

COMP2521 24T3

Tries

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Many applications require
searching through a set of strings

Examples:

Predictive text

Autocomplete

Approximate string matching

Spell checking

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Predictive text



For example, pressing “4663”
can be interpreted as the word
good, home, hood or hoof

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Autocomplete



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How can we implement a set of strings
using data structures covered so far?

AVL tree

Performance: $O(\log n)$ worst case

Hash table

Performance: $O(1)$ average case, $O(n)$ worst case

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AVL trees and hash tables are efficient, but...

...searching requires user to provide the full string...

...which is not always possible in the above applications
(or would be inefficient)

Possible solution: **tries**

A trie...

- is a tree data structure
- used to represent a set of strings
 - e.g., all the distinct words in a document, a dictionary, etc.
 - we will call these strings *keys* or *words*
- supports string matching queries in $O(m)$ time
 - where m is the length of the string being searched for

Note: the word *trie* comes from *retrieval*, but pronounced as “try” not “tree”

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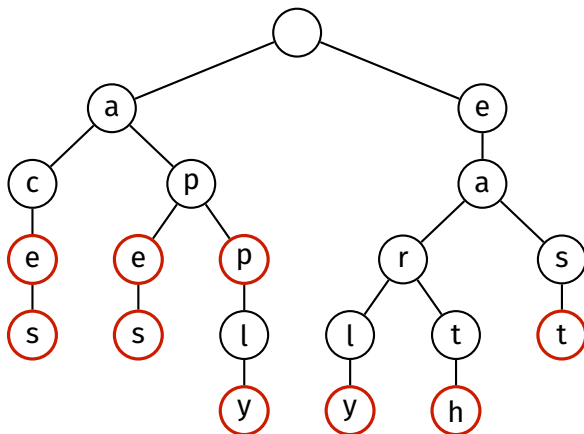
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Example:

*Keys in
the trie:*ace
aces
ape
apes
app
apply
early
earth
east

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Important features of tries:

- Each link represents an individual character
- A key is represented by a path in the trie
- Each node can be tagged as a “finishing” node
 - A “finishing” node marks the end of a key
- Each node may contain data associated with key
- Unlike a search tree, the nodes in a trie do not store their associated key
 - Instead, keys are implicitly defined by their position in the trie

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Assuming alphabetic strings:

```
#define ALPHABET_SIZE 26
```

```
struct node {  
    struct node *children[ALPHABET_SIZE];  
    bool finish; // marks the end of a key  
    Data data;   // data associated with key  
};
```

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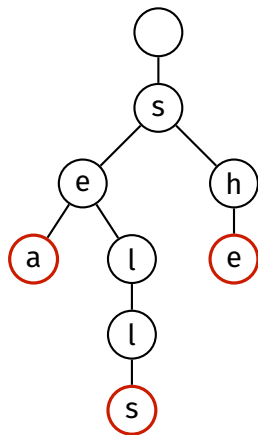
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Consider this trie:



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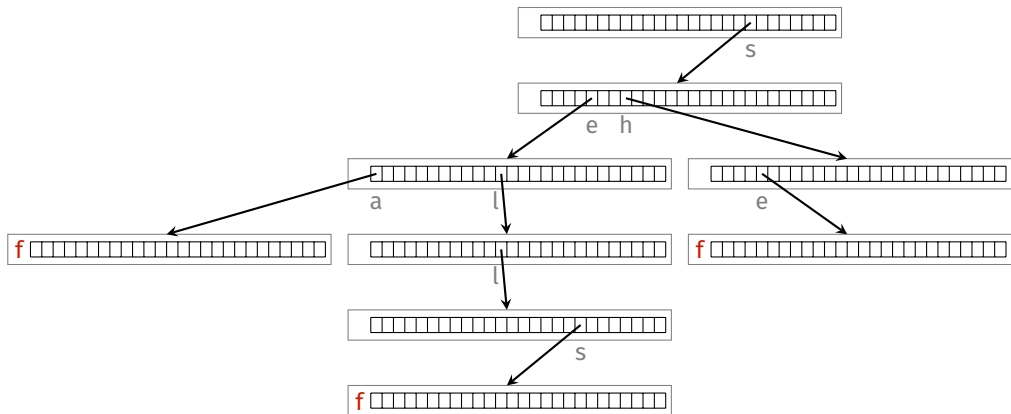
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Concrete representation:
(f = finishing node)



