COMP2521 24T1
Hash Tables

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associative arrays
hash tables
hashing
collision resolution

A commonly desired abstraction
in computer science and in the real world
is the ability to map one kind of data to another,
in other words, map keys to values

Examples:
Map words to definitions
Map student numbers to names
Map people to favourite colors

Motivation

Associative Arrays

An associative array is an abstract data type
that stores key-value pairs, where keys are unique.

It supports the following operations:

insert
insert a key-value pair

lookup
given a key, return its associated value

delete
given a key, delete its key-value pair

Note:
Associative arrays are also called maps, symbol tables, or dictionaries.

Motivation

Associative Arrays

How to implement an associative array?

unordered array

ordered array

balanced binary search tree

unordered array

[0]	[1]	[2]	[3]	[4]	[5]
jas green	andrew red	sasha purple	jake yellow	kevin blue	hayden red

Performance?
Insert: $O(n)$
Lookup: $O(n)$
Delete: $O(n)$

ordered array

[0]	[1]	[2]	[3]	[4]	[5]
andrew red	hayden red	jake yellow	jas green	kevin blue	sasha purple

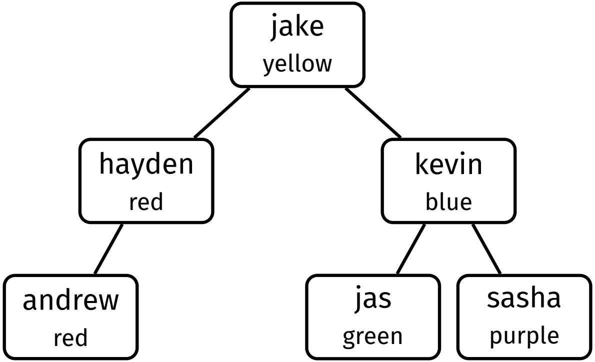
Performance?

Insert: $O(n)$

Lookup: $O(\log n)$

Delete: $O(n)$

balanced binary search tree



Performance?

Insert: $O(\log n)$

Lookup: $O(\log n)$

Delete: $O(\log n)$

Motivation

Associative Arrays

How to implement an associative array?

unordered array

ordered array

balanced binary search tree

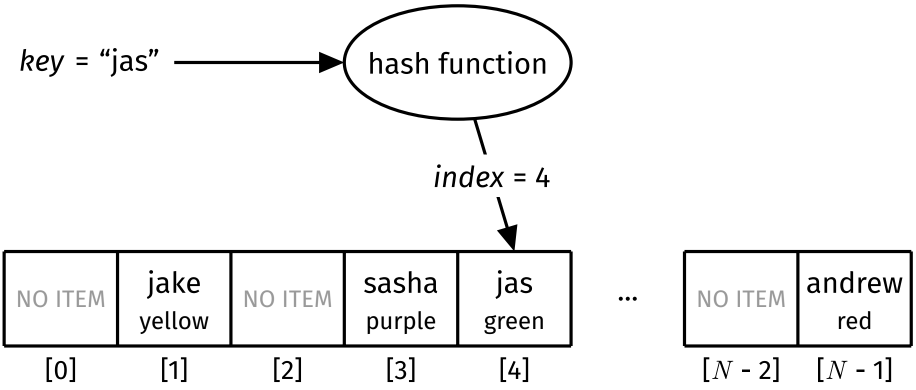
hash table

A hash table is a data structure
that implements an associative array.

It uses an array to store key-value pairs,
and a hash function that,
given a key, computes an index into the array
where the associated value can be found.

A good hash table implementation has an
average performance of $O(1)$ for
insertion, lookup and deletion!

