# COMP2521 24T3
## Binary Search Trees

Sushmita Ruj
cs2521@cse.unsw.edu.au

trees
binary search trees
binary search tree operations

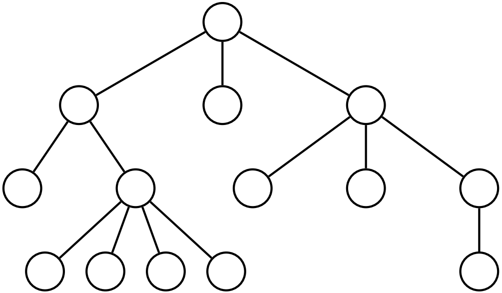
# Trees

# Trees

A tree is a hierarchical data structure
consisting of a set of connected nodes where:

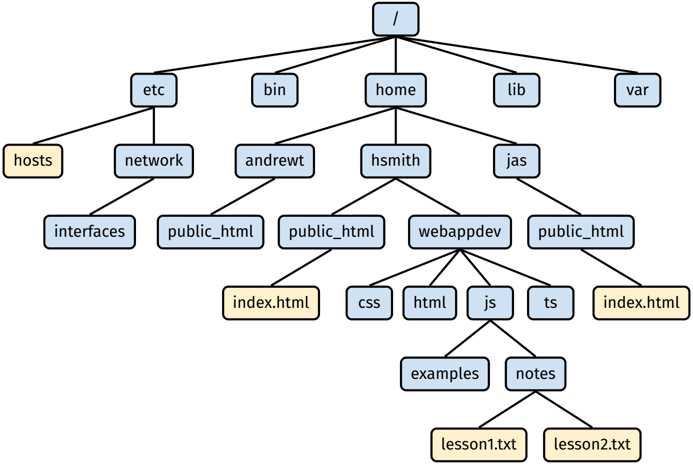
Each node may have multiple other nodes as children
(depending on the type of tree)

