

**Safety and Liveness;  
Progress and Preservation;  
Errors and Exceptions**

Thomas Sewell & Rob Sison  
UNSW  
Term 3 2025

# Trace Properties

Given a small step semantics  $\mapsto$ , a **trace** is sequence of states:

$$\sigma_1 \mapsto \sigma_2 \mapsto \sigma_3 \mapsto \cdots \mapsto \sigma_n$$

representing the evaluation of a program.

Some traces are finite, others infinite. To simplify things, we'll make all traces infinite by repeating the final state of any finite trace.

A trace property of a program is a **set of traces**.

## Safety vs Liveness

- ① A safety property: something **bad** ~~never~~ **never** happens, e.g.

*I will never run out of money. I will never have over \$1000.*

These are properties that may be violated by a **finite prefix** of a trace.

- ② A liveness property: something **good** ~~eventually~~ **eventually** happens, e.g.

*I will eventually have over \$1000. I will eventually run out of money.*

These are properties that cannot be violated by any **finite prefix** of a trace.

# Combining Properties

## Safety properties we've seen before

Static semantics properties      Partial correctness (Hoare Logic)

## Liveness properties we've seen before

Termination

### Theorem (Alpern & Schneider, 1985)

Every **trace** property is the **intersection** of a **safety** property and a **liveness** property.

Note: Not everything of interest is a (single-)trace property.  
(e.g. Confluence and confidentiality compare *multiple* traces.)

## Types

What sort of properties do **types** give us?

Adding types to  **$\lambda$ -calculus** eliminates terms with no normal forms.

$$\frac{(x : \tau) \in \Gamma}{\Gamma \vdash x : \tau} \quad \frac{x : \tau_1, \Gamma \vdash e : \tau_2}{\lambda x. e : \tau_1 \rightarrow \tau_2} \quad \frac{\Gamma \vdash e_1 : \tau_1 \rightarrow \tau_2 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1 e_2 : \tau_2}$$

Remember  $(\lambda x. x x)$   $(\lambda x. x x)$ ? Trying to type this requires an **infinite** type  $\tau_1 = \tau_1 \rightarrow \tau_2$ .

### Theorems

Every well-typed  $\lambda$ -term has a normal form (normalisation).

Furthermore, every reduction sequence for a well-typed  $\lambda$ -term has a normal form (strong normalisation).

This means that all typed  $\lambda$ -terms terminate!

## With Recursion

MinHS, unlike lambda calculus, has **built in recursion**. We can define terms like:

$$(\mathbf{recfun} \ f :: (\text{Int} \rightarrow \text{Int}) \ x = f \ x) \ 3$$

Which has no normal form or final state, despite being typed.

**What now?**

The **liveness** parts of the typing theorems can't be salvaged, but the **safety** parts can...

## Type Safety

Type safety is the property that states:

*Well-typed programs do not go wrong.*

By “go wrong”, we mean reaching a stuck state—a non-final state with no outgoing transitions.

What are some examples of stuck states?

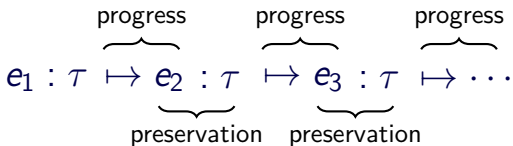
There are many other definitions of things called “type safety” on the internet. For our purposes, ignore them.

# Progress and Preservation

We want to prove that a well-typed program either goes on forever or reaches a final state. We prove this with two **lemmas**.

## How to prove type safety

- 1 **Progress**, which states that well-typed states are not stuck states. That is, if an expression  $e : \tau$  then either  $e$  is a final state or there exists a state  $e'$  such that  $e \mapsto e'$ .
- 2 **Preservation**, which states that evaluating one step preserves types. That is, if an expression  $e : \tau$  and  $e \mapsto e'$ , then  $e' : \tau$ .



## In the real world

Which of the following languages are type safe?

- C
- C++
- Haskell
- Java
- Python
- Rust
- MinHS

Why is MinHS **not type safe**?

## Division by Zero

We can assign a type to a division by zero:

$$\frac{\frac{}{(\text{Num } 3) : \text{Int}} \quad \frac{}{(\text{Num } 0) : \text{Int}}}{(\text{Quot } (\text{Num } 3) (\text{Num } 0)) : \text{Int}}$$

But there is no outgoing transition from this state (nor is it final)!

⇒ We have violated **progress**.

We have two options:

- 1 **Change the static semantics** to exclude division by zero.  
This reduces to the **halting problem**, so we would be forced to overapproximate.
- 2 **Change the dynamic semantics** so that the above state has an outgoing transition.

## Our Cop-Out

Add a new state, `Error`, that is the successor state for any partial function:

$$\frac{}{(\text{Quot } v \text{ (Num 0)}) \mapsto_M \text{Error}}$$

Any state containing `Error` evaluates to `Error`:

$$\frac{}{(\text{Plus } e \text{ Error}) \mapsto_M \text{Error}} \quad \frac{}{(\text{Plus Error } e) \mapsto_M \text{Error}}$$

$$\frac{}{(\text{If Error } t \text{ } e) \mapsto_M \text{Error}}$$

(and so on – this is much easier in the **C machine!**)

## Type Safety for Error

We've satisfied **progress** by making a successor state for partial functions, but how should we satisfy **preservation**?

---

**Error** :  $\tau$

That's right, we give **Error** any type.