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Process for insertion:

- Start at the root
- For each character c in the key (from left to right):
 - If there is no child node corresponding to c , create one
 - Descend into the child node corresponding to c
- Mark the resulting node as a finishing node and insert data (if any)

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Insert the following words into an initially empty trie:

sea shell sell shore she

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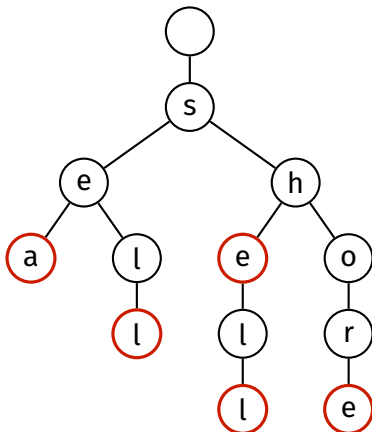
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Insert the following words into an initially empty trie:

sea shell sell shore she



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Trie insertion can be implemented recursively.

```
trieInsert(t, key, data):
```

```
    Inputs: trie t
```

```
             key of length m and associated data
```

```
    Output: t with key and data inserted
```

```
    if t is empty:
```

```
        t = new node
```

```
    if m = 0:
```

```
        t->finish = true
```

```
        t->data = data
```

```
    else:
```

```
        first = key[0]
```

```
        rest = key[1..m - 1] // i.e., slice off first character from key
```

```
        t->children[first] = trieInsert(t->children[first], rest, data)
```

```
    return t
```

EXERCISE Try writing an iterative version.

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Search is similar to insertion:

- Start at the root
- For each character c in the key (from left to right):
 - If there is no child node corresponding to c , return false
 - Descend into the child node corresponding to c
- If the resulting node is a finishing node, then return true, otherwise return false

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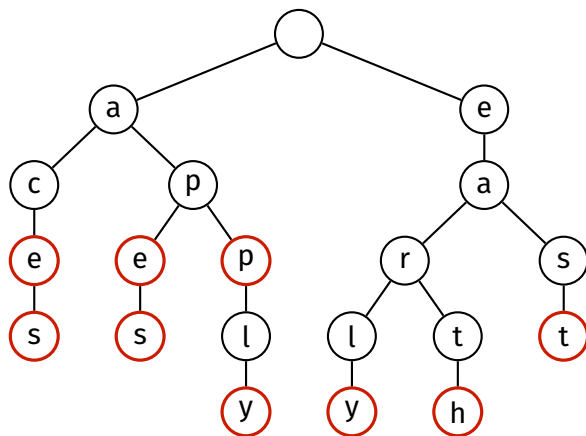
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Search for "early"



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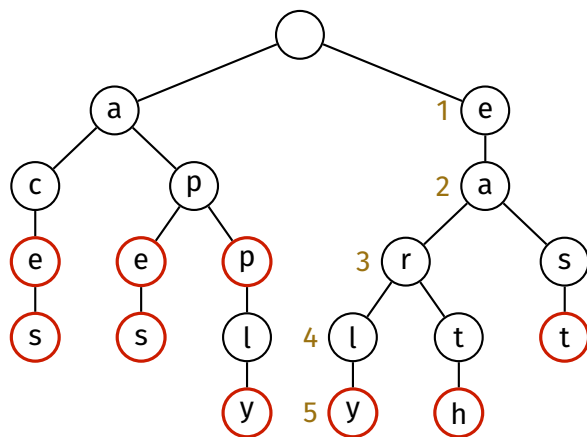
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Found!

