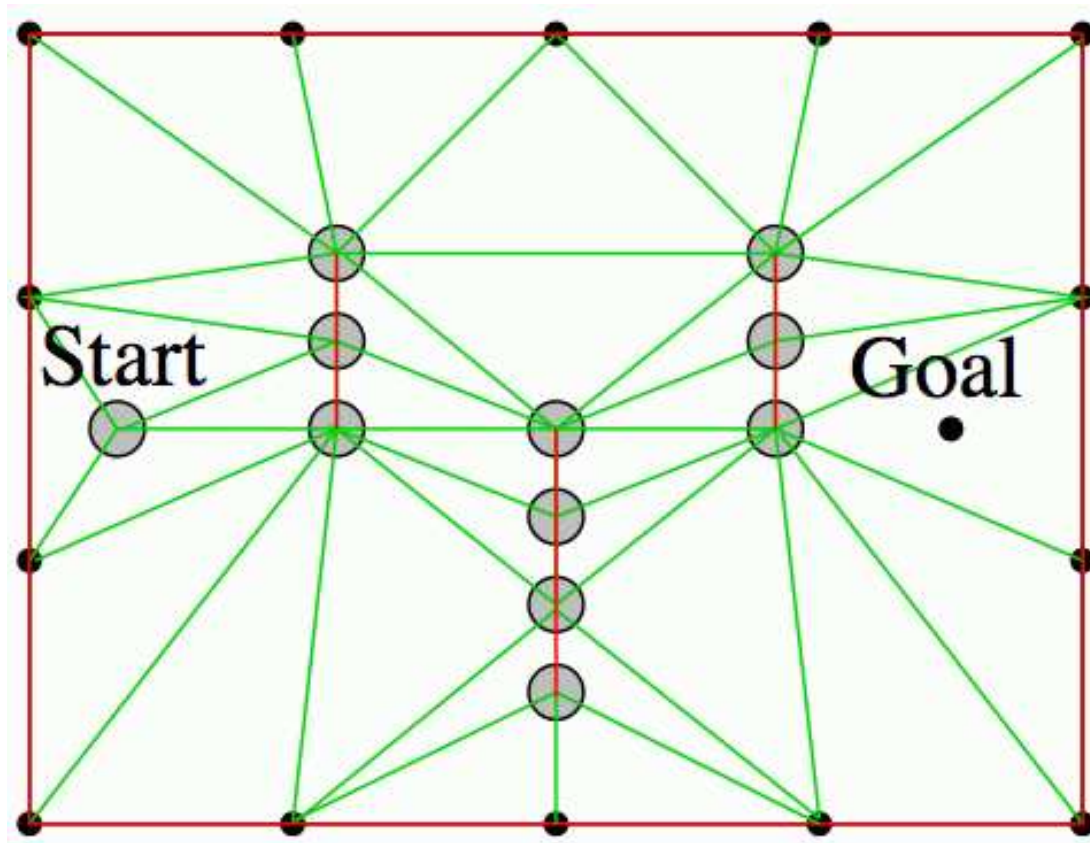
# Delaunay Triangulation

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# Delaunay Triangulation

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# Voronoi Diagram

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- dual of the Delaunay triangulation is called a Voronoi diagram
- prune arcs that are too small for the robot to traverse
- A\* Search can then be applied on the resulting graph
- path can be converted to a trajectory for the robot

