COMP2521 25T3
Binary Search Trees

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trees
binary search trees
binary search tree operations

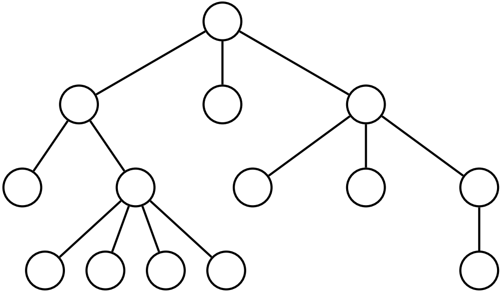
Trees

Trees

A tree is a hierarchical data structure
consisting of a set of connected nodes where:

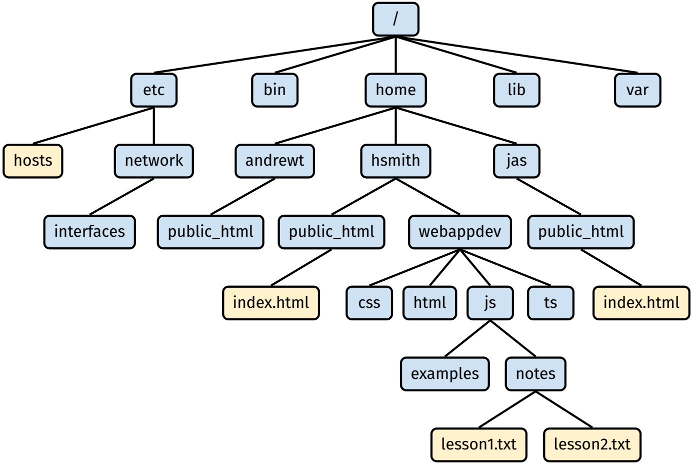
Each node may have multiple other nodes as children
(depending on the type of tree)