Hash Tables


key = "jas"
hash function
index = 4
NO ITEM
jake
yellow
NO ITEM
sasha
purple
jas
green
...
NO ITEM
andrew
red
[0]
[1]
[2]
[3]
[4]
[N - 2]
[N - 1]

```
/** Creates a new hash table */
HashTable HashTableNew(void);

/** Frees all memory allocated to the hash table */
void HashTableFree(HashTable ht);

/** Inserts a key-value pair into the hash table
    If the key already exists, replaces the value */
void HashTableInsert(HashTable ht, Key key, Value value);

/** Returns true if the hash table contains the given key,
    and false otherwise */
bool HashTableContains(HashTable ht, Key key);

/** Returns the value associated with the given key
    Assumes that the key exists */
Value HashTableGet(HashTable ht, Key key);

/** Deletes the key-value pair associated with the given key */
void HashTableDelete(HashTable ht, Key key);

/** Returns the number of key-value pairs in the hash table */
int HashTableSize(HashTable ht);
```

Hash Tables

Example Usage

```
HashTable ht = HashTableNew();

HashTableInsert(ht, "jas", "green");
HashTableInsert(ht, "andrew", "red");
HashTableInsert(ht, "sasha", "purple");
HashTableInsert(ht, "jake", "yellow");

printf("jas' fav colour is %s\n", HashTableGet(ht, "jas")); // green

HashTableInsert(ht, "jas", "orange");
printf("jas' fav colour is %s\n", HashTableGet(ht, "jas")); // orange

HashTableDelete(ht, "jas");
if (!HashTableContains(ht, "jas")) {
    printf("jas has no fav colour\n");
}

HashTableFree(ht);
```

Hashing

Hashing is the process of
mapping data of arbitrary size to fixed-size values
using a hash function

Applications:
Hash tables
Password storage and verification
Verifying integrity of messages and files
Database indexing
...many others

Hash Functions

A hash function:

- Maps a key to an index in the range $[0, N-1]$
  - where N is the size of the array
- Must be cheap to compute
- Is deterministic
  - Given the same key, will always return the same index
- Ideally, maps keys uniformly over the range of indices

Hash Functions

Basic mechanism of hash functions:

```
int hash(Key key, int N) {
    int val = convert key to 32-bit int
    return val % N;
}
```

Hash Functions

Example

Simple hash function for `int`s:

```
int hash(int key, int N) {
    return key % N;
}
```

Simple hash function for strings:

```
int hash(char *key, int N) {
    int sum = 0;
    for (int i = 0; key[i] != '\0'; i++) {
        sum += key[i];
    }
    return sum % N;
}
```

More robust hash function for strings:

```
int hash(char *key, int N) {
    int h = 0, a = 31415, b = 21783;
    for (char *c = key; *c != '\0'; c++) {
        a = a * b % (N - 1);
        h = (a * h + *c) % N;
    }
    return h;
}
```

A real hash function (from PostgreSQL DBMS)...

```
int hash_any(unsigned char *k, register int keylen, int N) {
    register uint32 a, b, c, len;

    // set up internal state
    len = keylen;
    a = b = 0x9e3779b9;
    c = 3923095;

    // handle most of the key, in 12-char chunks
    while (len >= 12) {
        a += (k[0] + (k[1] << 8) + (k[2] << 16) + (k[3] << 24));
        b += (k[4] + (k[5] << 8) + (k[6] << 16) + (k[7] << 24));
        c += (k[8] + (k[9] << 8) + (k[10] << 16) + (k[11] << 24));
        mix(a, b, c);
        k += 12; len -= 12;
    }

    // collect any data from remaining bytes into a,b,c
    mix(a, b, c);
    return c % N;
}
```

...where mix is defined as:

```
#define mix(a, b, c) \
{ \
  a -= b; a -= c; a ^= (c >> 13); \
  b -= c; b -= a; b ^= (a <<  8); \
  c -= a; c -= b; c ^= (b >> 13); \
  a -= b; a -= c; a ^= (c >> 12); \
  b -= c; b -= a; b ^= (a << 16); \
  c -= a; c -= b; c ^= (b >>  5); \
  a -= b; a -= c; a ^= (c >>  3); \
  b -= c; b -= a; b ^= (a << 10); \
  c -= a; c -= b; c ^= (b >> 15); \
}
```

