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COMP2521 24T3

Analysis of Algorithms

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Slides adapted from those by Kevin 24T1

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What are the requirements for an algorithm?

- Correctness
- Readability
- Robustness
- Efficiency

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- Program efficiency is critical for many applications:
 - Finance, robotics, games, database systems, AI, ...
- We may want to compare programs to decide which one to use
- We may want to determine whether a program will be “fast enough”

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- Each line of code we execute, takes time.
- Each variable we create, takes up space.

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For an application to be useful in real-world, it must:

- Run “fast enough”
- Not take up too much space

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In computer science:

- Sometimes you will need to make a tradeoff:
 - sacrifice space for speed
 - sacrifice speed for space
- Example
 - Array & Linked list
 - If you need fast access to elements by index and don't expect frequent insertions or deletions, an array might be better, favouring time over space.
 - If you expect frequent insertions and deletions and don't require fast access by index, a linked list might be better, favouring space over time.

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What determines how fast a program runs?

- The operating system?
- Compilers?
- Hardware?
 - E.g., CPU, GPU, cache
- Load on the machine?

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As software engineers, there are many factors influencing how usable our applications will be.

- Factors outside of our control:
 - The machine our code will be running on
 - How much data the application will need to handle
- Factors within our control:
 - Which data structure(s) we use → how much space is needed to store and manipulate the provided data
 - Which algorithm(s) we use → how much time (and space) it takes to process the provided data

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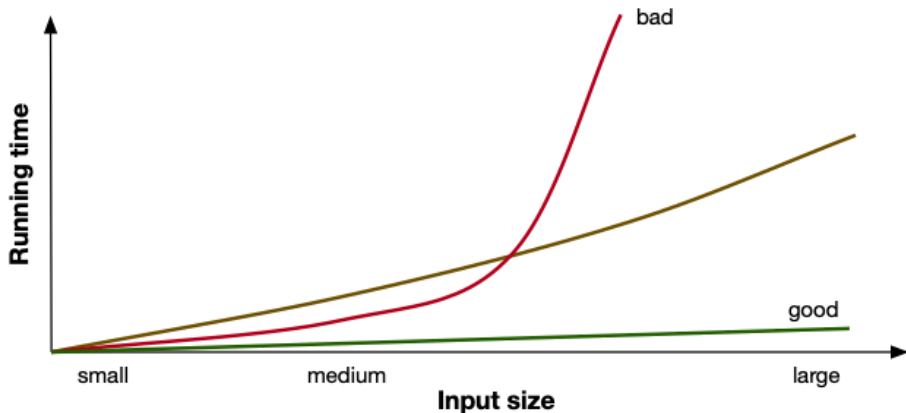
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- The running time of an algorithm tends to be a function of input size
- Typically: larger input \Rightarrow longer running time
 - Small inputs: fast running time, regardless of algorithm
 - Larger inputs: slower, but how much slower?



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- **Best-case performance**
 - Not very useful
 - Usually only occurs for specific types of input
- **Average-case performance**
 - Difficult; need to know how the program is used
- **Worst-case performance**
 - Most important; determines how long the program could possibly run

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Time complexity is
the amount of time it takes to run an algorithm,
as a function of the input size

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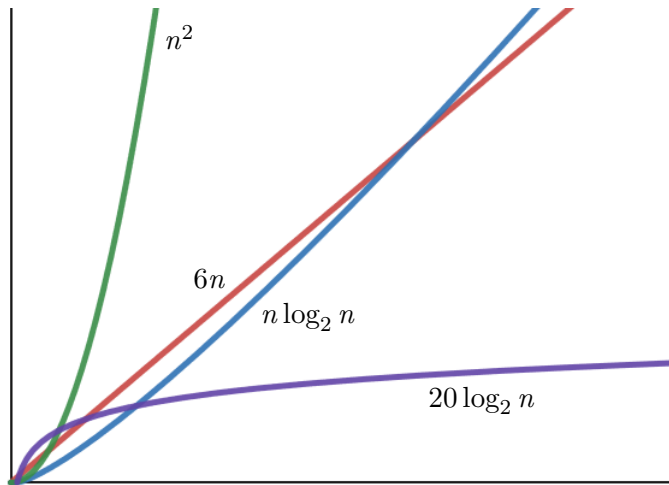
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Example functions:



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The time complexity of an algorithm can be analysed in two ways:

- Empirically: Measuring the time that a program implementing the algorithm takes to run
- Theoretically: Counting the number of operations or “steps” performed by the algorithm as a function of input size

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The search problem:

Given an array of size n and a value,
return the index containing the value if it exists,
otherwise return -1.