Each node is connected to one parent *except* the root node



Trees

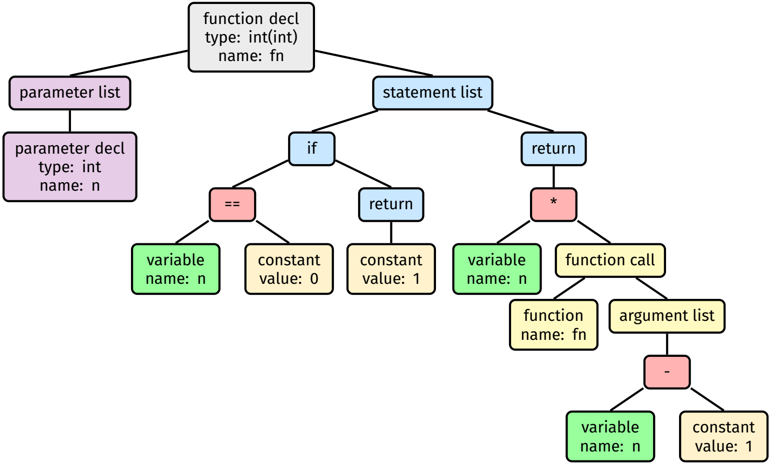
Example - File System Tree



Source: https://www.openbookproject.net/tutorials/getdown/unix/lesson2.html

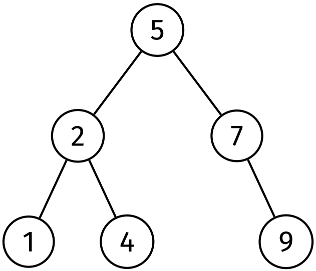
Trees

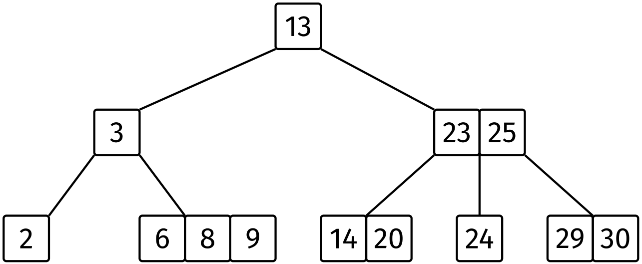
Example - Abstract Syntax Tree

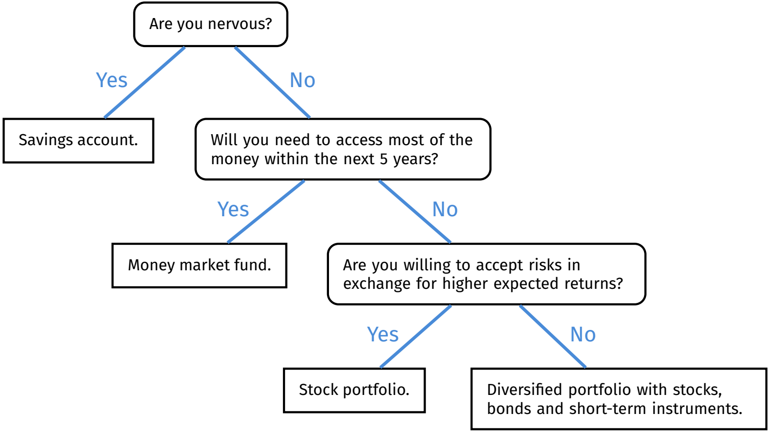


Trees

Example - Search Trees



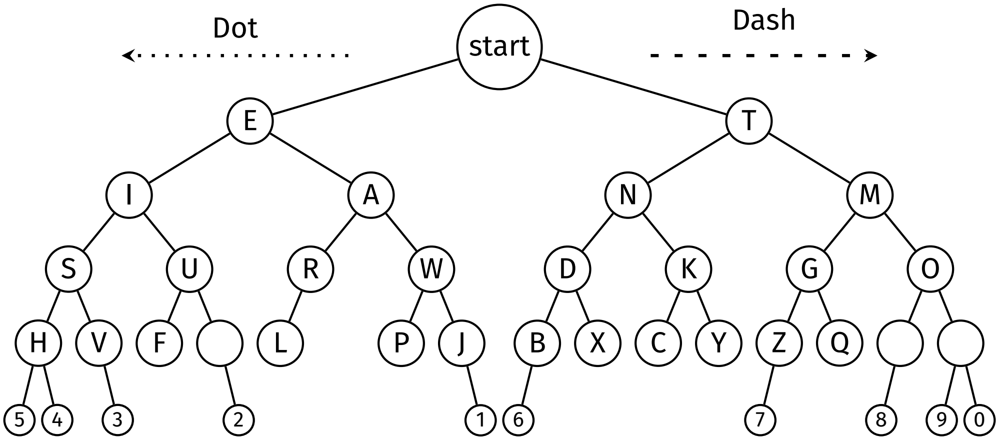




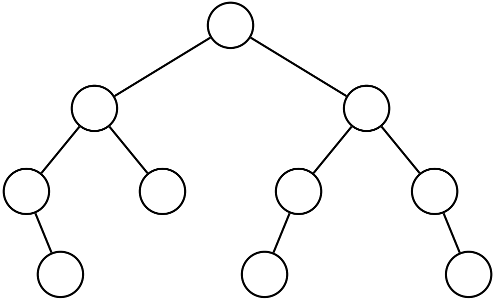
Source: “Data Structures and Algorithms in Java” (6th ed) by Goodrich et al.

Trees

Example - Decoding Morse Code



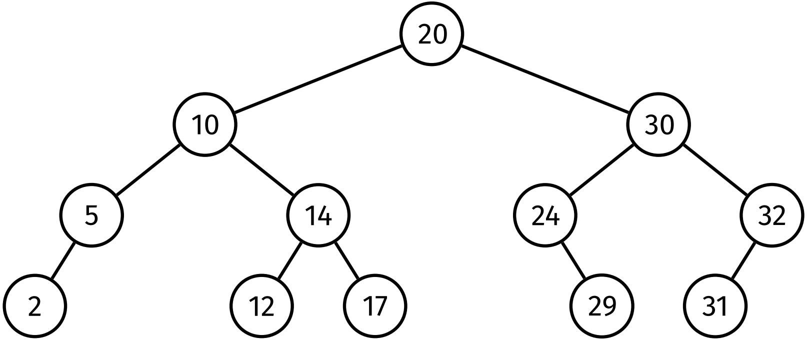
A binary tree is a tree where
each node can have up to two child nodes,
referred to as the left child and the right child.