Hashing
Example

Given a hash table with 11 slots
and the hash function $h(k) = k \,\%\, 11$,
insert the following keys:

4 8 15 16 23 42

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]

Given a hash table with 11 slots
and the hash function $h(k) = k \% 11$,
insert the following keys:

4 8 15 16 23 42

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]

Given a hash table with 11 slots
and the hash function $h(k) = k \ \% \ 11$,
insert the following keys:

4 8 15 16 23 42

$$h(4) = 4$$

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]

Given a hash table with 11 slots
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				4						

Hashing

Example

Given a hash table with 11 slots
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insert the following keys:

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[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
				4						

Given a hash table with 11 slots
and the hash function $h(k) = k \% 11$,
insert the following keys:

4 8 15 16 23 42

$$h(8) = 8$$

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
				4						

Hashing

Example

Given a hash table with 11 slots
and the hash function $h(k) = k \% 11$,
insert the following keys:

4 8 15 16 23 42

$$h(8) = 8$$

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
				4				8		

Hashing

Example

Given a hash table with 11 slots
and the hash function $h(k) = k \% 11$,
insert the following keys:

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[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
				4				8		

Hashing
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				4				8		

Hashing

Example

Given a hash table with 11 slots
and the hash function $h(k) = k \% 11$,
insert the following keys:

4 8 15 16 23 42

$$h(15) = 4$$

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
				4				8		

index 4 already contains an item $\Rightarrow$ collision!

Often, the range of possible key values is *much* larger than the range of indices ($[0, N-1]$), so collisions are inevitable.

A hash collision occurs when for two keys x and y, $x \neq y$, but $h(x) = h(y)$.

A hash table must have a method for resolving collisions.

Collision Resolution

Collision resolution methods:

- Separate chaining
  - Each array slot contains a list of the items hashed to that index
  - Allows multiple items in one slot
- Linear probing
  - Check rest of array slots consecutively until an empty slot is found
- Double hashing
  - Instead of checking slots consecutively, use an increment which is determined by a secondary hash

Collision Resolution

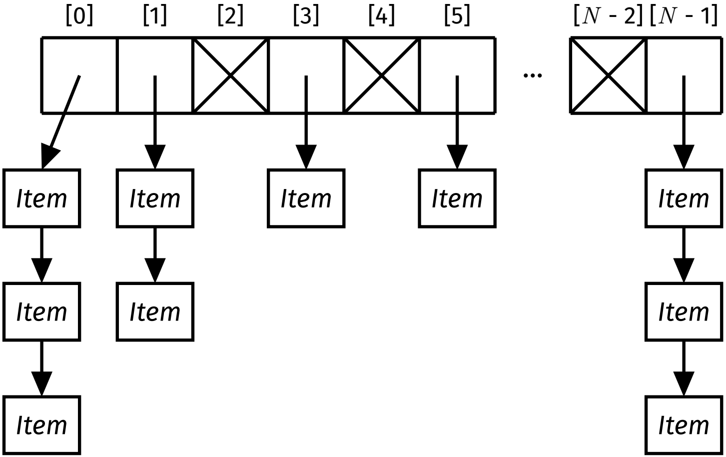
Important statistic: load factor (α)

- Ratio of items to slots; $\alpha = M/N$
- Useful when analysing collision resolution methods

Separate Chaining

Resolve collisions by having multiple items per array slot.

Each array slot contains a linked list of items that are hashed to that index.



Separate Chaining

Example

Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \,\%\, 7$, insert the following keys:

23 4 16 42 8 15

[0]	[1]	[2]	[3]	[4]	[5]	[6]

Separate Chaining

Example

Given a hash table with 7 slots that uses separate chaining
and the hash function $h(k) = k \,\%\, 7$,
insert the following keys:

23 4 16 42 8 15

[0]	[1]	[2]	[3]	[4]	[5]	[6]

Separate Chaining

Example

Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

23 4 16 42 8 15

$$h(23) = 23 \% 7 = 2$$

[0]	[1]	[2]	[3]	[4]	[5]	[6]

