COMP2521 25T3
Sorting Algorithms (II)
Elementary Sorting Algorithms

Sim Mautner
cs2521@cse.unsw.edu.au

selection sort
bubble sort
insertion sort

Selection Sort

Method:

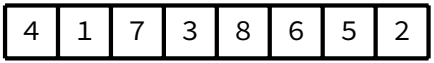
- Find the smallest element, swap it with the first element
- Find the second-smallest element, swap it with the second element
- ...
- Find the second-largest element, swap it with the second-last element

Each iteration improves the “sortedness” of the array by one element.

Selection Sort

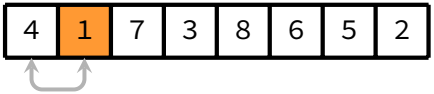
Example

Example



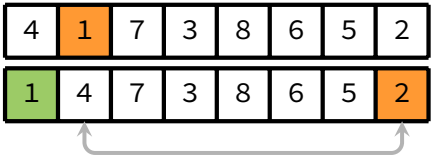
Selection Sort

Example



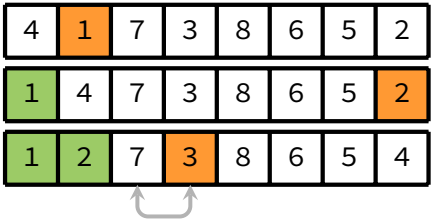
Selection Sort

Example



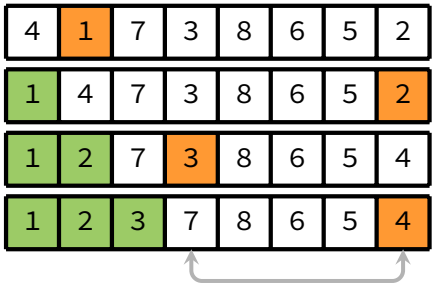
Selection Sort

Example



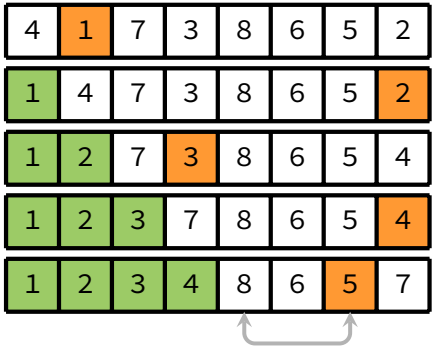
Selection Sort

Example



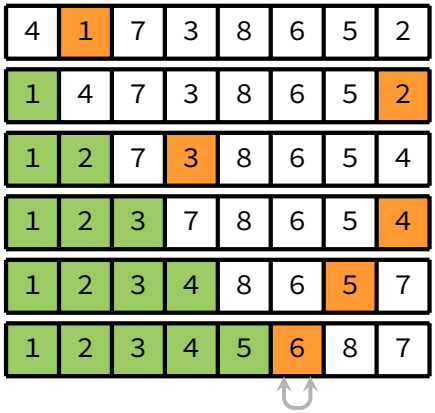
Selection Sort

Example



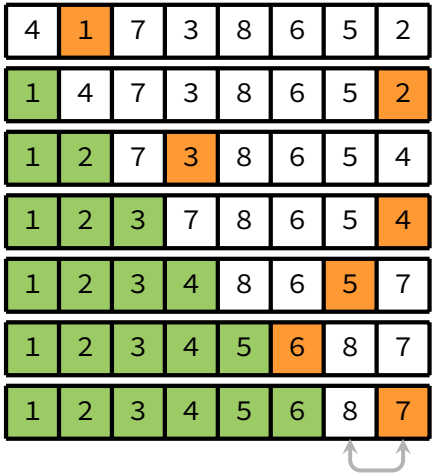
Selection Sort

Example



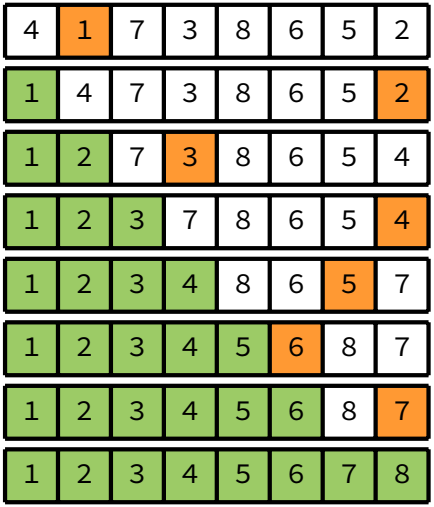
Selection Sort

Example



Selection Sort

Example



Selection Sort

C Implementation


```
void selectionSort(Item items[], int lo, int hi) {
    for (int i = lo; i < hi; i++) {
        int min = i;
        for (int j = i + 1; j <= hi; j++) {
            if (lt(items[j], items[min])) {
                min = j;
            }
        }
        swap(items, i, min);
    }
}
```

Selection Sort

Analysis

Cost analysis:

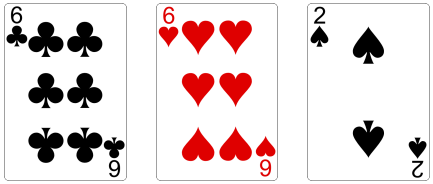
- In the first iteration, $n-1$ comparisons, 1 swap
- In the second iteration, $n-2$ comparisons, 1 swap
- ...
- In the final iteration, 1 comparison, 1 swap
- $C = (n-1) + (n-2) + \ldots + 1 = \frac{1}{2}n(n-1) \Rightarrow O(n^2)$
- $S = n-1$

Cost is the same, regardless of the sortedness of the original array.