Each node is connected to one parent *except* the root node



# Trees

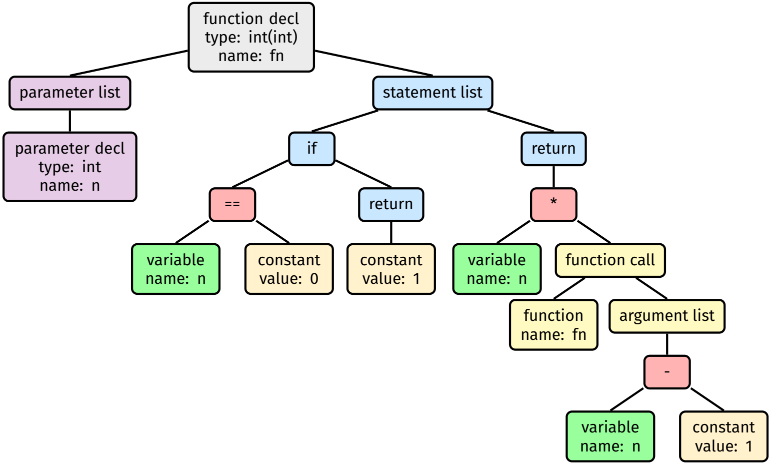
## Example - File System Tree

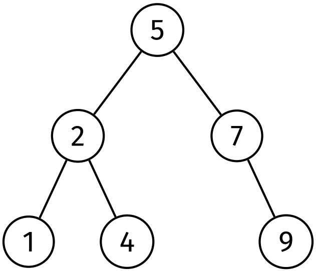


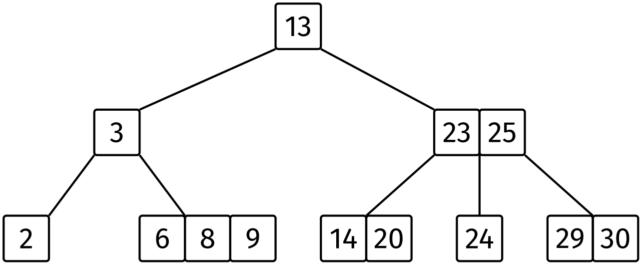

Source: https://www.openbookproject.net/tutorials/getdown/unix/lesson2.html

# Trees

## Example - Abstract Syntax Tree

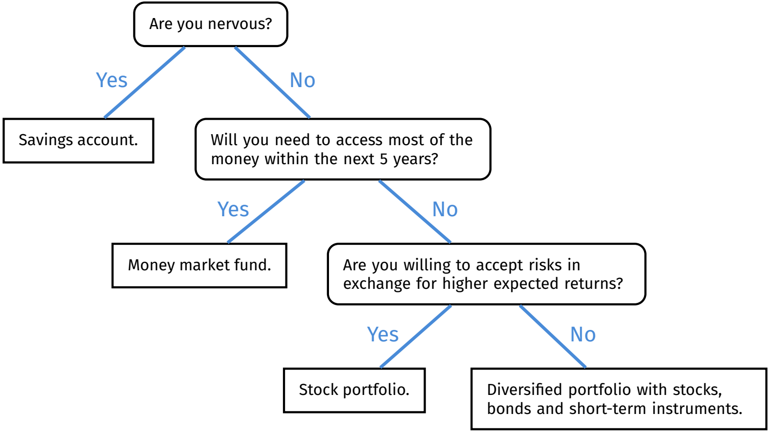



5
2
7
1
4
9


13
3
23 25
2
6 8 9
14 20
24
29 30

# Trees

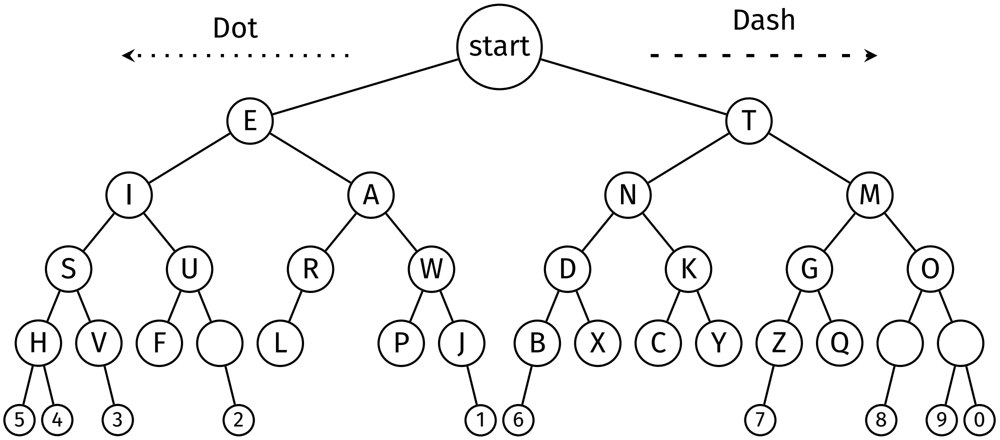
## Example - Decision Tree



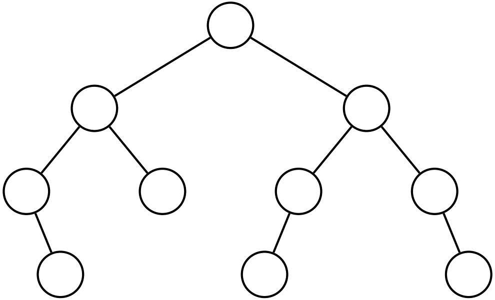

Source: “Data Structures and Algorithms in Java” (6th ed) by Goodrich et al.

# Trees

## Example - Decoding Morse Code



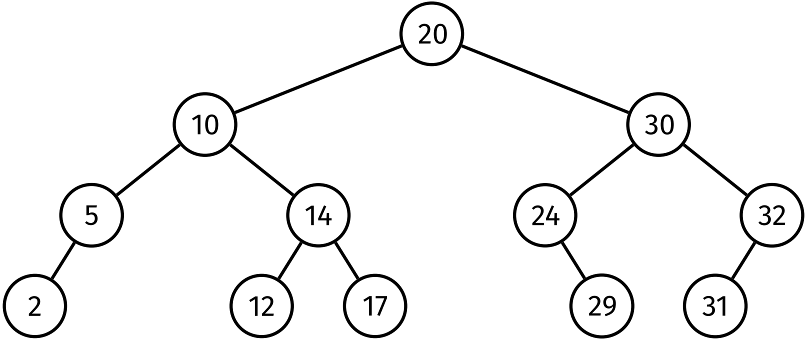
A binary tree is a tree where
each node can have up to two child nodes,
referred to as the left child and the right child.