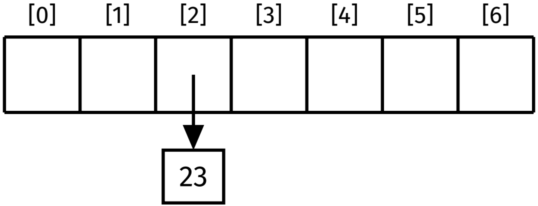
Separate Chaining

Example

Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

23 4 16 42 8 15

$$h(23) = 23 \% 7 = 2$$

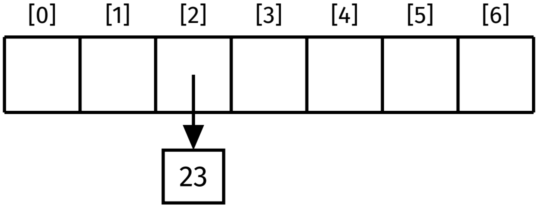


Separate Chaining

Example

Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \,\%\, 7$, insert the following keys:

23 4 16 42 8 15



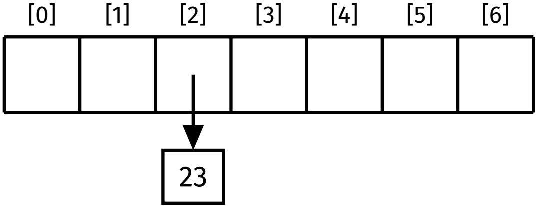
Separate Chaining

Example

Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

23 4 16 42 8 15

$$h(4) = 4 \% 7 = 4$$

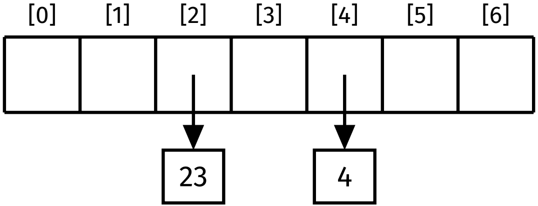


Separate Chaining
Example

Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

23 4 16 42 8 15

$$h(4) = 4 \% 7 = 4$$

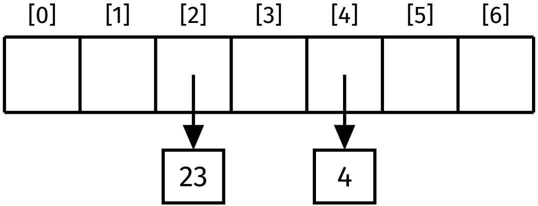


Separate Chaining

Example

Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

23 4 16 42 8 15



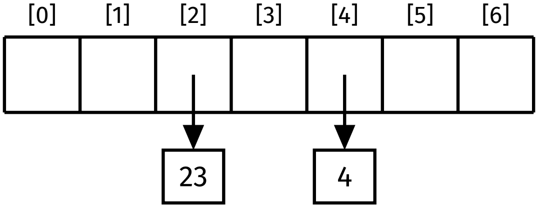
Separate Chaining

Example

Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

23 4 16 42 8 15

$$h(16) = 16 \% 7 = 2$$



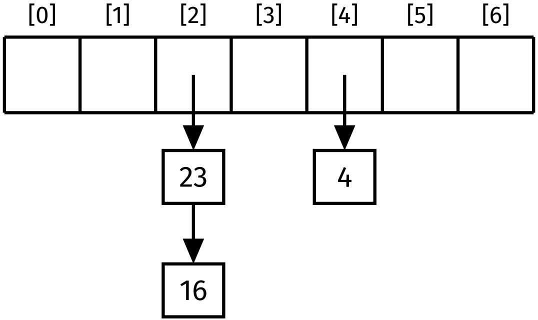
Separate Chaining

Example

Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

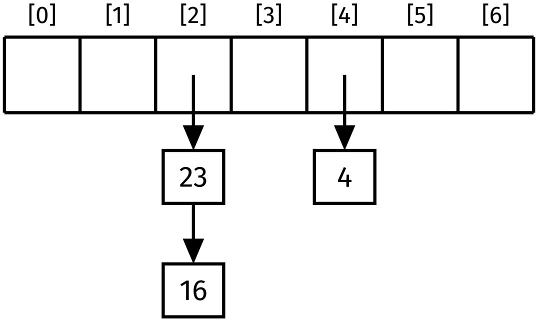
23 4 16 42 8 15

$$h(16) = 16 \% 7 = 2$$



Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

23 4 16 42 8 15



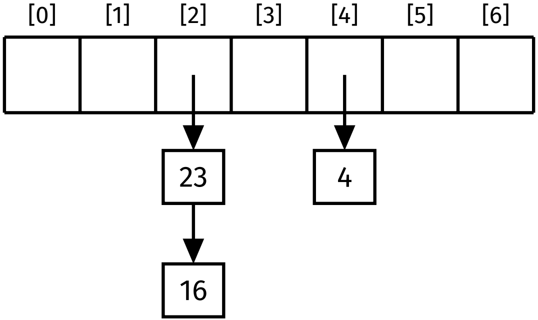