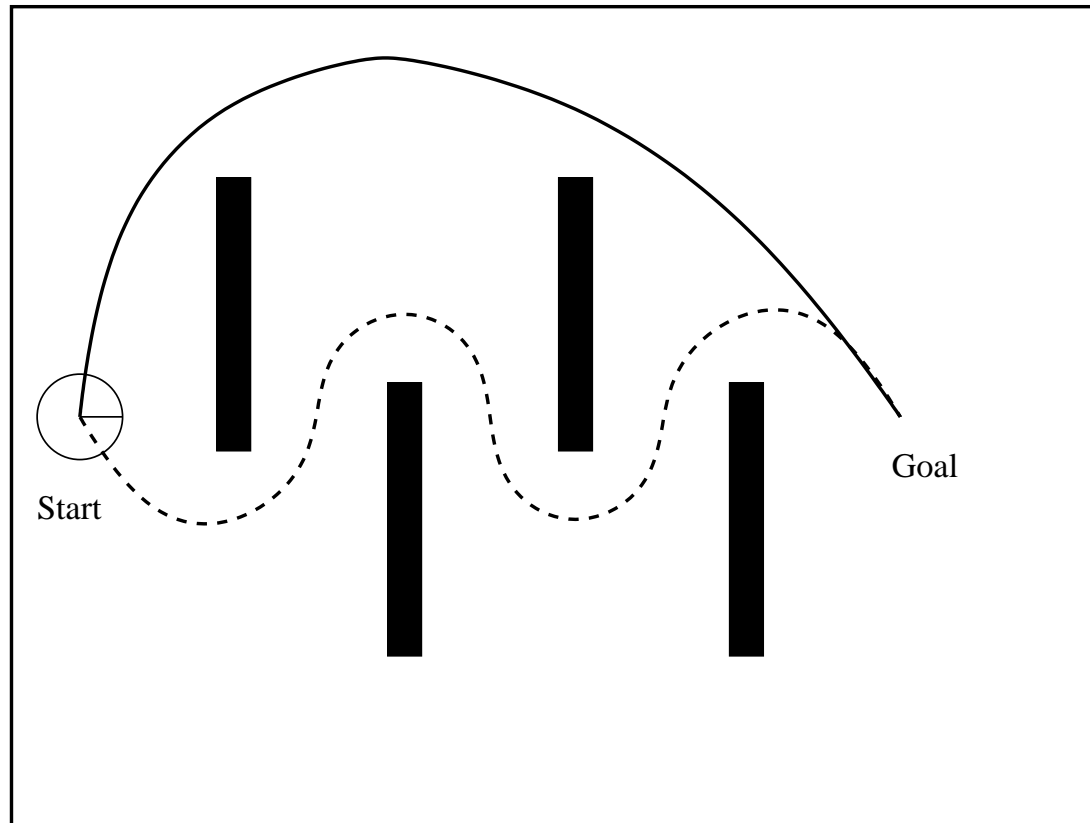
# Minimizing Time instead of Distance

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- For the problem of a soccer robot getting to the ball, or a wheeled robot navigating a maze, the “shortest distance” path might not be the same as the “shortest time” path.
- By speeding up and then slowing down, the robot could traverse a path with long straight stretches faster than a shorter path with lots of twists and turns.

# Minimizing Time instead of Distance

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How can we approach the problem of finding the “shortest time” path?

# Optimal Trajectory Planning

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Steps in the Algorithm:

- Delaunay Triangulation
- A\* Search, using paths composed of Parametric Cubic Splines
- Smooth entire curve with Waypoint Tuning by Gradient Descent

# Parameterized Cubic Splines

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Assume each path segment is of the form

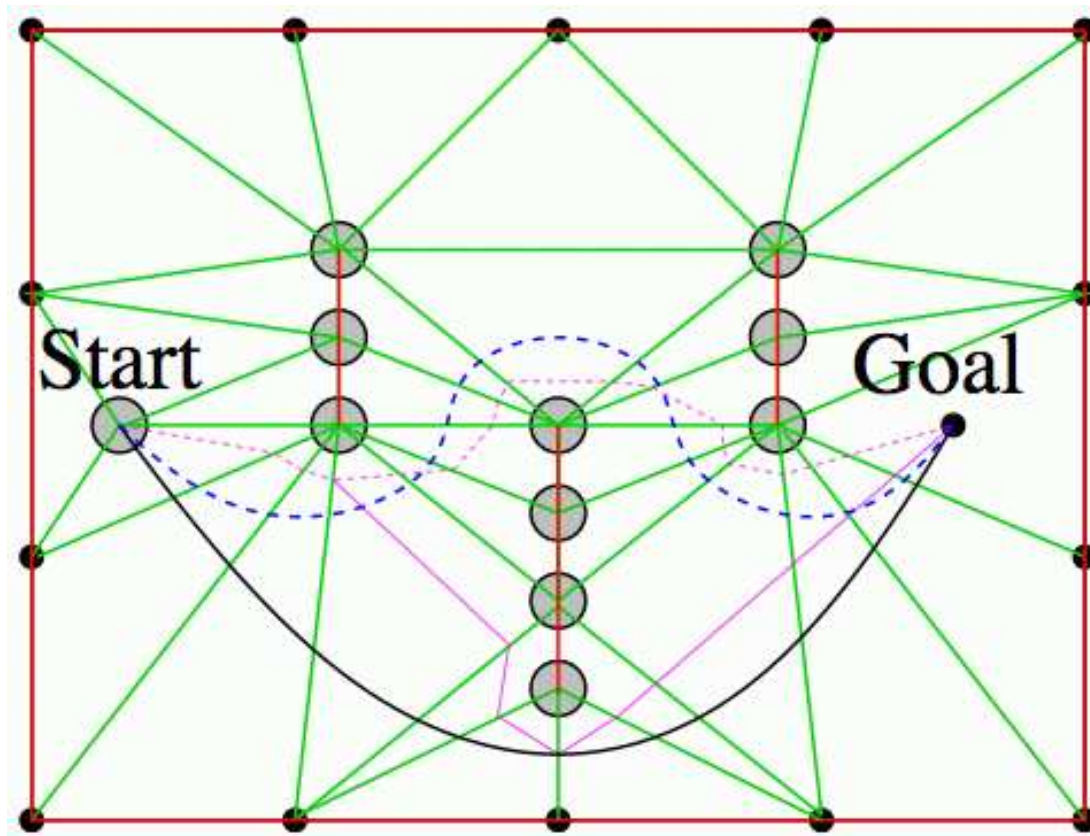
$$P(t) = \begin{pmatrix} P_x(t) \\ P_y(t) \end{pmatrix} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} t^3 + \begin{pmatrix} b_x \\ b_y \end{pmatrix} t^2 + \begin{pmatrix} c_x \\ c_y \end{pmatrix} t + \begin{pmatrix} d_x \\ d_y \end{pmatrix}$$