# Binary Search Trees

A binary search tree is an ordered binary tree, where *for each node*:

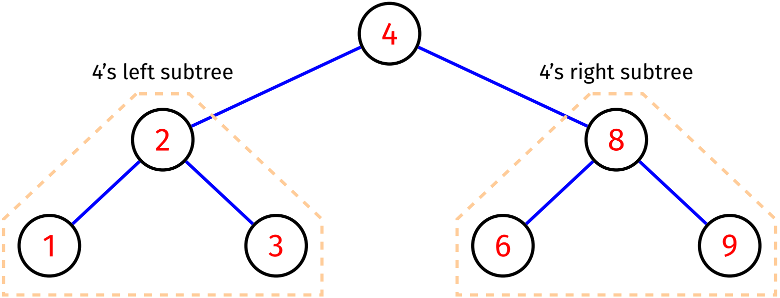
- All values in the left subtree are less than the value in the node
- All values in the right subtree are greater than the value in the node



# Binary Search Trees

A binary search tree is either:

- empty; or
- consists of a node with two subtrees
  - left and right subtrees are also BSTs (recursive)



Why use binary search trees?

Search is an extremely common operation in computing:

- selecting records in databases
- searching for pages on the web

Typically, there is a very large amount of data (very many items)

We need a more efficient way to search and maintain large amounts of data.

We've explored multiple approaches for searching:

- Ordered array
  - Searching/finding insertion point is $O(\log n)$ due to binary search
  - Inserting is $O(n)$ due to the need to shift items to preserve sortedness
- Ordered linked list
  - Searching/finding insertion point is $O(n)$ due to the nature of linked lists
  - Inserting *once we have found the insertion point* is $O(1)$ as there is no need to shift

# Binary Search Trees

## Why?

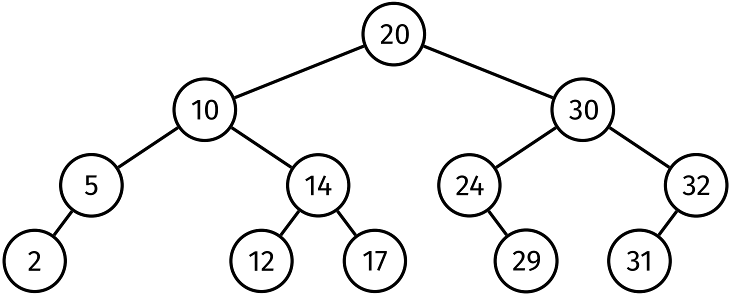
Binary search trees are efficient to search *and* maintain:

- Searching in a binary search tree is similar to how binary search works
- A binary search tree is a linked data structure (like a linked list), so there is no need to shift elements when inserting/deleting

# Binary Search Trees



| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 5 | 10 | 12 | 14 | 17 | 20 | 24 | 29 | 30 | 31 | 32 |



# Binary Search Trees

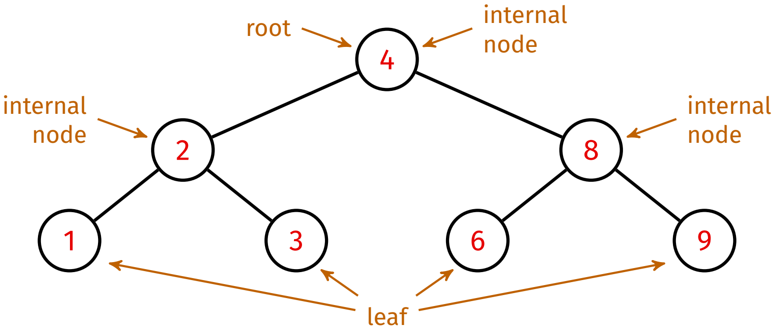
## Terminology



The root node is the node with no parent node.

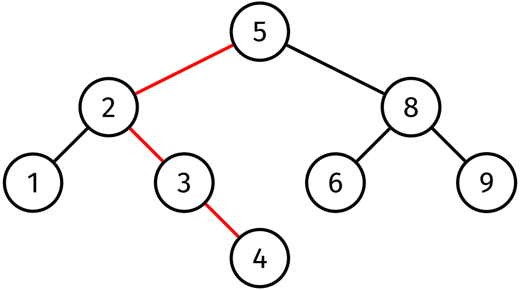
A leaf node is a node that has no child nodes.