Selection Sort
Properties

Selection sort is unstable

- Due to long-range swaps
- For example, sort these cards by value:



Selection Sort

Properties

Unstable
Due to long-range swaps

Non-adaptive
Performs same steps, regardless of sortedness of original array

In-place
Sorting is done within original array; does not use temporary arrays

Bubble Sort

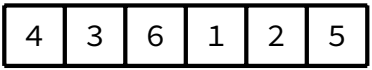
Method:

- Make multiple passes from left (`lo`) to right
- On each pass, swap any out-of-order adjacent pairs
- Elements “bubble up” until they meet a larger element
- Stop if there are no swaps during a pass
  - This means the array is sorted

Bubble Sort

Example

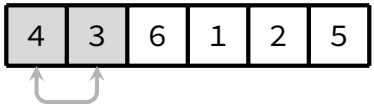
Example



Bubble Sort

Example

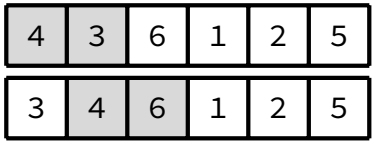
First pass



Bubble Sort

Example

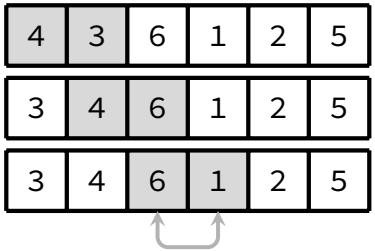
First pass



Bubble Sort

Example

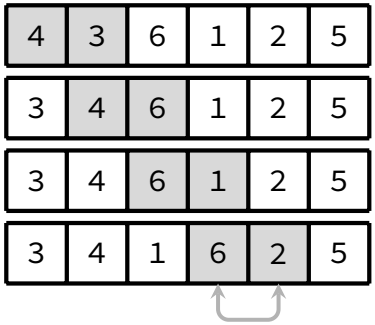
First pass



Bubble Sort

Example

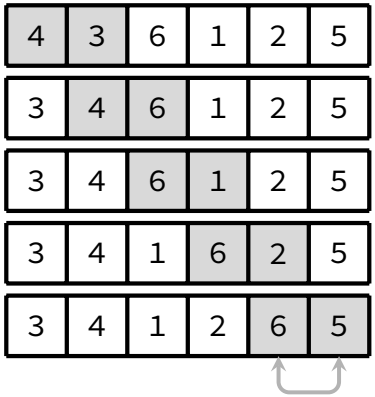
First pass



Bubble Sort

Example

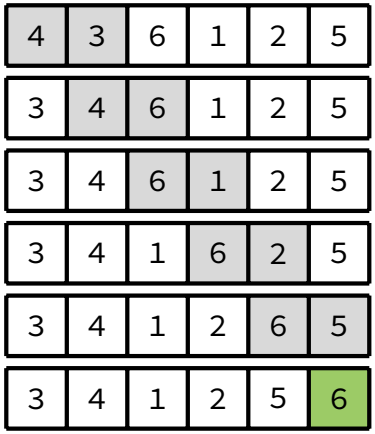
First pass



Bubble Sort

Example

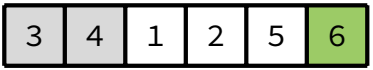
First pass



Bubble Sort

Example

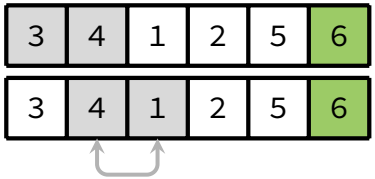
Second pass



Bubble Sort

Example

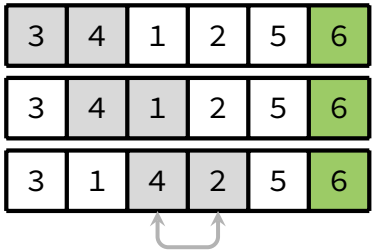
Second pass



Bubble Sort

Example

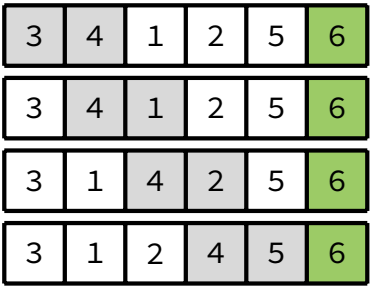
Second pass



Bubble Sort

Example

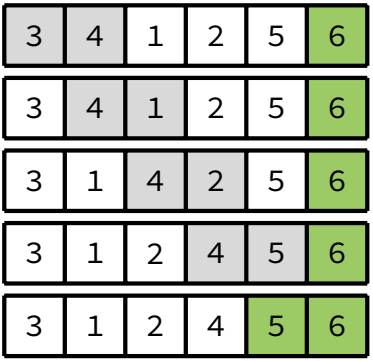
Second pass



Bubble Sort

Example

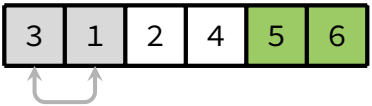
Second pass



Bubble Sort

Example

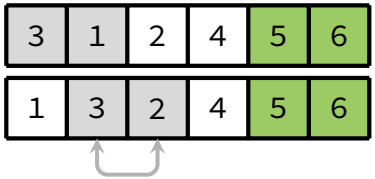
Third pass



Bubble Sort

Example

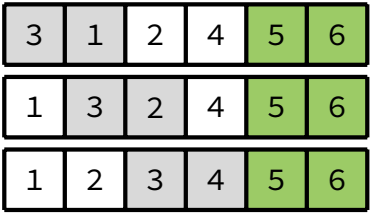
Third pass



Bubble Sort

Example

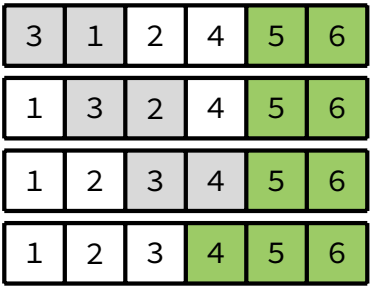
Third pass



Bubble Sort

Example

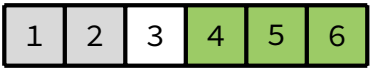
Third pass



Bubble Sort

Example

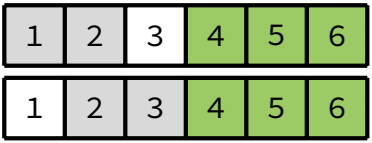
Fourth pass



Bubble Sort

Example