Solve these equations for a specified position and velocity at the beginning ( $t = 0$ ) and the end ( $t = s$ ) of the segment. Traditional method was to set  $s = 1$ . We instead try to minimize  $s$  (total time for segment) while satisfying the kinematic constraints:

$$\left| \frac{P''(t)}{A} + \frac{P'(t)}{V} \right|^2 \leq 1, \quad \text{for } 0 \leq t \leq s,$$

where  $A$  and  $V$  are the maximal acceleration and velocity of the robot.

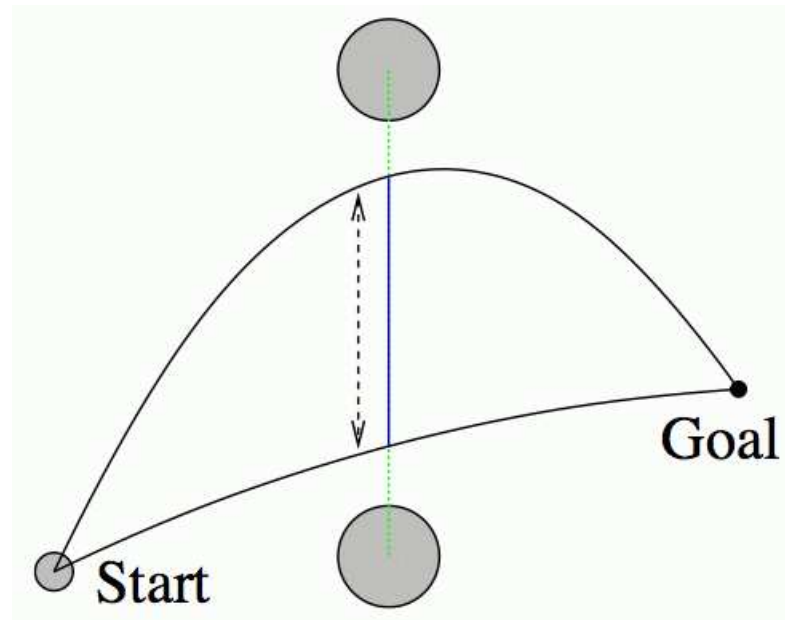
# A\* Search With and Without Smoothing





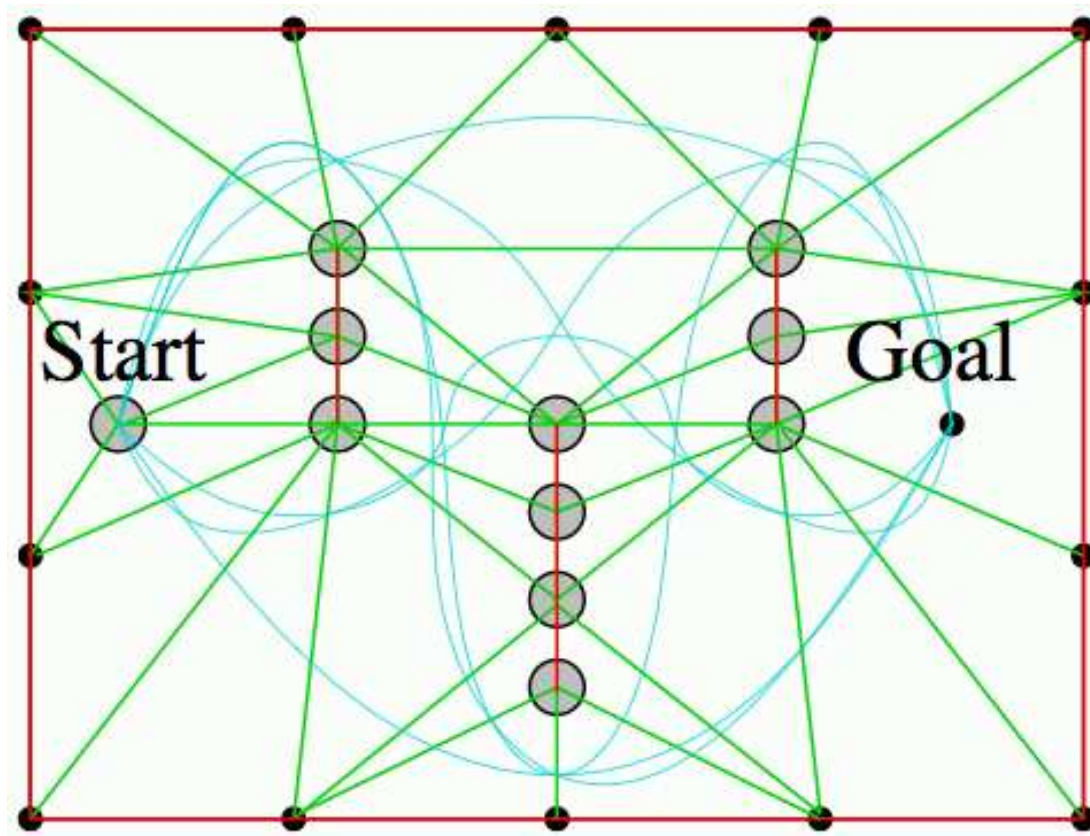