An internal node is a node that has at least one child node.



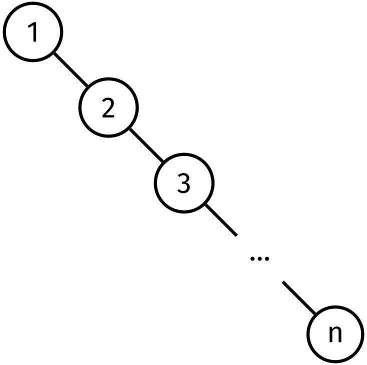
**Height** of a tree: Maximum path length from the root node to a leaf

- The height of an empty tree is considered to be -1
- The height of the following tree is 3



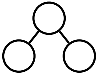
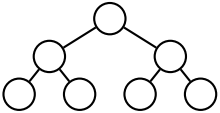
For a tree with $n$ nodes:

The maximum possible height is $n - 1$



For a tree with $n$ nodes:

The minimum possible height is $\lfloor \log_2 n \rfloor$

| $n$ | minimum height = $\lfloor \log_2 n \rfloor$ | tree |
|---|---|---|
| 1 | 0 |  |
| 2-3 | 1 |  |
| 4-7 | 2 |  |
| ... | ... | ... |

For a given number of nodes, a tree is said to be **balanced** if it has (close to) minimal height, and **degenerate** if it has (close to) maximal height.

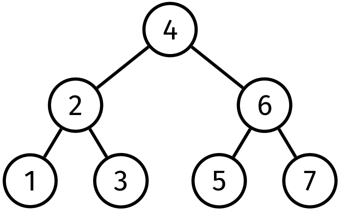
Binary trees are typically represented by node structures

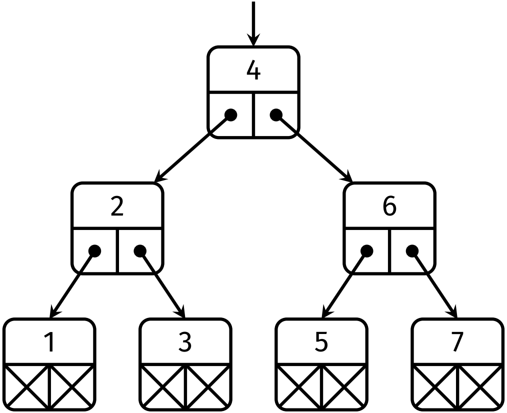
- Where each node contains a value and pointers to child nodes

```
struct node {
    int item;
    struct node *left;
    struct node *right;
};
```

# Binary Search Trees

## Concrete Representation



Key operations on binary search trees:

- Insert
- Search
- Traversal
- Join
- Delete

The height $h$ of a binary search tree determines the efficiency of many operations, so we will use both $n$ and $h$ in our analyses.

## Insertion

`bstInsert(`$t$`, `$v$`)`

Given a BST $t$ and a value $v$,
insert $v$ into the BST
and return the root of the updated BST

# BST Insertion

Insertion is straightforward:

- Start at the root
- Compare value to be inserted with value in the node
  - If value being inserted is less, descend to left child
  - If value being inserted is greater, descend to right child
- Repeat until...
  you have to go left/right but current node has no left/right child
  - Create new node and attach to current node

# BST Insertion

## Method

Recursive method:

- $t$ is empty
  $\Rightarrow$ make a new node with $v$ as the root of the new tree
- $v <$ `t->item`
  $\Rightarrow$ insert $v$ into $t$'s left subtree
- $v >$ `t->item`
  $\Rightarrow$ insert $v$ into $t$'s right subtree
- $v =$ `t->item`
  $\Rightarrow$ tree unchanged (assuming no duplicates)

EXERCISE Try writing an iterative version.

