Functional Programming Languages

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Functional Programming

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function maxOf(arr) {
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\begin{aligned}
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\mathit{maxOf} &= \mathit{foldr1}\ \mathit{max}
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What do they have in common?

Definitions

Unlike imperative languages, functional programming languages are not very crisply defined.

Attempt at a Definition

A *functional programming language* is a programming language derived from or inspired by the λ-calculus, or derived from or inspired by another functional programming language.

The result? If it has λ in it, you can call it functional.

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The result? If it has λ in it, you can call it functional.

In this course, we'll consider *purely functional* languages, which have a much better definition.

Why Study FP Languages?

Think of a major innovation in the area of programming languages.

Garbage Collection?

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Purely Functional Programming Languages

The term *purely functional* has a very crisp definition.

Definition

A programming language is *purely functional* if β-reduction (or evaluation in general) is actually a confluence.
In other words, functions have to be mathematical functions, and free of *side effects*.

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In other words, functions have to be mathematical functions, and free of *side effects*.

Consider what would happen if we allowed effects in a functional language:

$$\begin{aligned} &count = 0; \\ &f\ x\ = \{count := count + x; \textbf{return}\ count\}; \\ &m = (\lambda y.\ y + y)\ (f\ 3) \end{aligned}$$

If we evaluate $f\ 3$ first, we will get $m = 6$, but if we β-reduce m first, we will get $m = 9$. $\Rightarrow$ not confluent.

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Something not unlike this will appear in your Assignment 1.

Syntax

Integers	n	$::=$	$\cdots$
Identifiers	f, x	$::=$	$\cdots$
Literals	b	$::=$	$\texttt{True} \mid \texttt{False}$
Types	τ	$::=$	$\mathrm{Bool} \mid \mathrm{Int} \mid \tau_1 \rightarrow \tau_2$
Infix Operators	$\circledast$	$::=$	$\texttt{*} \mid \texttt{+} \mid \texttt{==} \mid \cdots$
Expressions	e	$::=$	$x \mid n \mid b \mid (e) \mid e_1 \circledast e_2$
		$\mid$	$\texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3$

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		$\mid$	$\texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3$
		$\mid$	$e_1\ e_2$
		$\mid$	$\texttt{recfun}\ f :: (\tau_1 \to \tau_2)\ x = e$
			$\uparrow$ Like λ, but with recursion.

As usual, this is ambiguous concrete syntax. But all the precedence and associativity rules apply as in Haskell. We assume a suitable parser.

Examples

Example (Stupid division by 5)

$$
\begin{array}{l}
\textbf{recfun } \mathit{divBy5} :: (\mathrm{Int} \to \mathrm{Int})\ x = \\
\quad \textbf{if } x < 5 \\
\qquad \textbf{then } 0 \\
\qquad \textbf{else } 1 + \mathit{divBy5}\ (x - 5)
\end{array}
$$

Example (Average Function)

$$
\begin{array}{l}
\textbf{recfun } \mathit{average} :: (\mathrm{Int} \to (\mathrm{Int} \to \mathrm{Int}))\ x = \\
\quad \textbf{recfun } \mathit{avX} :: (\mathrm{Int} \to \mathrm{Int})\ y = \\
\qquad (x + y)\ /\ 2
\end{array}
$$

As in Haskell, $(\mathit{average}\ 15\ 5) = ((\mathit{average}\ 15)\ 5)$.

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This language is so minimal, it doesn't even need **let** expressions.
How can we do without them?

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$$\textbf{let}\ x :: \tau_1 = e_1\ \textbf{in}\ e_2 :: \tau_2 \quad \equiv \quad (\textbf{recfun}\ f :: (\tau_1 \to \tau_2)\ x = e_2)\ e_1$$

Abstract Syntax

Moving to first order abstract syntax, we get:

Concrete Syntax	Abstract Syntax
n	(`Num` n)
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if c **then** t **else** e	(If c t e)
e_1 e_2	(Apply e_1 e_2)

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n	$(\texttt{Num}\ n)$
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if c **then** t **else** e	$(\texttt{If}\ c\ t\ e)$
$e_1\ e_2$	$(\texttt{Apply}\ e_1\ e_2)$
recfun $f :: (\tau_1 \rightarrow \tau_2)\ x = e$	$(\texttt{Recfun}\ \tau_1\ \tau_2\ f\ x\ e)$

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What changes when we move to higher order abstract syntax?