Binary Search Trees

A binary search tree is an ordered binary tree, where *for each node*:

- All values in the left subtree are less than the value in the node
- All values in the right subtree are greater than the value in the node



Binary Search Tree

Why?

We need a more efficient way to search and maintain large amounts of data.

We have already explored some approaches:

	Ordered array	Ordered linked list
Searching/finding the insertion/deletion point	$O(\log n)$	$O(n)$
Inserting/deleting *after* finding the insertion/deletion point	$O(n)$	$O(1)$

Binary Search Trees

Why?

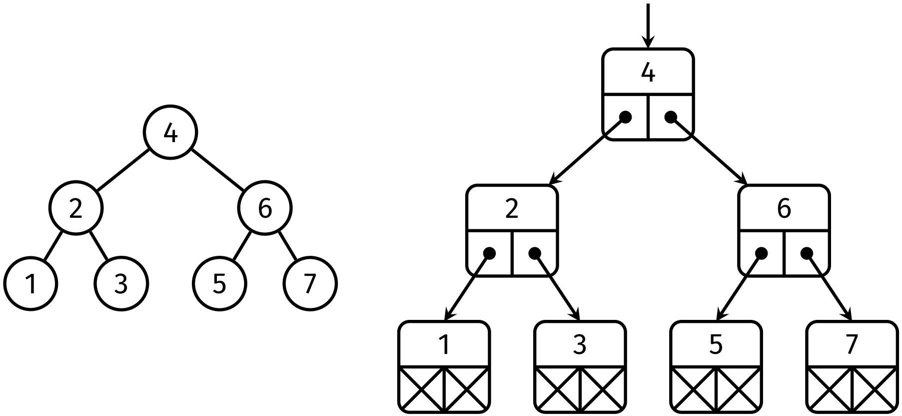
Binary search trees are efficient to search *and* maintain:

- Searching in a binary search tree is similar to how binary search works
- A binary search tree is a linked data structure (like a linked list), so there is no need to shift elements when inserting/deleting

Binary trees are typically represented by node structures

- Where each node contains a value and pointers to child nodes

```
struct node {
    int item;
    struct node *left;
    struct node *right;
};
```


4
2
6
1
3
5
7
4
2
6
1
3
5
7

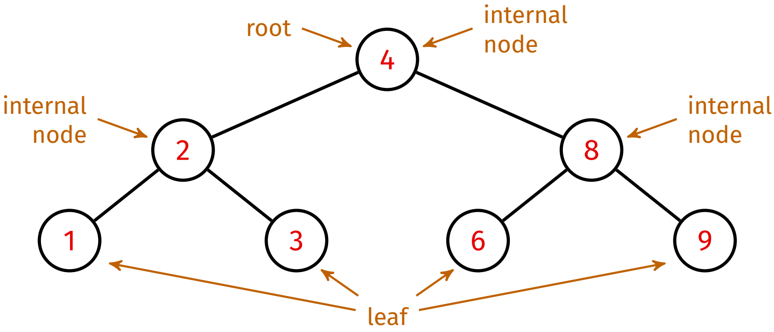
Binary Search Trees

Terminology

The root node is the node with no parent node.

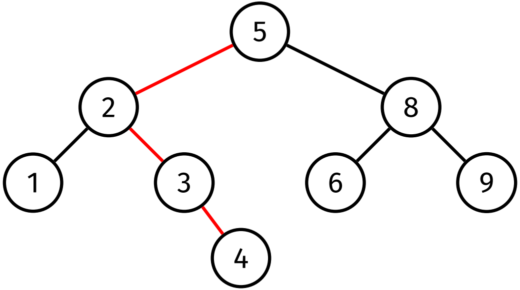
A leaf node is a node that has no child nodes.

An internal node is a node that has at least one child node.