# BST Insertion

## Example 1

Insert the following values into an empty tree:

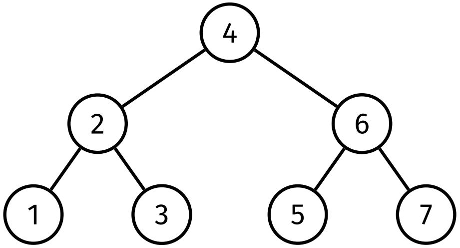
4 2 6 5 1 7 3

# BST Insertion

## Example 1

Insert the following values into an empty tree:

4 2 6 5 1 7 3



# BST Insertion

## Example 2

Insert the following values into an empty tree:

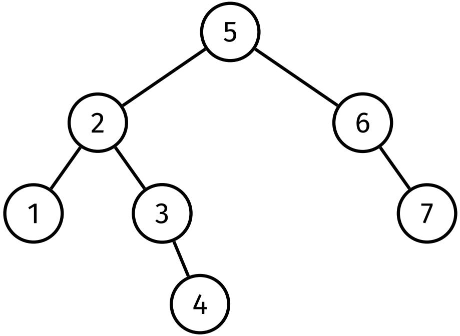
5 6 2 3 4 7 1

# BST Insertion

## Example 2

Insert the following values into an empty tree:

5 6 2 3 4 7 1



# BST Insertion

## Example 3

Insert the following values into an empty tree:

1 2 3 4 5 6 7

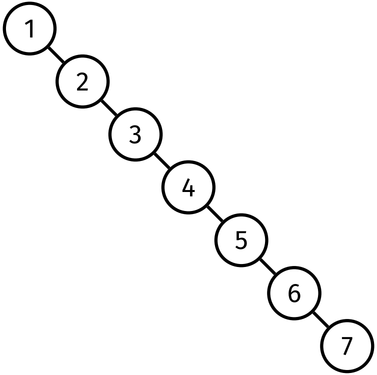
# BST Insertion

## Example 3



Insert the following values into an empty tree:

1 2 3 4 5 6 7



```
bstInsert(t, v):
    Input:  tree t, value v
    Output: t with v inserted

    if t is empty:
        return new node containing v
    else if v < t->item:
        t->left = bstInsert(t->left, v)
    else if v > t->item:
        t->right = bstInsert(t->right, v)

    return t
```

Analysis:

- At most one node is examined on each level
- Number of operations performed per node is constant
- Therefore, the worst-case time complexity of insertion is $O(h)$ where $h$ is the height of the BST

## Search

bstSearch($t$, $v$)

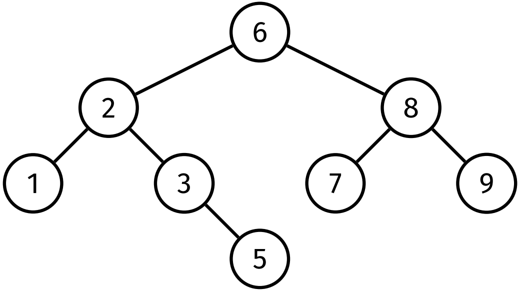
Given a BST $t$ and a value $v$,
return true if $v$ is in the BST
and false otherwise

Recursive method:

- $t$ is empty:
  $\Rightarrow$ return false
- $v <$ `t->item`
  $\Rightarrow$ search for $v$ in $t$'s left subtree
- $v >$ `t->item`
  $\Rightarrow$ search for $v$ in $t$'s right subtree
- $v =$ `t->item`
  $\Rightarrow$ return true

EXERCISE Try writing an iterative version.

Search for 4 and 7 in the following BST:



# BST Search

## Pseudocode

```
bstSearch(t, v):
    Input:  tree t, value v
    Output: true if v is in t
            false otherwise

    if t is empty:
        return false
    else if v < t->item:
        return bstSearch(t->left, v)
    else if v > t->item:
        return bstSearch(t->right, v)
    else:
        return true
```

Analysis:

- At most one node is examined on each level
- Number of operations performed per node is constant
- Therefore, the worst-case time complexity of search is $O(h)$ where $h$ is the height of the BST

**Traversal**

Given a BST,
visit every node of the tree

# Tree Traversal

There are 4 common ways to traverse a binary tree:

1. Pre-order (NLR):
   visit root, then traverse left subtree, then traverse right subtree
2. In-order (LNR):
   traverse left subtree, then visit root, then traverse right subtree
3. Post-order (LRN):
   traverse left subtree, then traverse right subtree, then visit root
4. Level-order:
   visit root, then its children, then their children, and so on

## Pseudocode:

```
preorder(t):
    Input: tree t

    if t is empty:
        return

    visit(t)
    preorder(t->left)
    preorder(t->right)
```