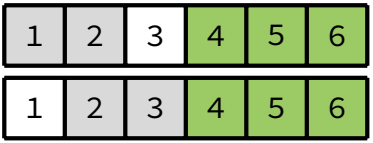
Fourth pass



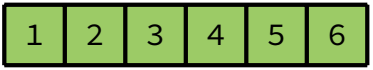
Bubble Sort

Example

Fourth pass



No swaps made; stop



Bubble Sort

C Implementation

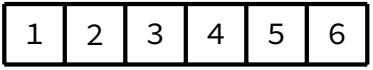

```
void bubbleSort(Item items[], int lo, int hi) {
    for (int i = hi; i > lo; i--) {
        bool swapped = false;
        for (int j = lo; j < i; j++) {
            if (gt(items[j], items[j + 1])) {
                swap(items, j, j + 1);
                swapped = true;
            }
        }
        if (!swapped) break;
    }
}
```

Bubble Sort

Analysis

Best case: Array is sorted

- Only a single pass required
- $n - 1$ comparisons, no swaps
- Best-case time complexity: $O(n)$

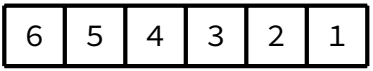


Bubble Sort

Analysis

Worst case: Array is reverse-sorted

- $n-1$ passes required
  - First pass: $n-1$ comparisons
  - Second pass: $n-2$ comparisons
  - ...
  - Final pass: 1 comparison
- Total comparisons: $(n-1)+(n-2)+\ldots+1 = \frac{1}{2}n(n-1)$
- Every comparison leads to a swap $\Rightarrow \frac{1}{2}n(n-1)$ swaps
- Worst-case time complexity: $O(n^2)$



Bubble Sort

Analysis

Average-case time complexity: $O(n^2)$

- It can be proven that for a randomly ordered array, bubble sort needs to perform $\frac{1}{4}n(n-1)$ swaps on average $\Rightarrow O(n^2)$
  - See appendix for details
- Can show empirically by generating random sequences and sorting them

Bubble Sort

Properties

Stable

Comparisons are between adjacent elements only
Elements are only swapped if out of order

Adaptive

Bubble sort is $O(n^2)$ on average, $O(n)$ if input array is sorted

In-place

Sorting is done within original array; does not use temporary arrays

Insertion Sort

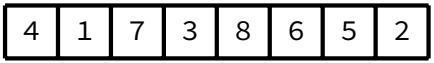
Method:

- Take first element and treat as sorted array (of length 1)
- Take next element and insert into sorted part of array so that order is preserved
  - This increases the length of the sorted part by one
- Repeat for remaining elements