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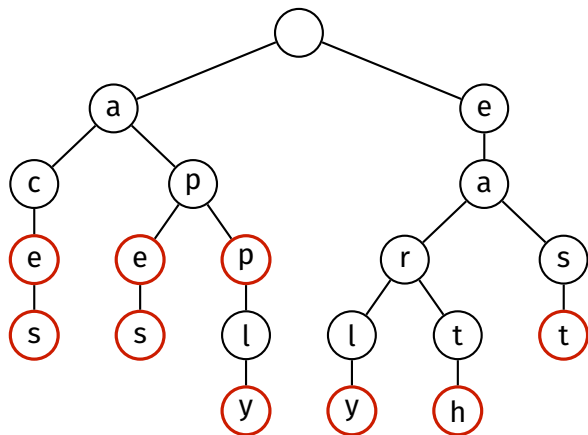
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Search for "apple"



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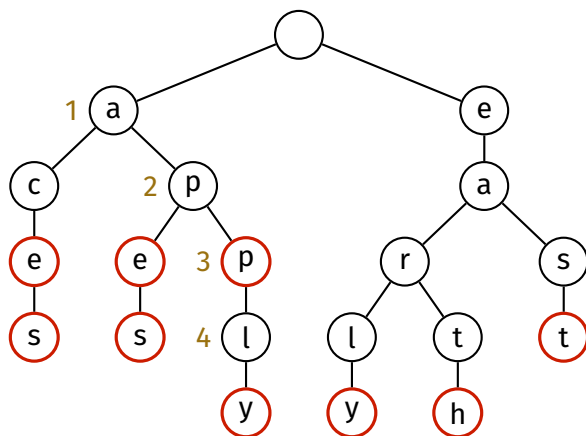
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Search for "apple"



Not found - node for "appl" has no child node for 'e'

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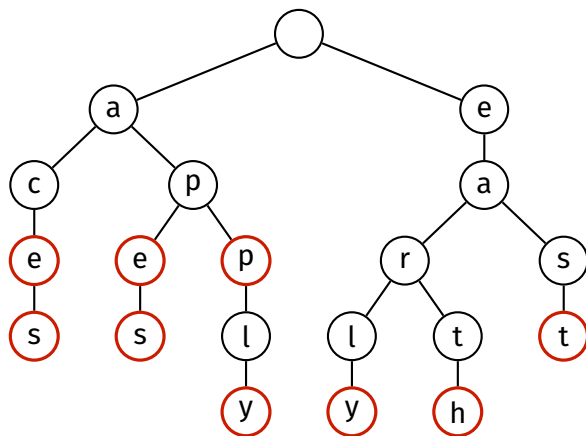
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Search for "ear"



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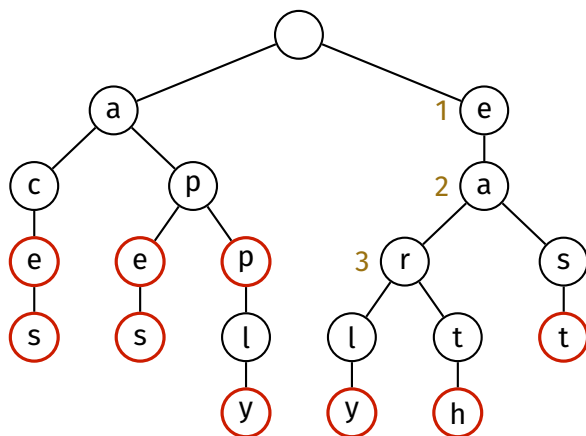
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Search for "ear"



Not found - node for "ear" is not a finishing node

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Trie search can be implemented recursively.

```
trieSearch(t, key):  
    Inputs: trie t  
              key of length m  
    Output: true if key is in t  
              false otherwise  
  
    if t is empty:  
        return false  
    else if m = 0:  
        return t->finish = true  
    else:  
        first = key[0]  
        rest = key[1..m - 1]  
        return trieSearch(t->children[first], rest)
```

EXERCISE Try writing an iterative version.

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Deletion is trickier...