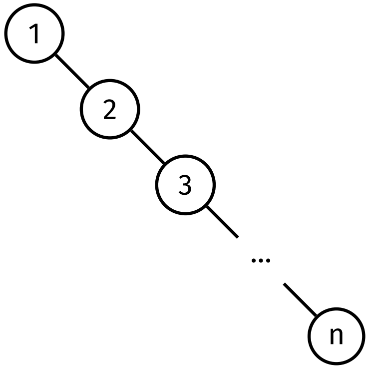
Height of a tree: Maximum path length from the root node to a leaf

- The height of an empty tree is considered to be -1
- The height of the following tree is 3



For a tree with n nodes:

The maximum possible height is $n - 1$

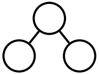
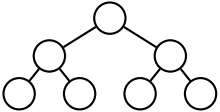


Binary Search Trees

Terminology

For a tree with n nodes:

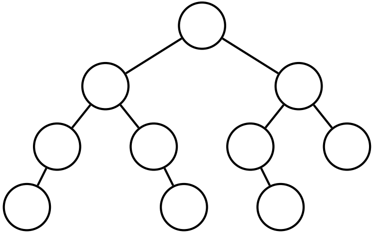
The minimum possible height is $\lfloor \log_2 n \rfloor$

n	minimum height = $\lfloor \log_2 n \rfloor$	tree
1	0	
2-3	1	
4-7	2	
...	...	...

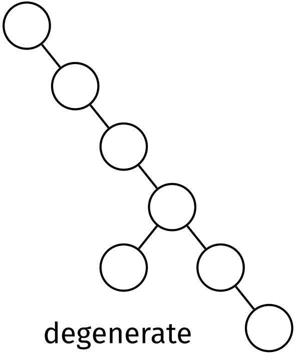
Binary Search Trees

Terminology

For a given number of nodes, a tree is said to be **balanced** if its height is minimal (or close to minimal), and **degenerate** if its height is maximal (or close to maximal).



balanced

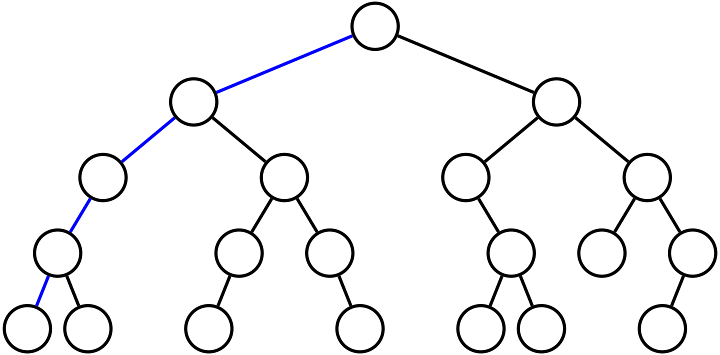


degenerate

Key operations on binary search trees:

- Insert
- Search
- Traverse
- Join
- Delete

The height h of a binary search tree determines the efficiency of many operations, so we will use both n and h in our analyses.

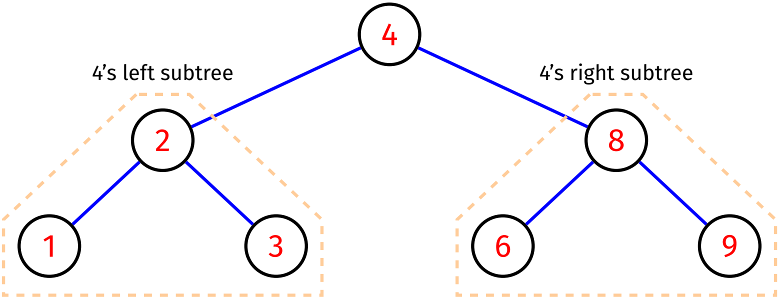


$n = 20 \quad h = 4$

Many BST operations can be implemented recursively.

A binary search tree is either:

- empty; or
- consists of a node with two subtrees
  - ...which are also binary search trees



BST Insertion

Insertion

`bstInsert(`t`, `v`)`

Given a BST t and a value v,
insert v into the BST
and return the root of the updated BST

BST Insertion

Insertion is straightforward:

- Start at the root
- Compare value to be inserted with value in the node
  - If value being inserted is less, descend to left child
  - If value being inserted is greater, descend to right child
- Repeat until...
  you have to go left/right but current node has no left/right child
  - Create new node and attach to current node

BST Insertion
Method

Recursive method:

- t is empty
  $\Rightarrow$ make a new node with v as the root of the new tree
- $v <$ t`->item`
  $\Rightarrow$ insert v into t's left subtree
- $v >$ t`->item`
  $\Rightarrow$ insert v into t's right subtree
- $v =$ t`->item`
  $\Rightarrow$ tree unchanged (assuming no duplicates)

EXERCISE Try writing an iterative version.