Insertion Sort

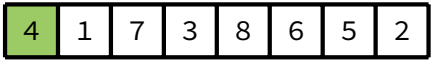
Example

Example



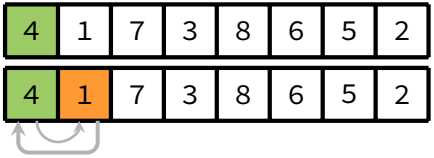
Insertion Sort

Example



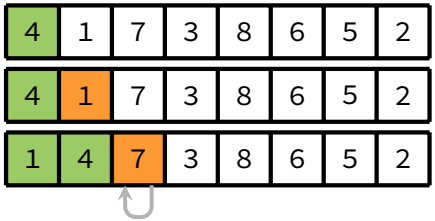
Insertion Sort

Example



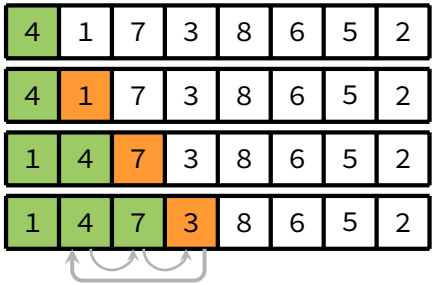
Insertion Sort

Example



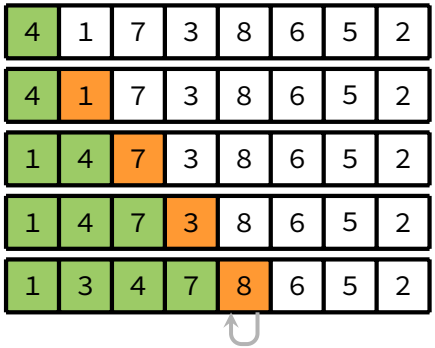
Insertion Sort

Example



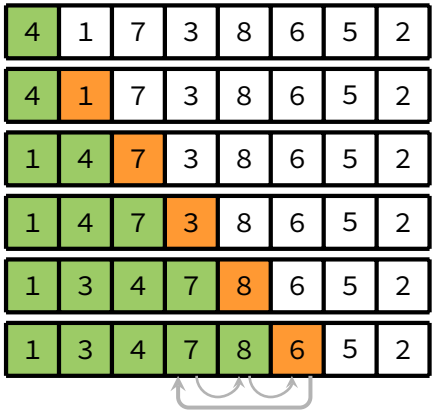
Insertion Sort

Example



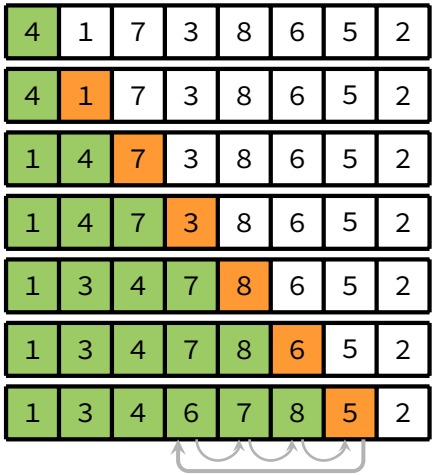
Insertion Sort

Example



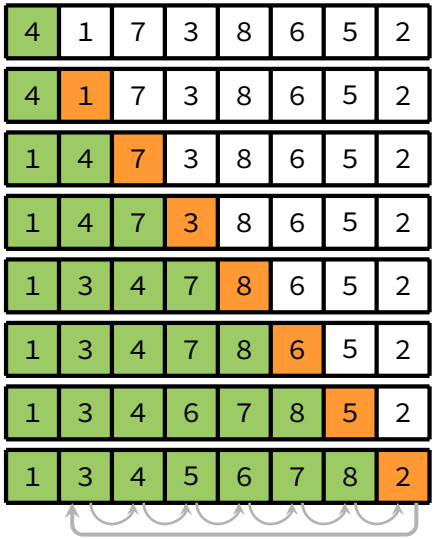
Insertion Sort

Example



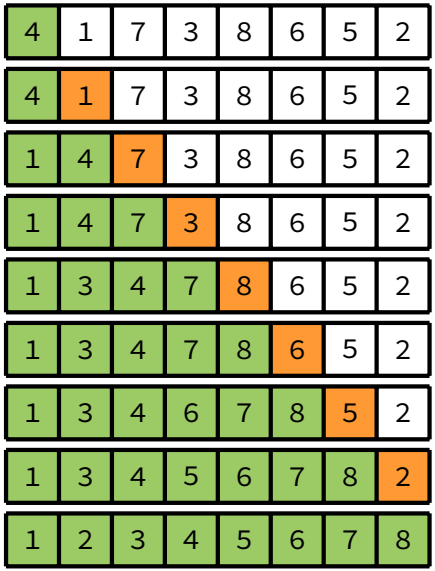
Insertion Sort

Example



Insertion Sort

Example



Insertion Sort

C Implementation

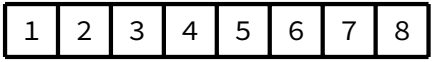

```c
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort

Analysis

Best case: Array is sorted

- Inserting each element requires one comparison
- $n - 1$ comparisons
- Best-case time complexity: $O(n)$

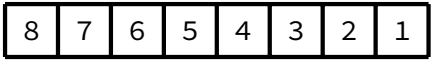


Insertion Sort

Analysis

Worst case: Array is reverse-sorted

- Inserting i-th element requires i comparisons
  - Inserting index 1 element requires 1 comparison
  - Inserting index 2 element requires 2 comparisons
  - ...
- Total comparisons: $1 + 2 + \ldots + (n-1) = \frac{1}{2}n(n-1)$
- Worst-case time complexity: $O(n^2)$



Insertion Sort

Analysis

Average-case time complexity: $O(n^2)$

- Same reason as for bubble sort
- Can show empirically by generating random sequences and sorting them

Insertion Sort

Properties

Stable

Elements are always inserted to the right of any equal elements

Adaptive

Insertion sort is $O(n^2)$ on average, $O(n)$ if input array is sorted

In-place