- Can simply find node corresponding to given key and mark it as a non-finishing node
- ...but this can leave behind dead branches
 - i.e., branches that don't contain any finishing nodes
 - dead branches waste memory

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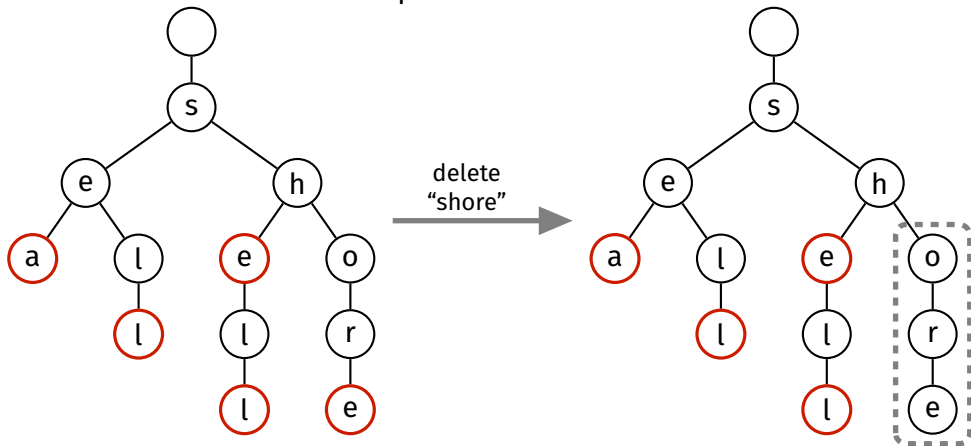
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Example of dead branch:



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Process for deletion:

- Find node corresponding to given key
 - If node doesn't exist, do nothing
- Mark the node as a non-finishing node
- While current node is not a finishing node and has no child nodes:
 - Delete current node and move up to parent
 - Handled recursively

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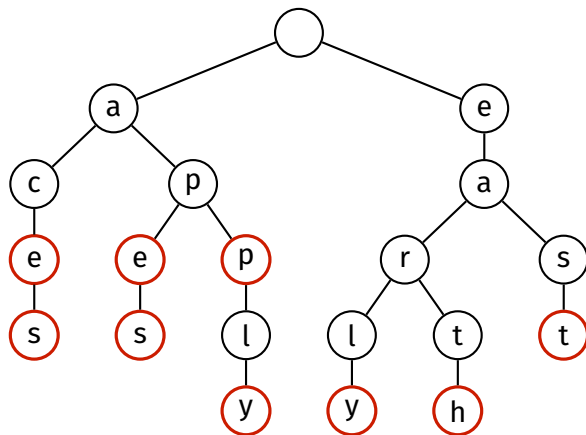
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Delete "ace"



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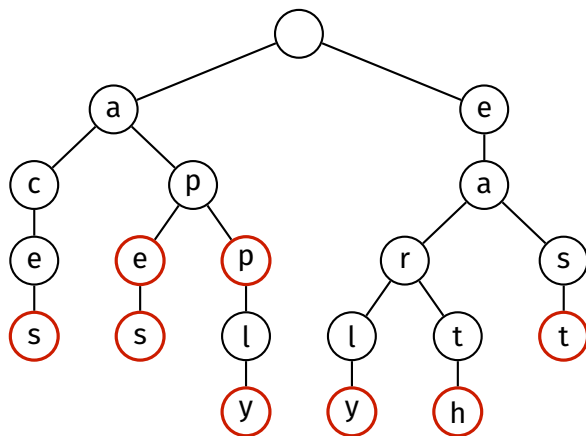
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Deleted - marked node for "ace" as a non-finishing node

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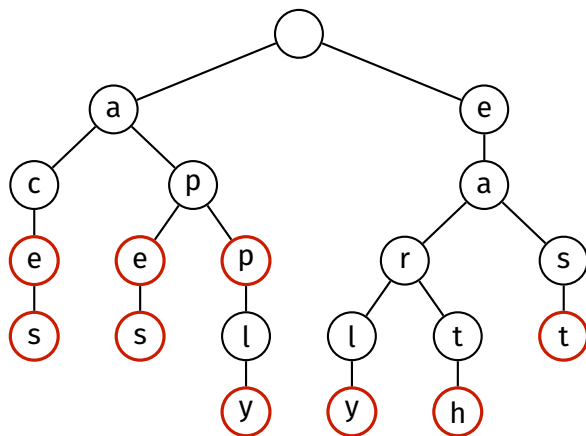
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Delete "apply"