# Waypoint Tuning by Gradient Descent

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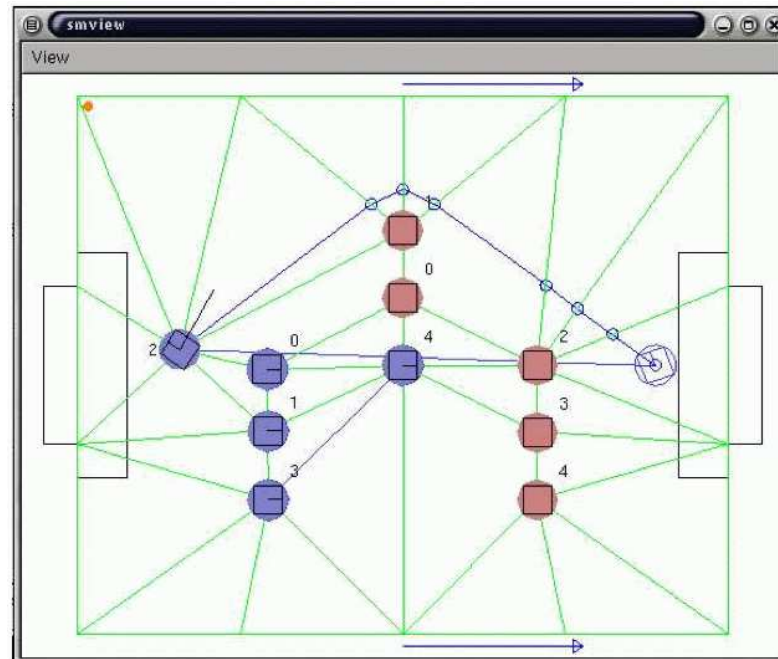
# Various Paths Explored

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# Robocup

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This system has been successfully deployed in the Robocup F180 League.

# References

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- S. Thrun, W. Burgard & D. Fox, [Probabilistic Robotics](#), MIT Press, 2005.
- J. Thomas, A. Blair & N. Barnes, “Towards an Efficient Optimal Trajectory Planner for Multiple Mobile Robots”, [2003 International Conference on Robotics and Systems \(IROS'03\)](#), 2291–2296.