Separate Chaining

Example

Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

23 4 16 42 8 15

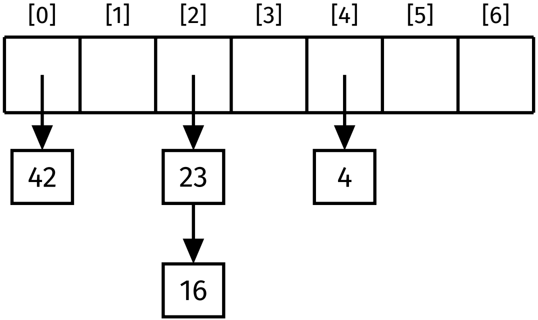
$$h(42) = 42 \% 7 = 0$$



Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

23 4 16 42 8 15

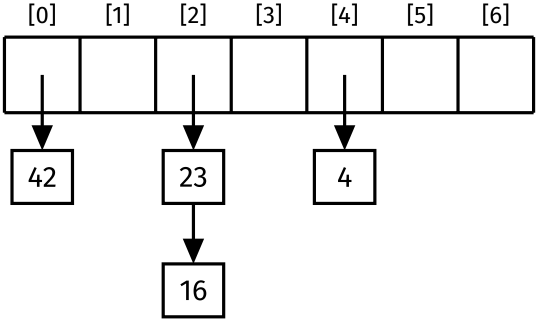
$$h(42) = 42 \% 7 = 0$$



Separate Chaining
Example

Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

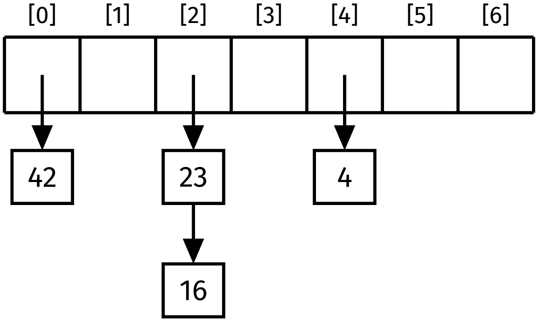
23 4 16 42 8 15



Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

23 4 16 42 8 15

$$h(8) = 8 \% 7 = 1$$



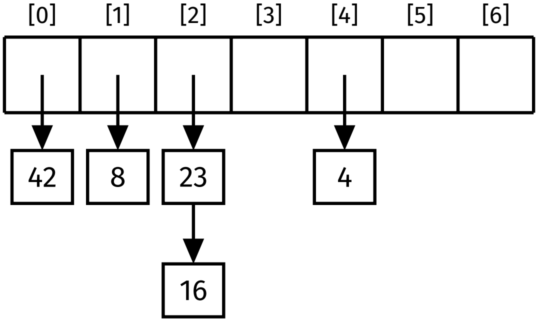
Separate Chaining

Example

Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

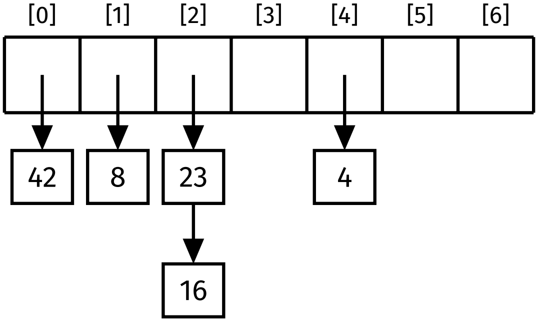
23 4 16 42 8 15

$$h(8) = 8 \% 7 = 1$$



Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \,\%\, 7$, insert the following keys:

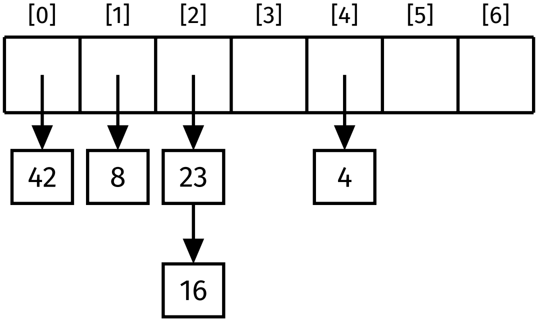
23 4 16 42 8 15



Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

23 4 16 42 8 15

$$h(15) = 15 \% 7 = 1$$



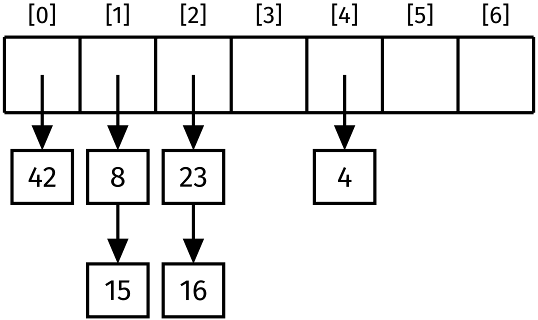
Separate Chaining

Example

Given a hash table with 7 slots that uses separate chaining and the hash function $h(k) = k \% 7$, insert the following keys:

23 4 16 42 8 15

$$h(15) = 15 \% 7 = 1$$



Assuming integer keys and values:

```
struct hashTable {