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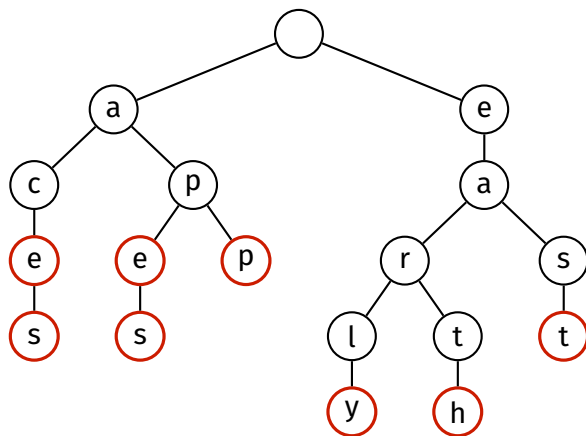
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Delete "apply"



Deleted - deleted nodes corresponding to "apply" and "appl"

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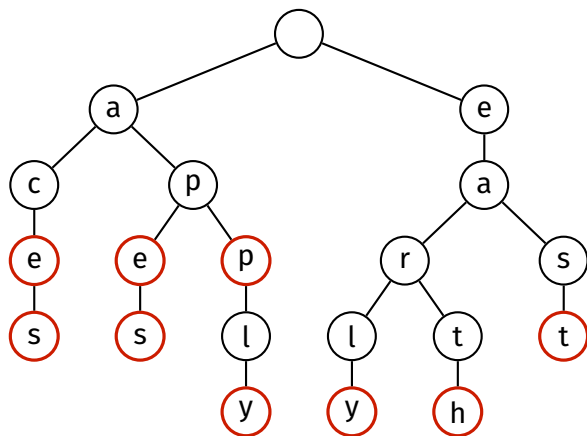
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Delete "earth"



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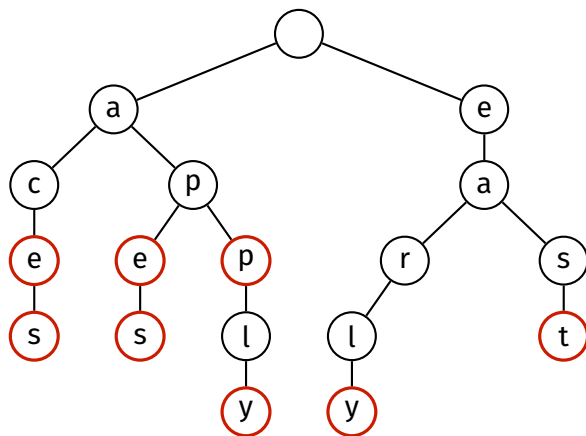
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Delete "earth"



Deleted - deleted nodes corresponding to "earth" and "earth"

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Trie deletion is implemented recursively.

```
trieDelete(t, key):  
    Inputs: trie t  
              key of length m  
    Output: t with key deleted  
  
    if t is empty:  
        return t  
    else if m = 0:  
        t->finish = false  
    else:  
        first = key[0]  
        rest = key[1..m - 1]  
        t->children[first] = trieDelete(t->children[first], rest)  
  
    if t->finish = false and t has no child nodes:  
        return NULL  
    else:  
        return t
```

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Analysis of standard trie:

- $O(m)$ insertion, search and deletion
 - where m is the length of the given key
 - each of these needs to examine at most m nodes
- $O(nR)$ space
 - where n is the total number of characters in all keys
 - where R is the size of the underlying alphabet (e.g., 26)

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Simple trie representation consumes an enormous amount of memory