Sorting is done within original array; does not use temporary arrays

Summary of Elementary Sorts

	Time complexity			Properties	
	Best	Average	Worst	Stable	Adaptive
Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	No	No
Bubble sort	$O(n)$	$O(n^2)$	$O(n^2)$	Yes	Yes
Insertion sort	$O(n)$	$O(n^2)$	$O(n^2)$	Yes	Yes

Aside: Sorting Linked Lists

Selection sort:

- Let L = original list, S = sorted list (initially empty)
- Repeat the following until L is empty:
  - Find the node V containing the largest value in L, and unlink it
  - Insert V at the front of S

Bubble sort:

- Traverse the list, comparing adjacent values
  - If value in current node is greater than value in next node, swap values
- Repeat the above until no swaps required in one traversal

Insertion sort:

- Let L = original list, S = sorted list (initially empty)
- For each node in L:
  - Insert the node into S in order

Appendix

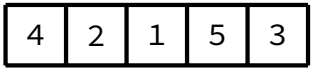
Bubble Sort - Proof of $O(n^2)$ Average Case

Note: Not required knowledge in COMP2521!

New concept: inversion

An inversion is a pair of elements from a sequence where the left element is greater than the right element.

For example, consider the following array:



The array contains 5 inversions:
(4, 2), (4, 1), (4, 3), (2, 1), (5, 3)

Bubble Sort - Proof of $O(n^2)$ Average Case
Continued

Observation:

- In bubble sort, every swap reduces the number of inversions by 1

The goal of the proof: Show that the average number of inversions in a randomly sorted array is $O(n^2)$.

- This implies the number of swaps required by bubble sort is $O(n^2)$ ...
- Which implies that the average-case time complexity of bubble sort is $O(n^2)$ or slower