    struct node **slots; // array of lists
    int numSlots;
    int numItems;
};

struct node {
    int key;
    int value;
    struct node *next;
};
```

Separate Chaining

Implementation

```
HashTable HashTableNew(void) {
    HashTable ht = malloc(sizeof(*ht));

    ht->slots = calloc(INITIAL_NUM_SLOTS, sizeof(struct node *));

    ht->numSlots = INITIAL_NUM_SLOTS;
    ht->numItems = 0;
    return ht;
}
```

```c
void HashTableInsert(HashTable ht, int key, int value) {
    if (/* load factor exceeds threshold */) {
        // resize hash table
    }

    int i = hash(key, ht->numSlots);
    ht->slots[i] = doInsert(ht, ht->slots[i], key, value);
}

struct node *doInsert(HashTable ht, struct node *list,
                      int key, int value) {
    if (list == NULL) {
        ht->numItems++;
        return newNode(key, value);
    } else if (list->key == key) {
        list->value = value; // replace value
    } else {
        list->next = doInsert(ht, list->next, key, value);
    }
    return list;
}
```

```c
bool HashTableContains(HashTable ht, int key) {
    int i = hash(key, ht->numSlots);

    struct node *curr = ht->slots[i];
    while (curr != NULL) {
        if (curr->key == key) {
            return true;
        }
        curr = curr->next;
    }

    return false;
}
```