- Each node contains ALPHABET_SIZE pointers
 - If keys are alphabetic, then this is 26 pointers...
 - ...which is $8 \times 26 = 208$ bytes on a 64-bit machine!
 - If keys can contain any ASCII character, then this is 128 pointers!
- Even if trie contains many keys, most child pointers will be unused

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Linked list of
children

Binary tree

Alphabet reduction

Compressed tries

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Different representations exist to reduce memory usage at the cost of increased running time:

- Use a singly linked list to store child nodes
- Alphabet reduction - break each character into smaller chunks, and treat these chunks as the characters

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One technique to reduce memory usage:

Have each node store a linked list of its children
instead of an array of `ALPHABET_SIZE` pointers

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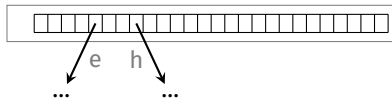
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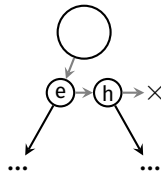
```
struct node {  
    struct child *children;  
    bool finish;  
    Data data;  
};
```

```
struct child {  
    char c;  
    struct node *node;  
    struct child *next;  
};
```

Instead of:



We have:



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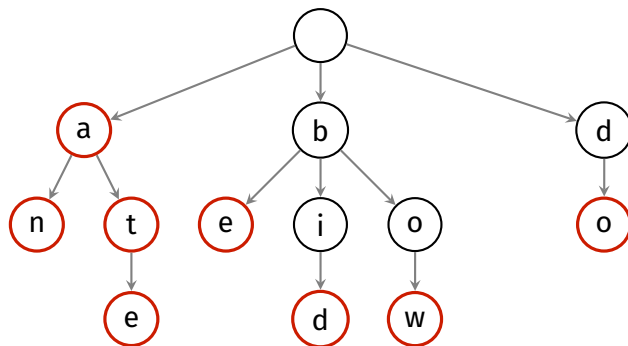
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Consider the following trie:



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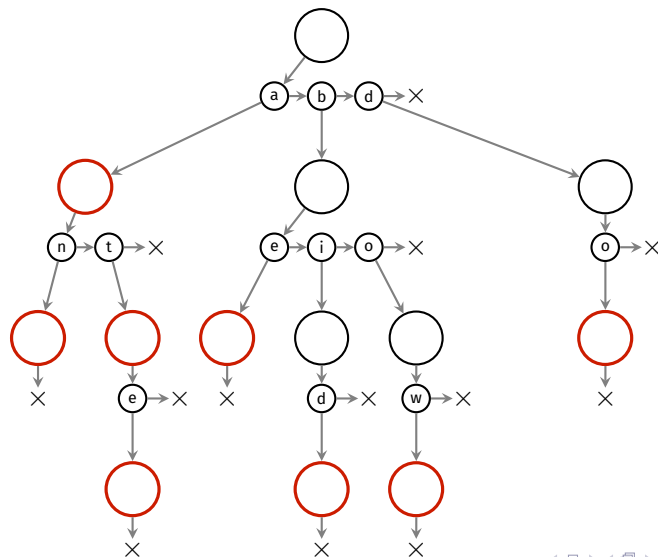
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Its concrete representation:



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We can simplify this representation
by merging each linked list node with its corresponding trie node

This produces the left-child right-sibling **binary tree** representation

```
struct node {  
    char c;  
    struct node *children;  
    struct node *sibling;  
    bool finish;  
    Data data;  
};
```

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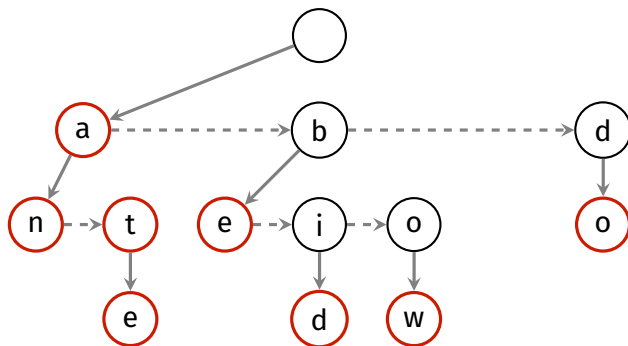
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Concrete representation of above trie:



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Analysis:

- This representation uses much less space
 - Each node just stores one extra pointer to its sibling instead of `ALPHABET_SIZE` pointers
- But this is at the expense of running time
 - Need to traverse up to `ALPHABET_SIZE` nodes before reaching desired child

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Another technique to reduce memory usage:
alphabet reduction

Break each 8-bit character into two 4-bit nybbles

This reduces the branching factor,
i.e., the number of pointers in each node

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For example, the word “sea” consists of the following bytes:

s	e	a
01110011	01100101	01100001

We break it into 4-bit nybbles like so:

s		e		a	
01110011		01100101		01100001	
0111	0011	0110	0101	0110	0001

Instead of storing the word “sea”, we now insert the following word:

0111 0011 0110 0101 0110 0001

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Analysis:

- This representation uses much less space
 - Much fewer pointers per node
- But this is at the expense of running time
 - Path to each key is twice as long - lookups need to visit twice as many nodes

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Another technique to reduce memory usage:
use a **compressed trie**

In a compressed trie, each node contains ≥ 1 character

Obtained by merging non-branching chains of nodes
Specifically, non-finishing nodes with only one child are merged with their child

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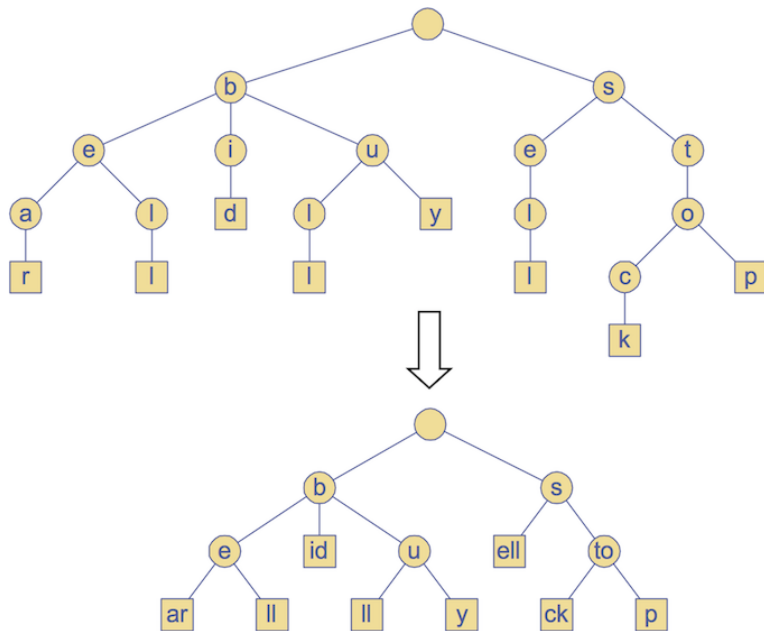
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Word finding

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Idea:

Given a document, preprocess it
by storing all words in a trie,
and for each word, store the location of all its occurrences

When user searches for a word,
can query the trie instead of scanning entire document

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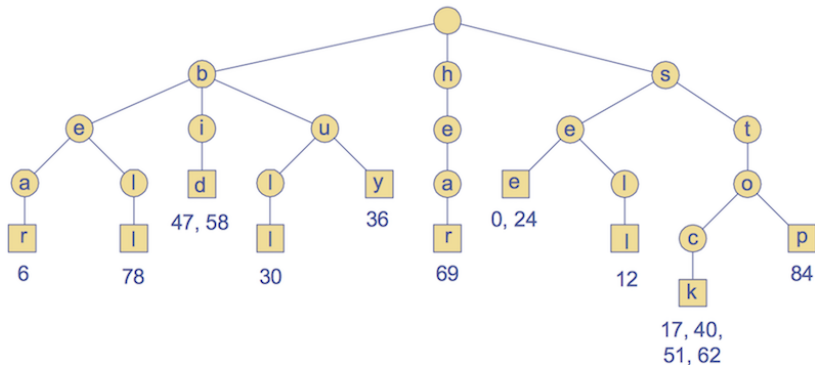
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Predictive text