  - (but we know that it can't be slower than $O(n^2)$ since the worst-case time complexity of bubble sort is $O(n^2)$)

Bubble Sort - Proof of $O(n^2)$ Average Case
Continued

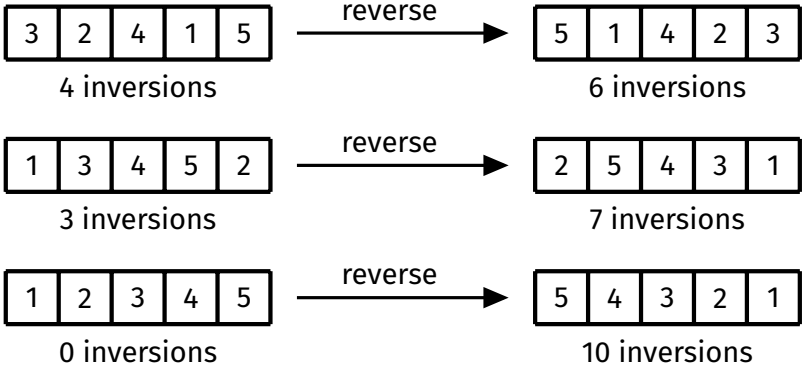
In a randomly sorted array:

- The minimum possible number of inversions is 0 (sorted array)
- The maximum possible number of inversions is $\frac{1}{2}n(n-1)$ (reverse-sorted array)

Bubble Sort - Proof of $O(n^2)$ Average Case
Continued

Let k be the number of inversions in a random permutation.
By reversing this permutation, one can obtain a permutation with $\frac{1}{2}n(n-1) - k$ inversions.

For example, suppose $n = 5$:



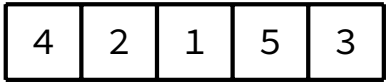
Bubble Sort - Proof of $O(n^2)$ Average Case
Continued

Thus, if we take all the possible permutations of an array and pair each permutation with its reverse, the total number of inversions in each pair is $\frac{1}{2}n(n-1)$.

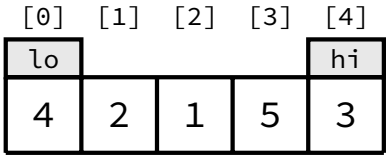
This implies that the average number of inversions across all permutations is $\frac{1}{4}n(n-1)$, which is $O(n^2)$.

Insertion Sort Walkthrough

Sort the following array:

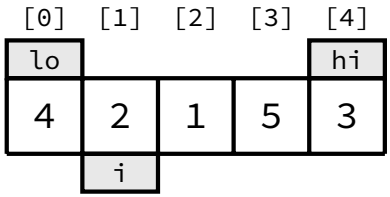


Insertion Sort Walkthrough



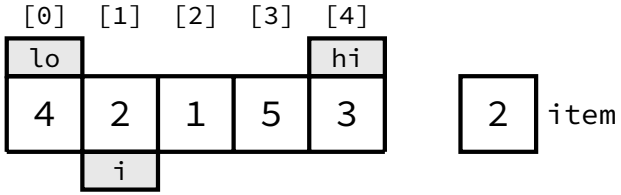

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



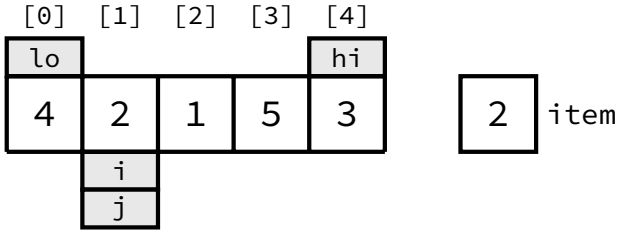

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



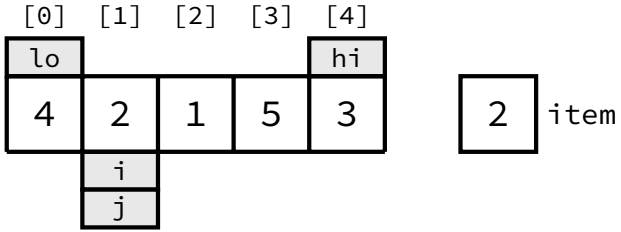

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



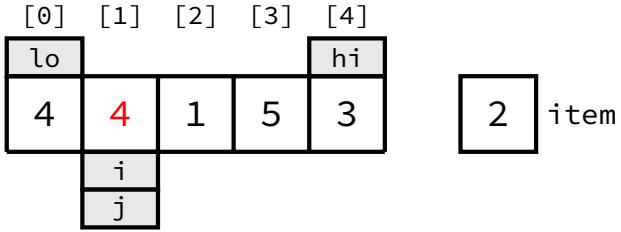

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



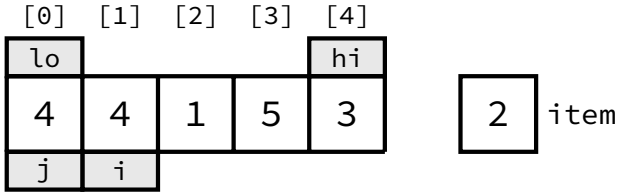