[0]	[1]	[2]	[3]	[4]	[5]	[6]
2	16	11	1	9	4	15

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- 1 Write a program that implements the algorithm
- 2 Run the program with inputs of varying size and composition
- 3 Measure the running time of the algorithm
- 4 Plot the results

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We can measure the running time of an algorithm using *clock(3)*.

- The *clock()* function determines the amount of processor time used since the start of the process.

```
#include <time.h>

clock_t start = clock();
// algorithm code here...
clock_t end = clock();

double seconds = (double)(end - start) / CLOCKS_PER_SEC;
```


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Absolute times will differ
between machines, between languages
...so we're not interested in absolute time.

We are interested in the *relative* change
as the input size increases

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Let's empirically analyse the following search algorithm:

```
// Returns the index of the given value in the array if it exists,  
// or -1 otherwise  
int linearSearch(int arr[], int size, int val) {  
    for (int i = 0; i < size; i++) {  
        if (arr[i] == val) {  
            return i;  
        }  
    }  
    return -1;  
}
```

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Sample results:

Input Size	Running Time
1,000,000	0.002
10,000,000	0.023
100,000,000	0.240
200,000,000	0.471
300,000,000	0.702
400,000,000	0.942
500,000,000	1.196
1,000,000,000	2.384

The worst-case running time of linear search grows linearly as the input size increases.

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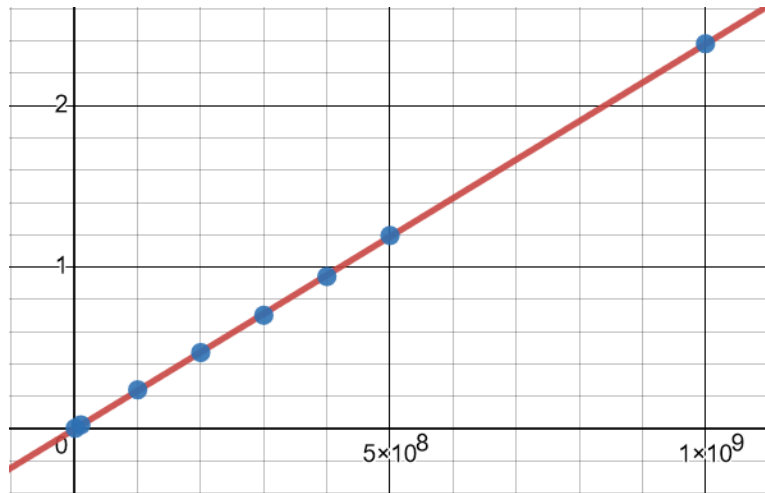
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- Requires implementation of algorithm
- Different choice of input data \Rightarrow different results
 - Choosing good inputs is extremely important
- Timing results affected by runtime environment
 - E.g., load on the machine
- In order to compare two algorithms...
 - Need “comparable” implementation of each algorithm
 - Must use same inputs, same hardware, same O/S, same load

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- Uses high-level description of algorithm (pseudocode)
 - Can use the code if it is implemented already
- Characterises running time as a function of input size
- Allows us to evaluate the efficiency of the algorithm
 - Independent of the hardware/software environment

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- Pseudocode is a plain language description of the steps in an algorithm
- Uses structural conventions of a regular programming language
 - if statements, loops
- Omits language-specific details
 - variable declarations
 - allocating/freeing memory

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Pseudocode for linear search:

```
linearSearch(A, val):  
    Input: array A of size n, value val  
    Output: index of val in A if it exists  
               -1 otherwise  
  
    for i from 0 up to n - 1:  
        if A[i] = val:  
            return i  
  
    return -1
```


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Every algorithm uses a core set of basic operations.

Examples:

- Assignment
- Indexing into an array
- Calling/returning from a function
- Evaluating an expression
- Increment/decrement

We call these operations **primitive** operations.

Assume that primitive operations take the same constant amount of time.