## Dynamic Types

Some languages (e.g. Python, JavaScript) are called dynamically typed. We call these untyped, as they achieve type safety with a trivial type system containing only one type, here written  $\star$ :

$$\frac{}{\Gamma \vdash e : \star}$$

### $\star$ vs. Dynamic Types

Things these languages call “types”, e.g. `int` or `dict` in Python, are part of a value’s runtime representation. They aren’t represented in the language’s static semantics.

They achieve type safety by defining execution for every syntactically valid expression, **even** those that are not well typed.

# Exceptions

Error may satisfy type safety, but it's not satisfying as a programming language feature. When an error occurs, we may want a way to recover. We will add more fine grained error control – exceptions – to MinHS.

## Example (Exceptions)

try/catch/throw in Java, setjmp/longjmp in C,  
try/except/raise in Python.

## Exceptions Syntax

	Raising an Exception	Handling an Exception
Concrete	<b>raise</b> $e$	<b>try</b> $e_1$ <b>handle</b> $x \Rightarrow e_2$
Abstract	(Raise $e$ )	(Try $e_1$ ( $x$ . $e_2$ ))

## Informal Semantics

### Example

```
try
  if  $y \leq 0$  then
    raise DivisorError
  else
    (x/y)
handle err  $\Rightarrow -1$ 
```

For an expression (**try**  $e_1$  **handle**  $x \Rightarrow e_2$ ) we

- 1 Evaluate  $e_1$
- 2 If **raise**  $v$  is encountered while evaluating  $e_1$ , we bind  $v$  to  $x$  and evaluate  $e_2$ .

Note that it is possible for **try** expressions to be **nested**.

- The inner-most **handle** will catch exceptions.
- Handlers may **re-raise** exceptions.

## Static Semantics

The type given to **exception values** is usually some specific blessed type  $\tau_E$  that is specifically intended for that purpose. For example, the `Throwable` type in Java. In dynamically typed languages, the type is just the same as everything else (i.e.  $\star$ ).

### Typing Rules

$$\frac{\Gamma \vdash e : \tau_E}{\Gamma \vdash (\text{Raise } e) : \tau}$$

$$\frac{\Gamma \vdash e_1 : \tau \quad x : \tau_E, \Gamma \vdash e_2 : \tau}{\Gamma \vdash (\text{Try } e_1(x. e_2)) : \tau}$$

## Dynamic Semantics

Easier to describe using the C Machine. We introduce a new type of state,  $s \Leftarrow v$ , that means an exception value  $v$  has been raised. The exception is bubbled up the stack  $s$  until a handler is found.

### Evaluating a Try Expression

$$s \succ (\text{Try } e_1 (x. e_2)) \mapsto_C (\text{Try } \square (x. e_2)) \triangleright s \succ e_1$$

### Returning from a Try without raising

$$(\text{Try } \square (x. e_2)) \triangleright s \Leftarrow v \mapsto_C s \Leftarrow v$$

### Evaluating a Raise expression

$$s \succ (\text{Raise } e) \mapsto_C (\text{Raise } \square) \triangleright s \succ e$$

### Raising an exception

$$(\text{Raise } \square) \triangleright s \Leftarrow v \mapsto_C s \Leftarrow v$$

### Catching an exception

$$(\text{Try } \square (x. e_2)) \triangleright s \Leftarrow v \mapsto_C s \succ e_2[x := v]$$

### Propagating an exception

$$f \triangleright s \Leftarrow v \mapsto_C s \Leftarrow v$$

## Efficiency Problems

The approach described above is highly **inefficient**. Throwing an exception takes linear time with respect to the depth of stack frames!

Only the most simplistic implementations work this way. A more efficient approach is to **keep a separate stack of handler frames**.

### Handler frames

A handler frame contains:

- 1 A copy of the control stack above the Try expression.
- 2 The exception handler that is given in the Try expression.

We write a handler frame that contains a control stack  $s$  and a handler  $(x. e_2)$  as  $(\text{Handle } s (x. e_2))$ .

## Efficient Exceptions

Evaluating a Try now pushes the handler onto the handler stack and a marker onto the control stack.

$$(h, s) \succ (\text{Try } e_1 (x. e_2)) \mapsto_C (\text{Handle } s (x. e_2) \triangleright h, (\text{Try } \square) \triangleright s) \succ e_1$$

Returning without raising in a Try block removes the handler again:

$$(\text{Handle } s (x. e_2) \triangleright h, (\text{Try } \square) \triangleright s) \prec v \mapsto_C (h, s) \prec v$$

Raising an exception now uses the handler stack to immediately jump to the handler:

$$(\text{Handle } s (x. e_2) \triangleright h, (\text{Raise } \square) \triangleright s') \prec v \mapsto_C (h, s) \succ e_2[x := v]$$

## Exceptions in Practice

Exceptions are useful, but they are a form of **non-local** control flow and should be used carefully.

In Haskell, exceptions tend to be avoided as they make a liar out of the type system:

$$\text{head} :: [a] \rightarrow a$$

In Java, **checked exceptions** allow the possibility of exceptions to be tracked in the type system.

### Monads

One of the most common uses of the Haskell **monad** construct is for a kind of error handling that is honest about what can happen in the types.

## Tail Recursion

Consider this drop function:

```
drop2 :: Int -> [a] -> [a]
drop2 n xs = if n == 0 then xs
             else case xs of
                   [] -> []
                   (y : ys) -> drop2 (n - 1) ys
```

How large a runtime stack do we need to compute the result of `drop 10000 [1 .. 100000]`?