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What changes when we move to higher order abstract syntax?

1. `Var` terms go away – we use the meta-language's variables.
2. $(\texttt{Recfun}\ \tau_1\ \tau_2\ f\ x\ e)$ now uses meta-language abstraction: $(\texttt{Recfun}\ \tau_1\ \tau_2\ (f.\ x.\ e))$.

Working Statically with HOAS

To Code

We're going to write code for an AST and pretty-printer for MinHS with HOAS.
Seeing as this requires us to look under abstractions without evaluating the term, we have to extend the AST with special "tag" values.

Static Semantics

To check if a MinHS program is well-formed, we need to check:

1. **Scoping** – all variables used must be well defined
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Our judgement is an extension of the scoping rules to include types:

Under this context of assumptions

$$\Gamma \vdash e : \tau$$

The expression is assigned this type

The context Γ includes typing assumptions for the variables:

$$x : \mathrm{Int}, y : \mathrm{Int} \vdash (\texttt{Plus}\ x\ y) : \mathrm{Int}$$

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Let's implement a *type checker*.

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Structural Operational Semantics (Small-Step)

Initial states:

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Initial states: All well typed expressions.
Final states:

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Final states: $(\texttt{Num}\ n)$, $(\texttt{Lit}\ b)$,

Dynamic Semantics

Structural Operational Semantics (Small-Step)

Initial states: All well typed expressions.
Final states: (`Num` n), (`Lit` b), `Recfun` too!

Evaluation of built-in operations:

$$\frac{e_1 \mapsto e_1'}{(\texttt{Plus}\ e_1\ e_2) \mapsto (\texttt{Plus}\ e_1'\ e_2)}$$

(and so on as per arithmetic expressions)

Specifying If

$$\frac{e_1 \mapsto e_1'}{(\texttt{If}\ e_1\ e_2\ e_3) \mapsto (\texttt{If}\ e_1'\ e_2\ e_3)}$$

$$\frac{}{(\texttt{If}\ (\texttt{Lit True})\ e_2\ e_3) \mapsto e_2}$$

$$\frac{}{(\texttt{If}\ (\texttt{Lit False})\ e_2\ e_3) \mapsto e_3}$$

How about Functions?

Recall that `Recfun` is a final state – we don't need to evaluate it unless it's applied to an argument.

Evaluating function application requires us to:

1. Evaluate the left expression to get a `Recfun`;
2. evaluate the right expression to get an argument value; and
3. evaluate the function's body, after supplying substitutions for the abstracted variables.

$$\frac{e_1 \mapsto e_1'}{(\texttt{Apply}\ e_1\ e_2) \mapsto (\texttt{Apply}\ e_1'\ e_2)}$$

$$\frac{e_2 \mapsto e_2'}{(\texttt{Apply}\ (\texttt{Recfun} \ldots)\ e_2) \mapsto (\texttt{Apply}\ (\texttt{Recfun} \ldots)\ e_2')}$$

How about Functions?

Recall that $\texttt{Recfun}$ is a final state – we don't need to evaluate it unless it's applied to an argument.

Evaluating function application requires us to:

1. Evaluate the left expression to get a $\texttt{Recfun}$;
2. evaluate the right expression to get an argument value; and
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$$\frac{e_1 \mapsto e_1'}{(\texttt{Apply}\ e_1\ e_2) \mapsto (\texttt{Apply}\ e_1'\ e_2)}$$

$$\frac{e_2 \mapsto e_2'}{(\texttt{Apply}\ (\texttt{Recfun} \ldots)\ e_2) \mapsto (\texttt{Apply}\ (\texttt{Recfun} \ldots)\ e_2')}$$

$$\frac{v \in F}{(\texttt{Apply}\ (\texttt{Recfun}\ \tau_1\ \tau_2\ (f.x.\ e))\ v) \mapsto e[x := v, f := (\texttt{Recfun}\ \tau_1\ \tau_2\ (f.x.\ e))]}$$

MinHS To Come

We've now looked at MinHS (0.10: beta).

More MinHS content will come soon as part of Assignment 1 and Assignment 2.