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How many primitive operations are performed by this line of code?

```
for (int i = 0; i < n; i++)
```

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How many primitive operations are performed by this line of code?

```
for (int i = 0; i < n; i++)
```

The assignment $i = 0$ occurs 1 time

The comparison $i < n$ occurs $n + 1$ times

The increment $i++$ occurs n times

Total: $1 + (n + 1) + n$ primitive operations

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By inspecting the pseudocode, we can determine the maximum number of primitive operations executed by an algorithm as a function of the input size.

```
linearSearch(A, val):  
    Input: array A of size n, value val  
    Output: index of val in A if it exists  
              -1 otherwise  
  
    for i from 0 up to n - 1:      1 + (n + 1) + n  
        if A[i] = val:              2n  
            return i  
  
    return -1                          1  
  
                                         -----  
                                         4n + 3 (total)
```

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Linear search requires $4n + 3$ primitive operations in the worst case.

If the time taken by a primitive operation is c , then the time taken by linear search in the worst case is $c(4n + 3)$.

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We are mainly interested in
how the running time of an algorithm changes
as the input size increases.

This is called the **asymptotic behaviour** of the running time.

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Asymptotic behaviour is not affected by lower-order terms.

- For example, suppose the running time of an algorithm is $4n + 100$.
- As n increases, the lower-order term (i.e., 100) becomes less significant (i.e., becomes a smaller proportion of the running time)

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Asymptotic behaviour is not affected by constant factors.

Example: Suppose the running time $T(n)$ of an algorithm is n^2 .

- What happens when we double the input size?

$$\begin{aligned}T(2n) &= (2n)^2 \\ &= 4n^2 \\ &= 4T(n)\end{aligned}$$

When we double the input size, the time taken quadruples.

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Example: Now suppose the running time $T(n)$ of an algorithm is $10n^2$.

- Now what happens when we double the input size?

$$\begin{aligned}T(2n) &= 10 \times (2n)^2 \\ &= 10 \times 4n^2 \\ &= 4 \times 10n^2 \\ &= 4T(n)\end{aligned}$$

When we double the input size, the time taken also quadruples!

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To summarise:

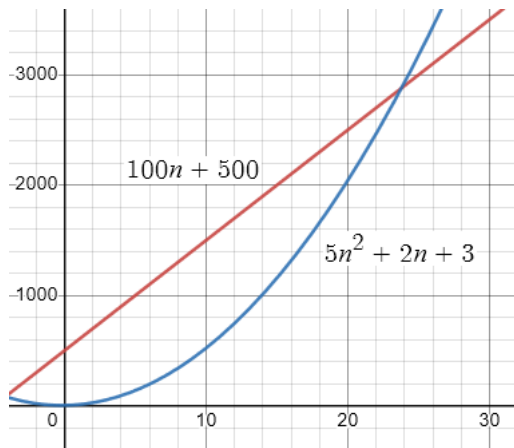
- Asymptotic behaviour is unaffected by lower-order terms
- Asymptotic behaviour is unaffected by constant factors

This means we can ignore lower-order terms and constant factors when characterising the asymptotic behaviour of an algorithm.

Examples:

- If $T(n) = 100n + 500$, ignoring lower-order terms and constant factors gives n
- If $T(n) = 5n^2 + 2n + 3$, ignoring lower-order terms and constant factors gives n^2

This also means that for sufficiently large inputs, the algorithm that has the running time with the highest-order term will always take longer.



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Big-Oh notation

is used to classify the asymptotic behaviour of an algorithm,
and this is how we usually express time complexity in this course.

For example, linear search is $O(n)$ in the worst case.

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Big-Oh notation allows us to easily compare the efficiency of algorithms

- For example, if algorithm A has a time complexity of $O(n)$ and algorithm B has a time complexity of $O(n^2)$, then we can say that for sufficiently large inputs, algorithm A will perform better.

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Formally, big-Oh is actually a notation used to describe the asymptotic relationship between functions.

Formally:

Given functions $f(n)$ and $g(n)$, we say that $f(n)$ is $O(g(n))$ if:

- There are positive constants c and n_0 such that:
 - $f(n) \leq c \cdot g(n)$ for all $n \geq n_0$

Informally:

Given functions $f(n)$ and $g(n)$, we say that $f(n)$ is $O(g(n))$ if for sufficiently large n , $f(n)$ is bounded above by some multiple of $g(n)$.