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



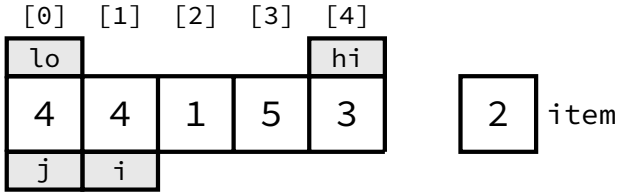

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



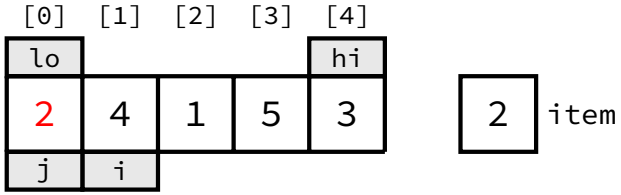

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



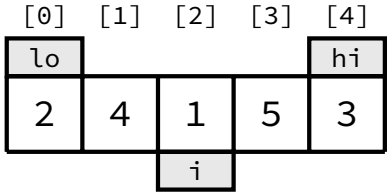

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



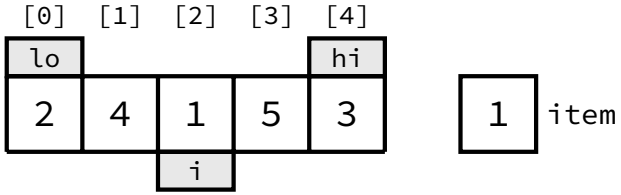

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



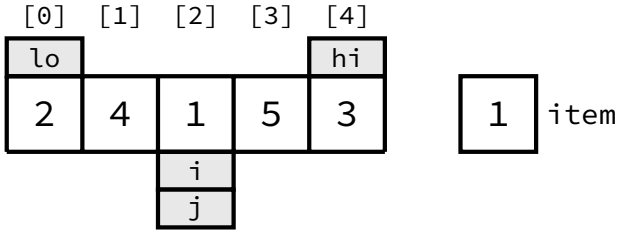

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



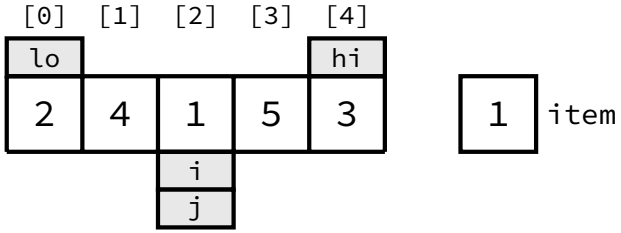

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



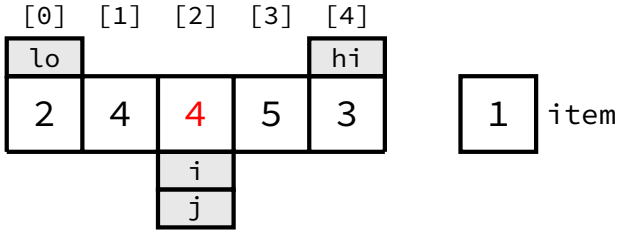

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



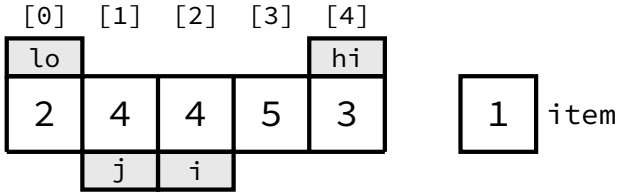

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



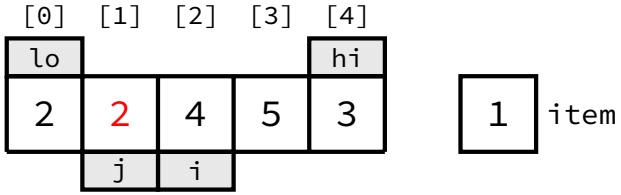

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



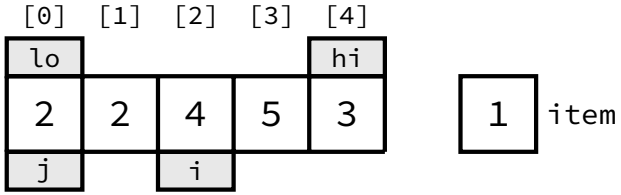

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



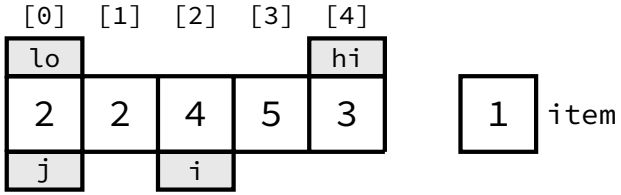

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



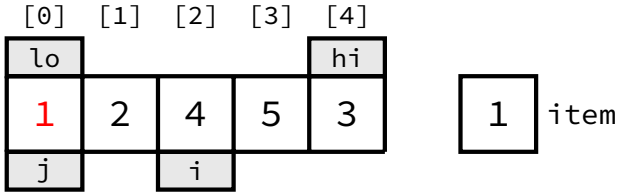

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



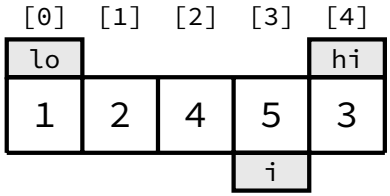

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



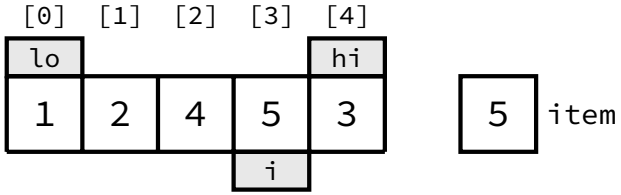

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



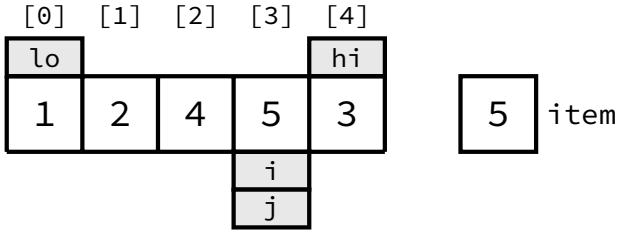

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



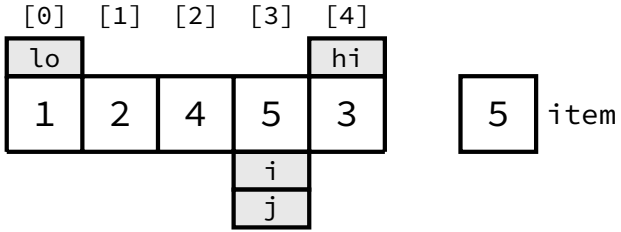

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



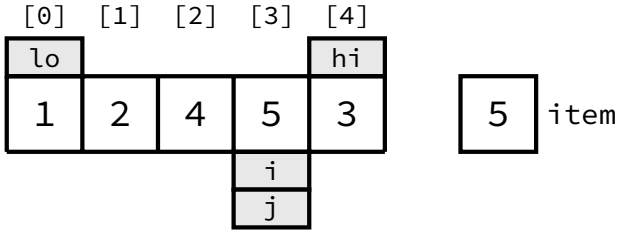