Separate Chaining

Implementation

```
int HashTableGet(HashTable ht, int key) {
    int i = hash(key, ht->numSlots);

    struct node *curr = ht->slots[i];
    while (curr != NULL) {
        if (curr->key == key) {
            return curr->value;
        }
        curr = curr->next;
    }

    error;
}
```

```
void HashTableDelete(HashTable ht, int key) {
    int i = hash(key, ht->numSlots);
    ht->slots[i] = doDelete(ht, ht->slots[i], key);
}

struct node *doDelete(HashTable ht, struct node *list,
                      int key) {
    if (list == NULL) {
        return NULL;
    } else if (list->key == key) {
        struct node *newHead = list->next;
        free(list);
        ht->numItems--;
        return newHead;
    } else {
        list->next = doDelete(ht, list->next, key);
        return list;
    }
}
```

Separate Chaining

Analysis

Cost analysis:

- N array slots, M items
- Average list length $L = M/N$
- Best case: Items evenly distributed, so maximum list length is $\lceil M/N \rceil$
  - Cost of insert/lookup/delete: $O(M/N)$
- Worst case: One list of length M
  - Cost of insert/lookup/delete: $O(M)$

Average costs:

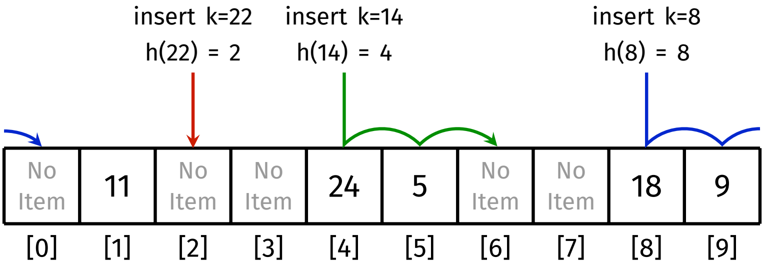
- If good hash and $\alpha \leq 1$, cost is $O(1)$
- If good hash and $\alpha > 1$, cost is $O(M/N)$
  - To avoid degrading perfomance, hash table should be resized when $\alpha \approx 1$

Linear Probing

Resolve collisions by finding a new slot for the item

- Each array slot stores a single item (unlike separate chaining)
- On a hash collision, try next slot, then next, until an empty slot is found
- Insert item into empty slot

Example: $h(k) = k \% 10$



Assuming integer keys and values:

```
struct hashTable {
    struct slot *slots;
    int numSlots;
    int numItems;
};

struct slot {
    int key;
    int value;
    bool empty;
};
```

Linear Probing

Constructor


```
HashTable HashTableNew(void) {
    HashTable ht = malloc(sizeof(*ht));
    ht->slots = malloc(INITIAL_CAPACITY * sizeof(struct slot));
    for (int i = 0; i < ht->numSlots; i++) {
        ht->slots[i].empty = true;
    }

    ht->numSlots = INITIAL_CAPACITY;
    ht->numItems = 0;
    return ht;
}
```

Linear Probing

Insertion

Process for insertion:

1. If load factor exceeds threshold, resize
   - Whether to do this or not is a design decision
2. Hash given key to get an index
3. Starting from this index, find first slot that either:
   - Contains the given key, or
   - Is empty
4. If the slot is empty, store the key and value, otherwise just replace the value

This will be a task in the week 9 lab exercise!

Process for lookup:

1. Hash given key to get an index
2. Starting from this index, find first slot that either:
    - Contains the given key, or
    - Is empty
3. If the slot contains the given key, return the value, otherwise error
    - This is a design decision

Linear Probing

Lookup - Implementation


```
int HashTableGet(HashTable ht, int key) {
    int i = hash(key, ht->numSlots);

    for (int j = 0; j < ht->numSlots; j++) {
        if (ht->slots[i].empty) break;
        if (ht->slots[i].key == key) {
            return ht->slots[i].value;
        }

        i = (i + 1) % ht->numSlots;
    }

    error;
}
```

How to delete an item?

We can't simply remove the item and be done,
as this can break the probe paths for other items,
for example:

$$h(k) = k \,\%\, 10$$

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
No Item	11	No Item	No Item	24	5	14	4	18	No Item

deleting 24 (incorrectly)

No Item	11	No Item	No Item	No Item	5	14	4	18	No Item

Probe path for 14 and 4 is broken!