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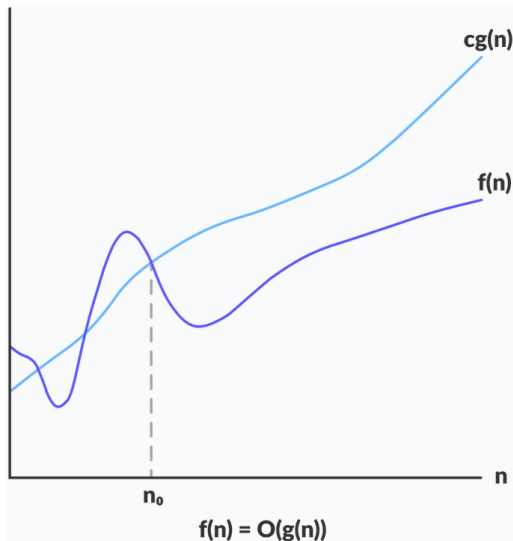
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 $f(n)$ is $O(g(n))$ if $f(n)$ is asymptotically **less than or equal** to $g(n)$ $f(n)$ is $\Omega(g(n))$ if $f(n)$ is asymptotically **greater than or equal** to $g(n)$ $f(n)$ is $\Theta(g(n))$ if $f(n)$ is asymptotically **equal** to $g(n)$

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Since time complexity is not affected by constant factors, instead of counting primitive operations, we can simply count line executions.

```
linearSearch(A, value):  
    Input: array  $A$  of size  $n$ , value  
    Output: index of value in  $A$  if it exists  
              -1 otherwise  
  
    for  $i$  from 0 up to  $n-1$ :       $n$   
        if  $A[i] = \text{value}$ :           $n$   
            return  $i$   
  
    return -1                        1  
                                     -----  
                                      $2n + 1$  (total)
```

Worst-case time complexity: $O(n)$

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To determine the worst-case time complexity of an algorithm:

- Determine the number of line executions performed in the worst case in terms of the input size
- Discard lower-order terms and constant factors
- The worst-case time complexity is then the big-Oh of the term that remains

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Commonly encountered functions in algorithm analysis:

- **Constant:** 1
- **Logarithmic:** $\log n$
- **Linear:** n
- **N-Log-N:** $n \log n$
- **Quadratic:** n^2
- **Cubic:** n^3
- **Exponential:** 2^n
- **Factorial:** $n!$

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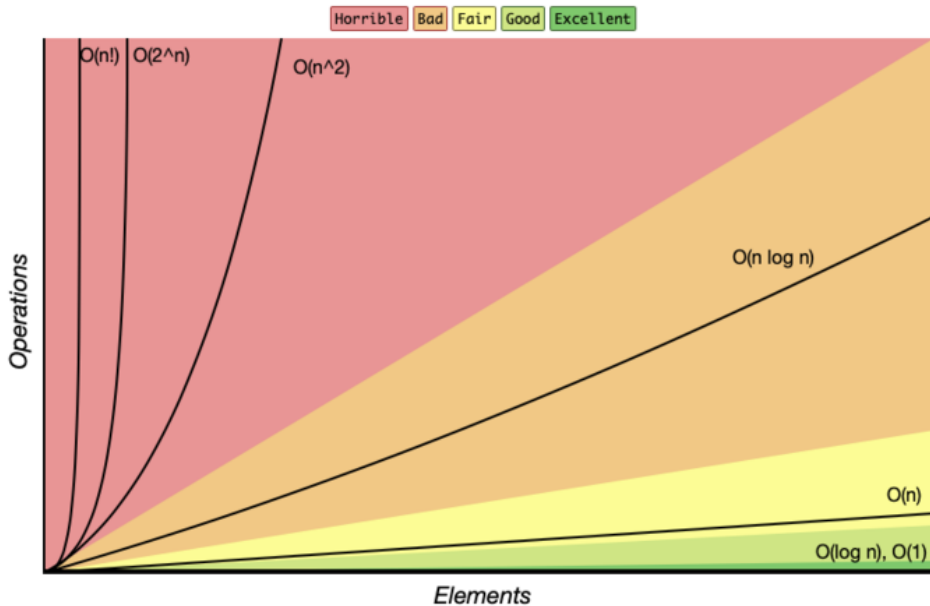
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Linear search requires $4n + 3$ primitive operations in the worst case.

Therefore, linear search is $O(n)$ in the worst case.

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Is there a faster algorithm for searching an array?

Yes... if the array is sorted.

[0]	[1]	[2]	[3]	[4]	[5]	[6]
1	2	4	9	11	15	16

Let's start in the **middle**.