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



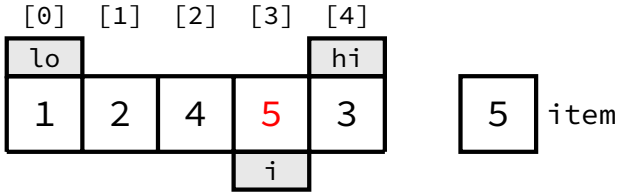

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



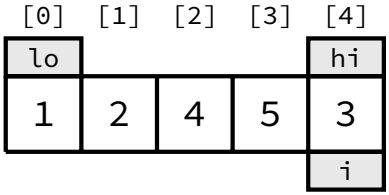

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



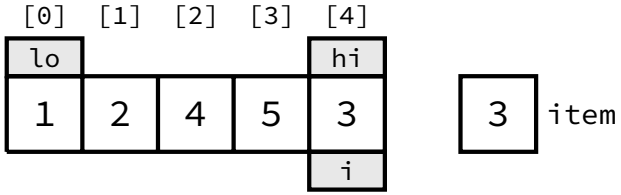

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



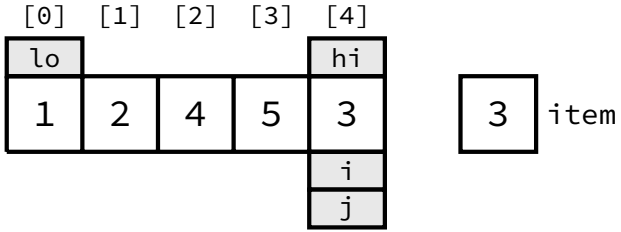

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



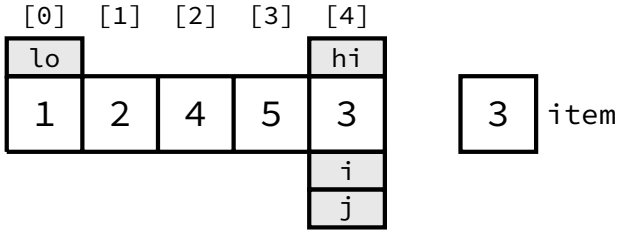

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



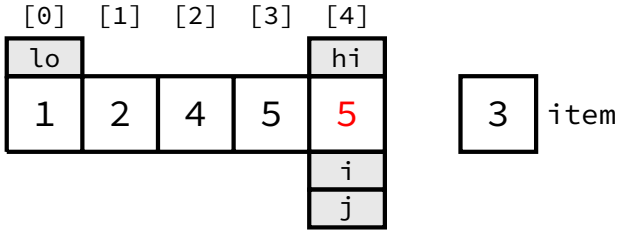

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



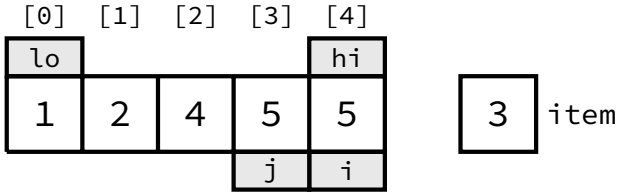

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



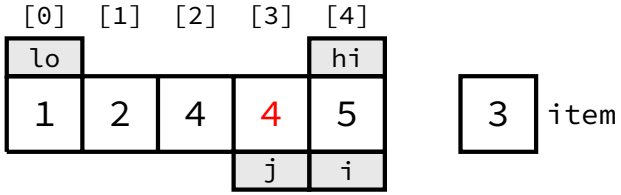

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



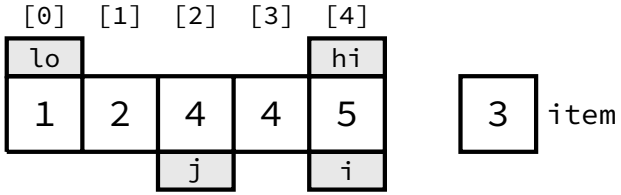

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



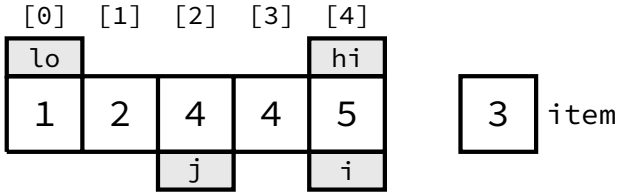

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



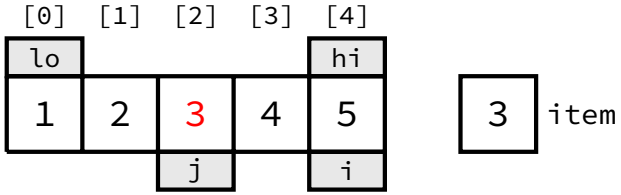

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



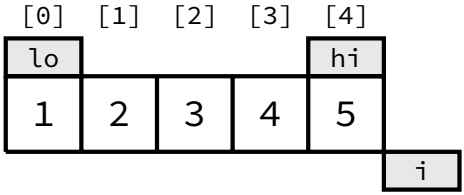

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough



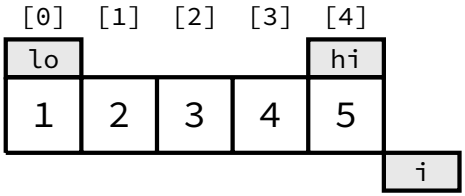

```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough




```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Insertion Sort Walkthrough




```
void insertionSort(Item items[], int lo, int hi) {
    for (int i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        int j = i;
        for (; j > lo && lt(item, items[j - 1]); j--) {
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```