BST Insertion

Example 1

Insert the following values into an empty tree:

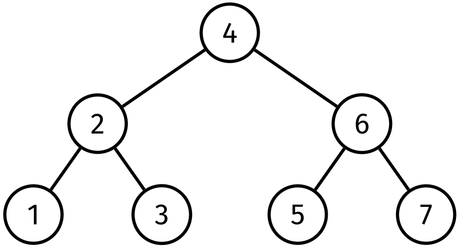
4 2 6 5 1 7 3

BST Insertion

Example 1

Insert the following values into an empty tree:

4 2 6 5 1 7 3

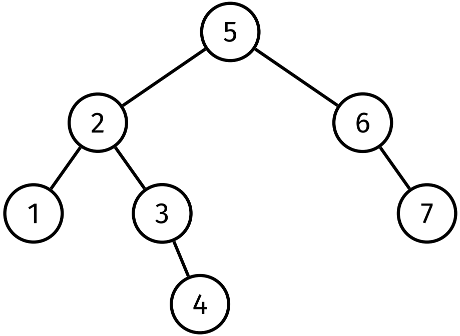


Insert the following values into an empty tree:

5 6 2 3 4 7 1

Insert the following values into an empty tree:

5 6 2 3 4 7 1



BST Insertion

Example 3

Insert the following values into an empty tree:

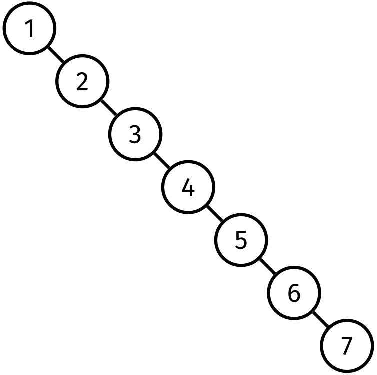
1 2 3 4 5 6 7

BST Insertion

Example 3

Insert the following values into an empty tree:

1 2 3 4 5 6 7



BST Insertion

Pseudocode

```
bstInsert(t, v):
    Input:  tree t, value v
    Output: t with v inserted

    if t is empty:
        return new node containing v
    else if v < t->item:
        t->left = bstInsert(t->left, v)
    else if v > t->item:
        t->right = bstInsert(t->right, v)

    return t
```

Analysis:

- At most one node is visited per level
- Number of operations performed per node is constant
- Therefore, the worst-case time complexity of insertion is $O(h)$ where h is the height of the BST

Search

`bstSearch(`t`, `v`)`

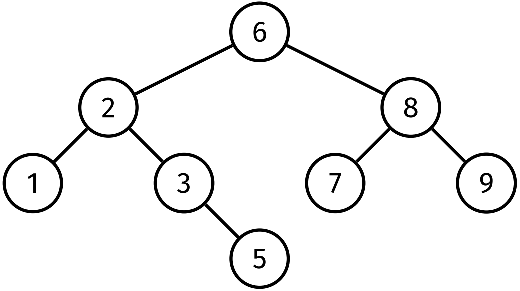
Given a BST t and a value v,
return true if v is in the BST
and false otherwise

Recursive method:

- t is empty:
  $\Rightarrow$ return false
- $v <$ t`->item`
  $\Rightarrow$ search for v in t's left subtree
- $v >$ t`->item`
  $\Rightarrow$ search for v in t's right subtree
- $v =$ t`->item`
  $\Rightarrow$ return true

EXERCISE Try writing an iterative version.

Search for 4 and 7 in the following BST:



BST Search

Pseudocode

```
bstSearch(t, v):
    Input:  tree t, value v
    Output: true if v is in t
            false otherwise

    if t is empty:
        return false
    else if v < t->item:
        return bstSearch(t->left, v)
    else if v > t->item:
        return bstSearch(t->right, v)
    else:
        return true
```

Analysis:

- At most one node is visited per level
- Number of operations performed per node is constant
- Therefore, the worst-case time complexity of search is $O(h)$ where h is the height of the BST

Traversal

Given a BST,
visit every node of the tree

Tree Traversal

There are 4 common ways to traverse a binary tree:

1. Pre-order (NLR):
   visit root, then traverse left subtree, then traverse right subtree
2. In-order (LNR):
   traverse left subtree, then visit root, then traverse right subtree
3. Post-order (LRN):
   traverse left subtree, then traverse right subtree, then visit root
4. Level-order:
   visit root, then its children, then their children, and so on

Tree Traversal

Pseudocode

Pseudocode:

```
preorder(t):
    Input: tree t

    if t is empty:
        return

    visit(t)
    preorder(t->left)
    preorder(t->right)
```

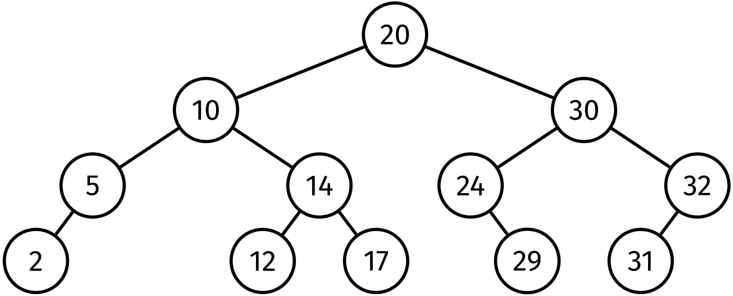
```
inorder(t):
    Input: tree t

    if t is empty:
        return

    inorder(t->left)
    visit(t)
    inorder(t->right)
```

```
postorder(t):
    Input: tree t

    if t is empty:
        return

    postorder(t->left)
    postorder(t->right)
    visit(t)
```

Note:
Level-order traversal is difficult to implement recursively.
It is typically implemented using a queue.

Tree Traversal

Example: Binary Search Tree

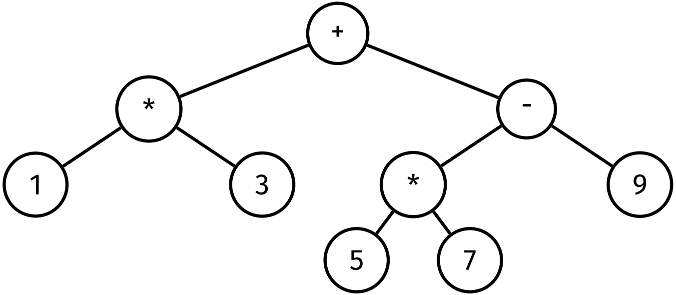


Pre-order	20 10 5 2 14 12 17 30 24 29 32 31
In-order	2 5 10 12 14 17 20 24 29 30 31 32
Post-order	2 5 12 17 14 10 29 24 31 32 30 20
Level-order	20 10 30 5 14 24 32 2 12 17 29 31

Tree Traversal

Example: Expression Tree

Expression tree for 1 * 3 + (5 * 7 - 9)



Pre-order	+ * 1 3 - * 5 7 9
In-order	1 * 3 + 5 * 7 - 9
Post-order	1 3 * 5 7 * 9 - +

Tree Traversal

Applications

Pre-order traversal:

- Useful for reconstructing a tree

In-order traversal:

- Useful for traversing a BST in ascending order

Post-order traversal:

- Useful for evaluating an expression tree
- Useful for freeing a tree

Level-order traversal:

- Useful for printing a tree

Analysis:

- Each node is visited once
- Hence, time complexity of tree traversal is $O(n)$, where n is the number of nodes

BST Join

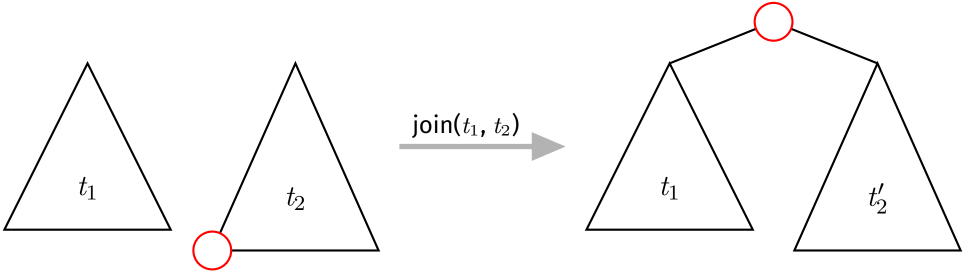
Join

`bstJoin(`t_1`, `t_2`)`

Given two BSTs t_1 and t_2
where $\max(t_1) < \min(t_2)$
return a BST containing all items from t_1 and t_2