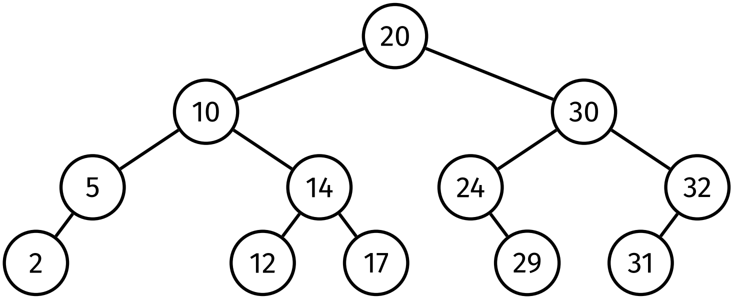
```
inorder(t):
    Input: tree t

    if t is empty:
        return

    inorder(t->left)
    visit(t)
    inorder(t->right)
```

```
postorder(t):
    Input: tree t

    if t is empty:
        return

    postorder(t->left)
    postorder(t->right)
    visit(t)
```

Note:
Level-order traversal is difficult to implement recursively.
It is typically implemented using a queue.

# Tree Traversal

## Example: Binary Search Tree



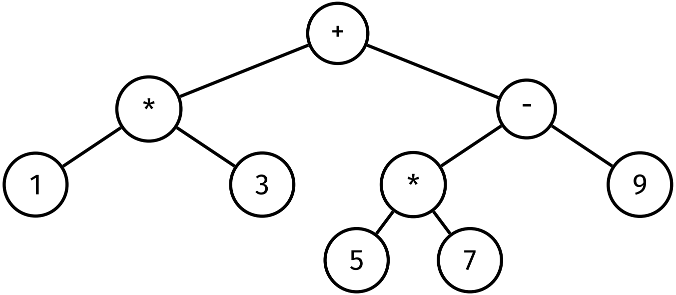

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Pre-order** | 20 | 10 | 5 | 2 | 14 | 12 | 17 | 30 | 24 | 29 | 32 | 31 |
| **In-order** | 2 | 5 | 10 | 12 | 14 | 17 | 20 | 24 | 29 | 30 | 31 | 32 |
| **Post-order** | 2 | 5 | 12 | 17 | 14 | 10 | 29 | 24 | 31 | 32 | 30 | 20 |
| **Level-order** | 20 | 10 | 30 | 5 | 14 | 24 | 32 | 2 | 12 | 17 | 29 | 31 |

# Tree Traversal

## Example: Expression Tree



Expression tree for 1 * 3 + (5 * 7 - 9)




| | |
|---|---|
| **Pre-order** | + * 1 3 - * 5 7 9 |
| **In-order** | 1 * 3 + 5 * 7 - 9 |
| **Post-order** | 1 3 * 5 7 * 9 - + |

# Tree Traversal

## Applications

Pre-order traversal:

- Useful for reconstructing a tree

In-order traversal:

- Useful for traversing a BST in ascending order

Post-order traversal:

- Useful for evaluating an expression tree
- Useful for freeing a tree

Level-order traversal:

- Useful for printing a tree

Analysis:

- Each node is visited once
- Hence, time complexity of tree traversal is $O(n)$, where $n$ is the number of nodes

# BST Join

## Join

`bstJoin(`$t_1$`, `$t_2$`)`

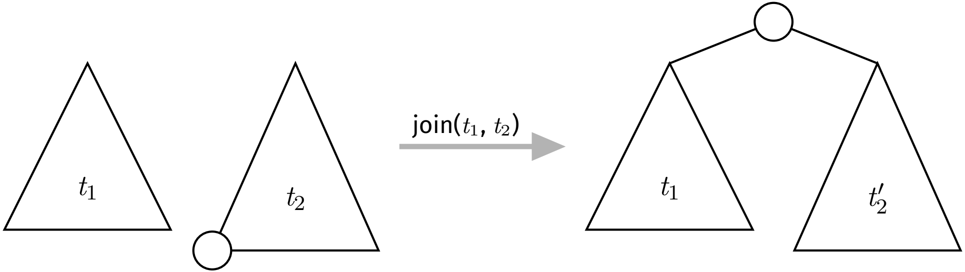
Given two BSTs $t_1$ and $t_2$
where $\max(t_1) < \min(t_2)$
return a BST containing all items from $t_1$ and $t_2$