Given a series of button presses (e.g., on a keypad), where each button can represent multiple letters, find all possible matching words



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<https://forms.office.com/r/aPF09YHZ3X>



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Insertion example

Insert the following words into an initially empty trie:

sea shell sell shore she

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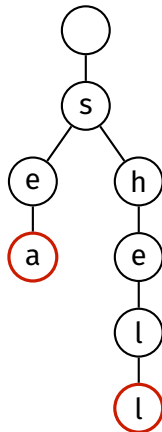
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Insert the following words into an initially empty trie:

sea shell sell shore she



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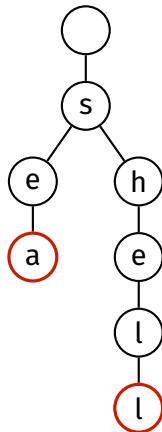
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Insert the following words into an initially empty trie:

sea shell **sell** shore she



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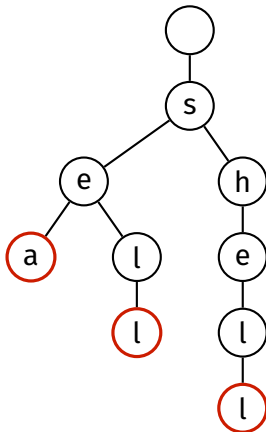
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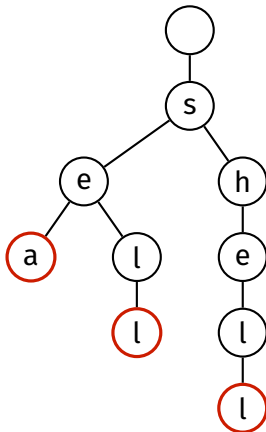
Insert the following words into an initially empty trie:

sea shell **sell** shore she



Insert the following words into an initially empty trie:

sea shell sell **shore** she



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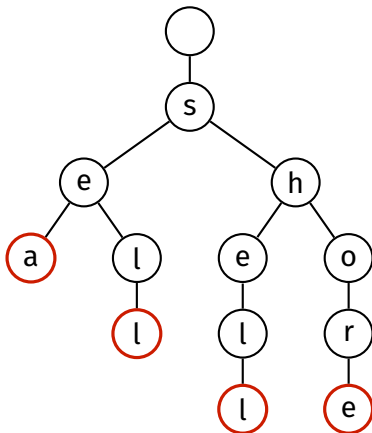
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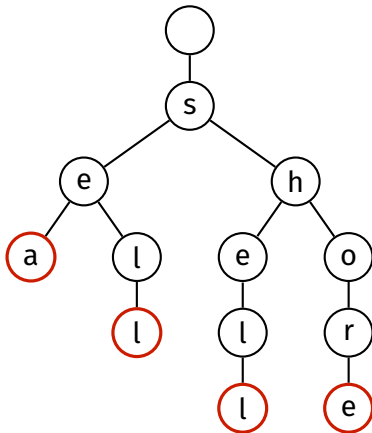
Insert the following words into an initially empty trie:

sea shell sell **shore** she



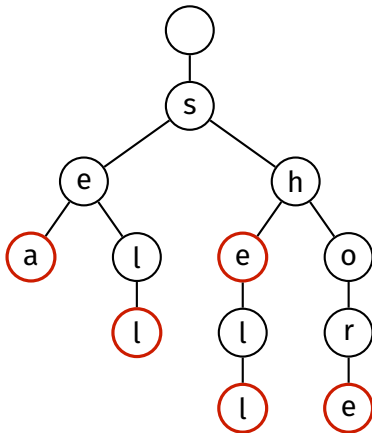
Insert the following words into an initially empty trie:

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<https://forms.office.com/r/zEqxUXvmLR>