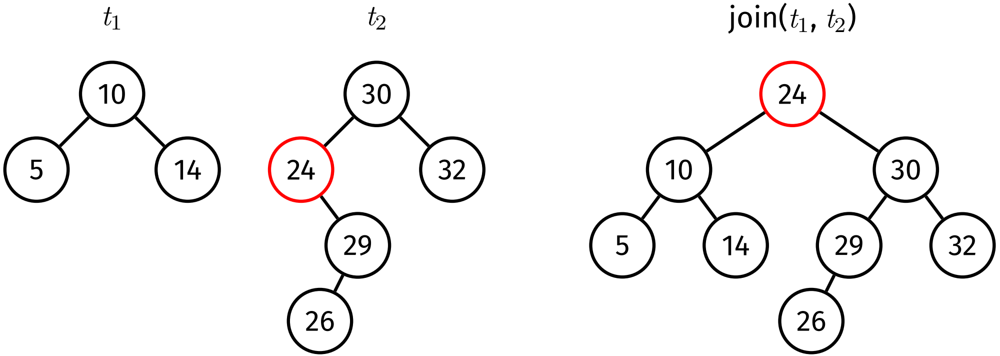
Method:

1. Find the minimum node min in t_2
2. Replace min by its right subtree (if it exists)
3. Elevate min to be the new root of t_1 and t_2



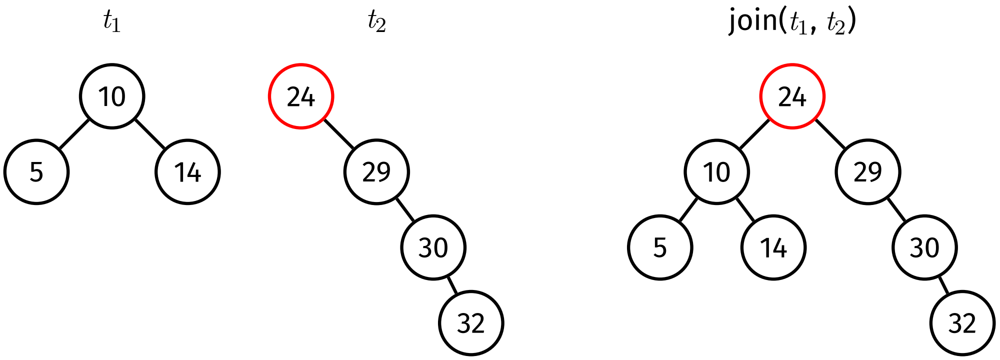
BST Join

Example 1



BST Join

Example 2



BST Join
Pseudocode


```
bstJoin(t1, t2):
    Input:  trees t1, t2
    Output: t1 and t2 joined together

    if t1 is empty:
        return t2
    else if t2 is empty:
        return t1
    else if t2->left is empty:
        t2->left = t1
        return t2
    else:
        curr = t2
        parent = NULL
        while curr->left ≠ NULL:
            parent = curr
            curr = curr->left

        parent->left = curr->right
        curr->left = t1
        curr->right = t2
        return curr
```

Analysis:

- The join algorithm simply finds the minimum node in t_2
- Thus, at most one node is visited per level of t_2
- Therefore, the worst-case time complexity of join is $O(h_2)$ where h_2 is the height of t_2

Deletion

bstDelete(t, v)

Given a BST t and a value v
delete v from the BST
and return the root of the updated BST

BST Deletion
Method

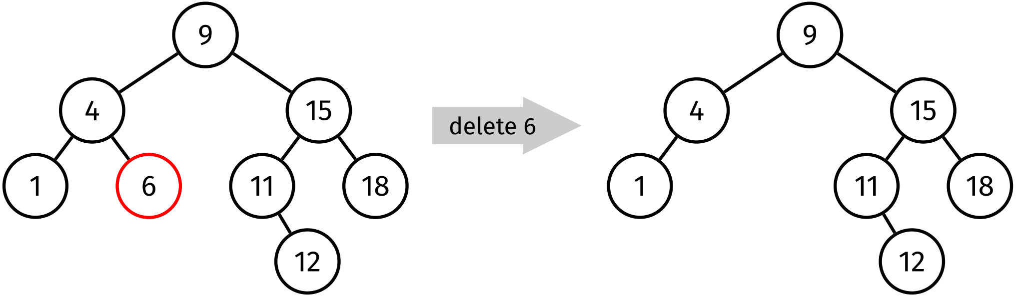
Recursive method:

- t is empty:
  $\Rightarrow$ result is empty
- $v <$ t`->item`
  $\Rightarrow$ delete v from t's left subtree
- $v >$ t`->item`
  $\Rightarrow$ delete v from t's right subtree
- $v =$ t`->item`
  $\Rightarrow$ three sub-cases:
  - t is a leaf
    $\Rightarrow$ result is empty tree
  - t has one subtree
    $\Rightarrow$ replace with subtree
  - t has two subtrees
    $\Rightarrow$ join the two subtrees

BST Deletion

Examples

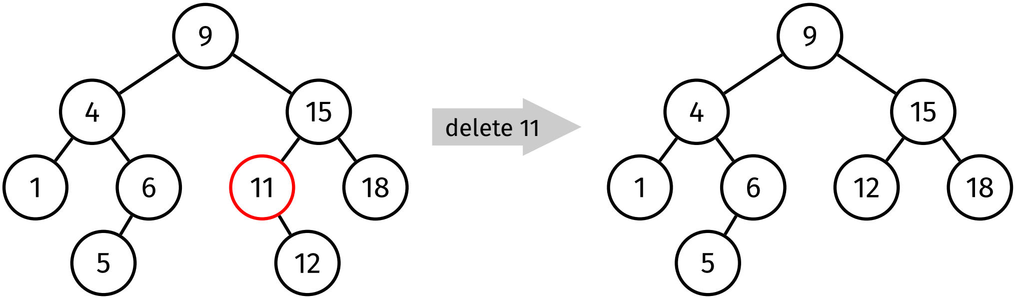
If the node being deleted is a leaf, then the result is an empty tree



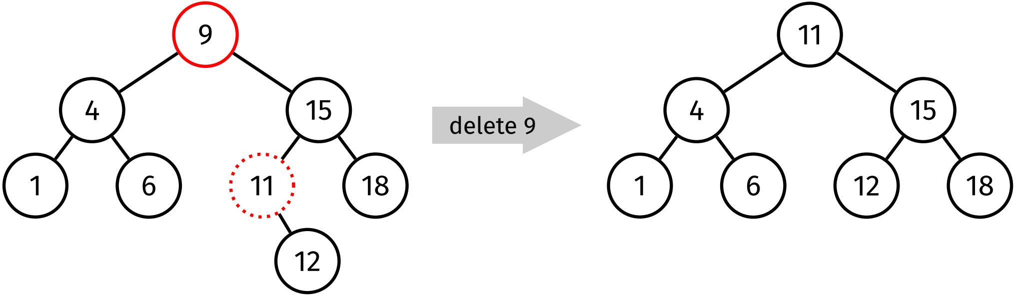
BST Deletion

Examples

Node to be deleted has one subtree



Node to be deleted has two subtrees



BST Deletion

Pseudocode

```
bstDelete(t, v):
    Input:  tree t, value v
    Output: t with v deleted

    if t is empty:
        return empty tree
    else if v < t->item:
        t->left = bstDelete(t->left, v)
    else if v > t->item:
        t->right = bstDelete(t->right, v)
    else:
        if t->left is empty:
            new = t->right
        else if t->right is empty:
            new = t->left
        else:
            new = bstJoin(t->left, t->right)

        free(t)
        t = new

    return t
```

BST Deletion

Analysis

Analysis:

- The deletion algorithm traverses down just one branch
  - First, the item being deleted is found
  - If the item exists and has two subtrees, its successor is found
- Thus, at most one node is visited per level
- Therefore, the worst-case time complexity of deletion is $O(h)$ where h is the height of the BST

BST Exercises

- `bstFree`
  free all nodes of a tree
- `bstSize`
  return the size of a tree
- `bstHeight`
  return the height of a tree
- `bstPrune`
  given values lo and hi, remove all values outside the range $[lo, hi]$