# BST Join

## Method

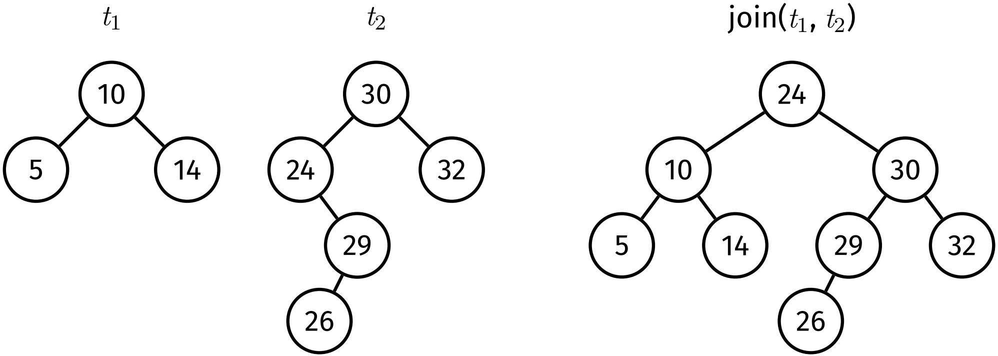
Method:

1. Find the minimum node $min$ in $t_2$
2. Replace $min$ by its right subtree (if it exists)
3. Elevate $min$ to be the new root of $t_1$ and $t_2$



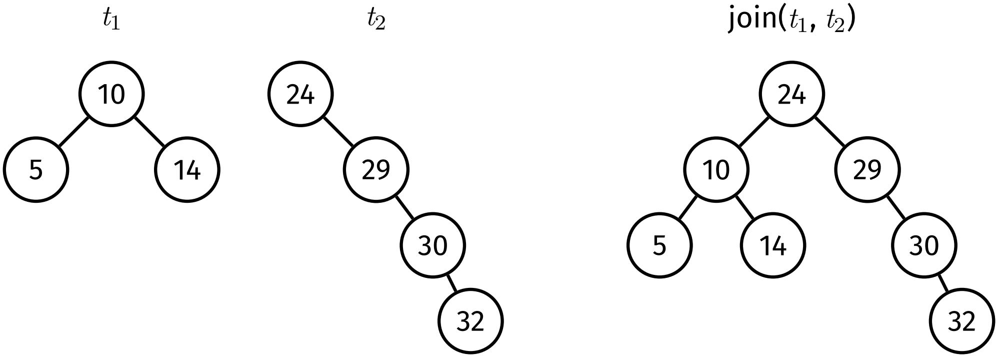
# BST Join

## Example 1



# BST Join

## Example 2



```
bstJoin(t1, t2):
    Input:  trees t1, t2
    Output: t1 and t2 joined together

    if t1 is empty:
        return t2
    else if t2 is empty:
        return t1
    else:
        curr = t2
        parent = NULL
        while curr->left ≠ NULL:
            parent = curr
            curr = curr->left

        if parent ≠ NULL:
            parent->left = curr->right
            curr->right = t2

        curr->left = t1
        return curr
```

Analysis:

- The join algorithm simply finds the minimum node in $t_2$
- Thus, at most one node is visited per level of $t_2$
- Therefore, the worst-case time complexity of join is $O(h_2)$ where $h_2$ is the height of $t_2$

## Deletion

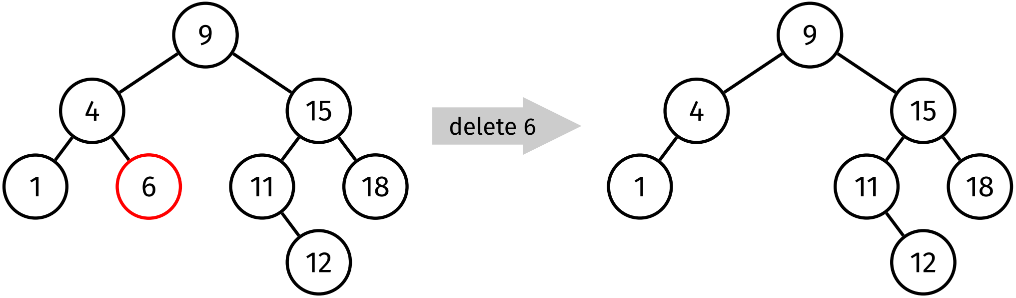
`bstDelete(`$t$`,` $v$`)`

Given a BST $t$ and a value $v$
delete $v$ from the BST
and return the root of the updated BST