- If $a[N/2] = val$, we found val ; we're done!
- Otherwise, we split the array:
 - ... if $val < a[N/2]$, we search the left half ($a[0]$ to $a[(N/2) - 1]$)
 - ... if $val > a[N/2]$, we search the right half ($a[(N/2) + 1]$ to $a[N - 1]$)

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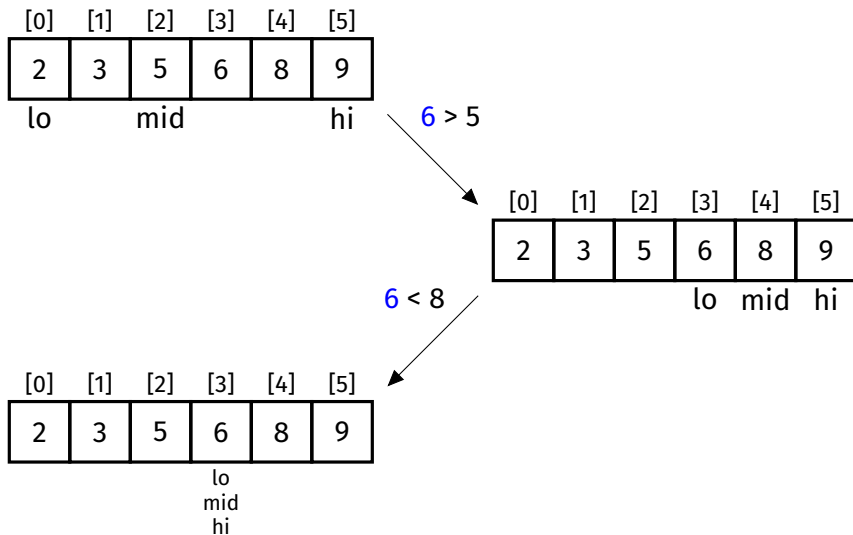
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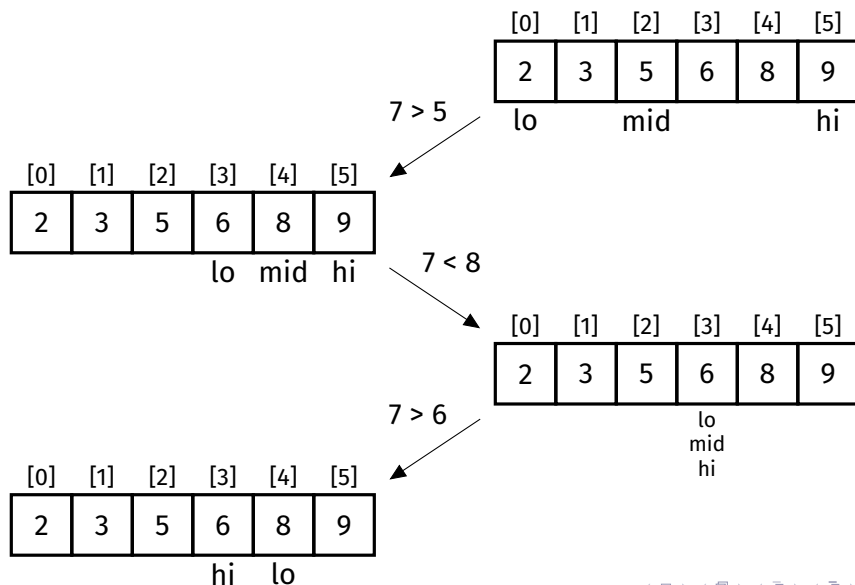
Binary search is a more efficient search algorithm for **sorted arrays**:

```
int binarySearch(int arr[], int size, int val) {  
    int lo = 0;  
    int hi = size - 1;  
  
    while (lo <= hi) {  
        int mid = (lo + hi) / 2;  
  
        if (val < arr[mid]) {  
            hi = mid - 1;  
        } else if (val > arr[mid]) {  
            lo = mid + 1;  
        } else {  
            return mid;  
        }  
    }  
  
    return -1;  
}
```

Successful search for 6:



Unsuccessful search for 7:



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How many iterations of the loop?

- Best case: 1 iteration
 - Item is found right away
- Worst case: $\log_2 n$ iterations
 - Item does not exist
 - Every iteration, the size of the subarray being searched is halved

Thus, binary search is $O(\log_2 n)$ or simply $O(\log n)$

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$$O(\log_2 n) = O(\log n)$$

Why drop the base?