Recursive method:

- $t$ is empty:
  $\Rightarrow$ result is empty
- $v <$ $t$`->item`
  $\Rightarrow$ delete $v$ from $t$'s left subtree
- $v >$ $t$`->item`
  $\Rightarrow$ delete $v$ from $t$'s right subtree
- $v =$ $t$`->item`
  $\Rightarrow$ three sub-cases:
  - $t$ is a leaf
    $\Rightarrow$ result is empty tree
  - $t$ has one subtree
    $\Rightarrow$ replace with subtree
  - $t$ has two subtrees
    $\Rightarrow$ join the two subtrees

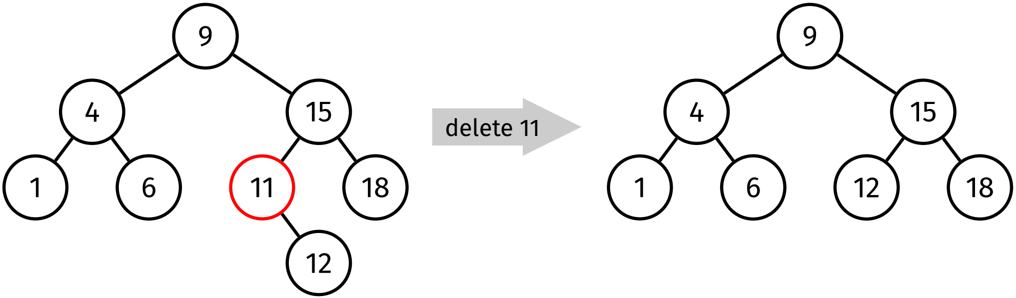
If the node being deleted is a leaf, then the result is an empty tree

# BST Deletion

## Examples

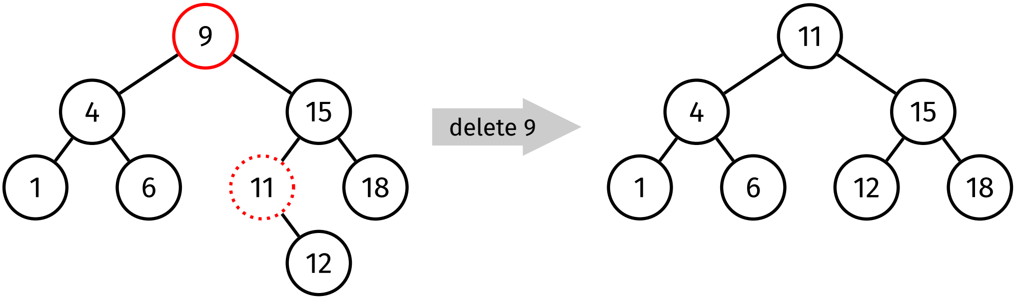
Node to be deleted has one subtree



# BST Deletion

## Examples

Node to be deleted has two subtrees



# BST Deletion

## Pseudocode

```
bstDelete(t, v):
    Input:  tree t, value v
    Output: t with v deleted

    if t is empty:
        return empty tree
    else if v < t->item:
        t->left = bstDelete(t->left, v)
    else if v > t->item:
        t->right = bstDelete(t->right, v)
    else:
        if t->left is empty:
            new = t->right
        else if t->right is empty:
            new = t->left
        else:
            new = bstJoin(t->left, t->right)

        free(t)
        t = new

    return t
```

Analysis:

- The deletion algorithm traverses down just one branch
  - First, the item being deleted is found
  - If the item exists and has two subtrees, its successor is found
- Thus, at most one node is visited per level
- Therefore, the worst-case time complexity of deletion is $O(h)$ where $h$ is the height of the BST

# BST Exercises

- `bstFree`
  free a tree
- `bstSize`
  return the size of a tree
- `bstHeight`
  return the height of a tree
- `bstPrune`
  given values $lo$ and $hi$, remove all values outside the range $[lo, hi]$

# Feedback

https://forms.office.com/r/zEqxUXvmLR