According to the change of base formula:

$$\log_a n = \frac{\log_b n}{\log_b a}$$

If a and b are constants,
 $\log_a n$ and $\log_b n$ differ by a constant factor

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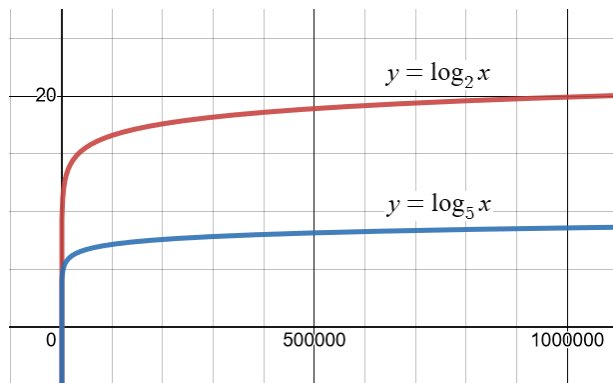
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For example:

$$\log_2 n = \frac{\log_5 n}{\log_5 2}$$
$$\approx 2.32193 \log_5 n$$



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What if an algorithm takes multiple arrays as input?

If there is no constraint on the relative sizes of the arrays, their sizes would be given as two variables, usually n and m

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Example time complexities with two variables:

$$O(n + m)$$

$$O(nm)$$

$$O(\max(n, m))$$

$$O(\min(n, m))$$

$$O(n \log m)$$

$$O(n \log m + m \log n)$$

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Problem:

Given two arrays, where each array contains no repeats,
find the number of elements in common

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```
numCommonElements(A, B):  
    Input: array  $A$  of size  $n$   
            array  $B$  of size  $m$   
    Output: number of elements in common  
  
    numCommon = 0  
    for  $i$  from 0 up to  $n - 1$ :  
        for  $j$  from 0 up to  $m - 1$ :  
            if  $A[i] = B[j]$ :  
                numCommon = numCommon + 1  
  
    return numCommon
```

Time complexity: $O(nm)$

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<https://forms.office.com/r/zEqxUXvmLR>



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More Examples

If I know my algorithm is quadratic (i.e., $O(n^2)$),
and I know that for a dataset of 1000 items,
it takes 1.2 seconds to run ...

- how long for 2000?
- how long for 10,000?
- how long for 100,000?
- how long for 1,000,000?

(answers on the next slide)

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If I know my algorithm is quadratic (i.e., $O(n^2)$),
and I know that for a dataset of 1000 items,
it takes 1.2 seconds to run ...

- how long for 2000? **4.8 seconds**
- how long for 10,000? **120 seconds** (2 mins)
- how long for 100,000? **12000 seconds** (3.3 hours)
- how long for 1,000,000? **1200000 seconds** (13.9 days)

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More Examples

```
1  
2  p = 0;  
3  
4  for (i = 1; p <= n; i++) {  
5      p = p + i;  
6  }
```

Figure: Example 1

- Big O?

(answers on the next slide)

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```
1  
2  p = 0;  
3  
4  for (i = 1; p <= n; i++) {  
5      p = p + i;  
6  }
```

Figure: Example 1

- Big O? $O(\sqrt{n})$

```
1  
2  
3 for (i = 1; i < n; i = i * 2) {  
4     some statement  
5 }
```

Figure: Example 2

- Big O?

(answers on the next slide)

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```
1  
2  
3 for (i = 1; i < n; i = i * 2) {  
4     some statement  
5 }
```

Figure: Example 2

- Big O? $O(\log n)$


```
for (i = 1; i <= n; i++) {  
    for (j = 1; j <= n; j++) {  
        for (k = 1; k <= n; k++) {  
            statement  
        }  
    }  
}
```

Figure: Example 3

- Big O?

(answers on the next slide)

```
for (i = 1; i <= n; i++) {  
    for (j = 1; j <= n; j++) {  
        for (k = 1; k <= n; k++) {  
            statement  
        }  
    }  
}
```

Figure: Example 3

- Big O? $O(n^3)$

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```
for (i = 1; i <= n; i++) {  
    for (j = 1; j <= i; j++) {  
        statement  
    }  
}
```

Figure: Example 4

- Big O?

(answers on the next slide)

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```
for (i = 1; i <= n; i++) {  
    for (j = 1; j <= i; j++) {  
        statement  
    }  
}
```

Figure: Example 4

- Big O? $O(n^2)$