Linear Probing

Deletion

Two primary methods for deletion:

1. Backshift
    - Remove and re-insert all items between the deleted item and the next empty slot
2. Tombstone
    - Replace the deleted item with a “deleted” marker (AKA a tombstone) that:
        - Is treated as empty during insertion
        - Is treated as occupied during lookup

Linear Probing

Backshift Deletion - Example

Using the backshift method, delete 24 from this hash table:

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
No Item	11	No Item	No Item	24	5	14	4	18	No Item

Linear Probing

Backshift Deletion - Example

Step 1: Remove 24

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
No Item	11	No Item	No Item	No Item	5	14	4	18	No Item

Step 2: Re-insert 5

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
No Item	11	No Item	No Item	No Item	5	14	4	18	No Item

Step 3: Re-insert 14

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
No Item	11	No Item	No Item	14	5	No Item	4	18	No Item

Linear Probing

Backshift Deletion - Example

Step 4: Re-insert 4

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
No Item	11	No Item	No Item	14	5	4	No Item	18	No Item

Step 5: Re-insert 18

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
No Item	11	No Item	No Item	14	5	4	No Item	18	No Item

This will be a task in the week 9 lab exercise!

Linear Probing

Tombstone Deletion - Example

Using the tombstone method, delete 14 from this hash table:

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
No Item	11	No Item	No Item	24	5	14	4	18	No Item

Linear Probing

Tombstone Deletion - Example

After deleting 14:

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
No Item	11	No Item	No Item	24	5	DEL	4	18	No Item

Search for 4:

h(4) = 4

No Item	11	No Item	No Item	24	5	DEL	4	18	No Item

Linear Probing

Tombstone Deletion - Example

Insert 15:

h(15) = 5

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
No Item	11	No Item	No Item	24	5	DEL	4	18	No Item

Result:

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
No Item	11	No Item	No Item	24	5	15	4	18	No Item

Backshift method:

- Moves items closer to their hash index
  - Thus reducing the length of their probe path
- Deletion becomes more expensive

Tombstone method:

- Fast
- But does not reduce probe path length
- Large number of deletions will cause tombstones to build up

Problem with linear probing: clustering

- Items tend to cluster together into long runs
  - i.e., long contiguous regions that don't contain empty slots
- Long runs are a problem:
  - Insertions must travel to the end of a run
  - Lookups of non-existent keys must travel to the end of a run

Causes of clustering:

- The longer a run becomes, the more likely it is to accrue additional items
- Two long runs can be connected together into an even longer run due to the insertion of an item between them

Example ($h(k) = k \ \% \ 15$):

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]

Insert 1, 2, 3, 17, 18

	1	2	3	17	18									

Insert 7, 9, 22, 24, 37, 11

	1	2	3	17	18		7	22	9	24	37	11		

What happens if we insert/search for 8? How about if we insert 6?

Analysis of lookup:

- Hash function is $O(1)$
- Subsequent cost depends on probe path length
  - Affected by load factor $\alpha = M/N$
  - Analysed by Donald Knuth in 1963
  - Average cost for successful search $= \frac{1}{2}\left(1 + \frac{1}{1-\alpha}\right)$
  - Average cost for unsuccessful search $= \frac{1}{2}\left(1 + \frac{1}{(1-\alpha)^2}\right)$

Example costs (assuming large hash table):

load factor (α)	0.50	0.67	0.75	0.90
search hit	1.5	2.0	3.0	5.5
search miss	2.5	5.0	8.5	55.5

Double Hashing

Double hashing improves on linear probing:

- By using an increment which...
  - is based on a secondary hash of the key
  - ensures that all slots will be visited
    (by using an increment which is relatively prime to N)
- Tends to reduce clustering $\Rightarrow$ shorter probe paths

To generate relatively prime number:

- Set table size to prime, e.g., $N = 127$
- Ensure secondary hash function returns number in range $[1, N-1]$

Double Hashing

Example: Insert 22

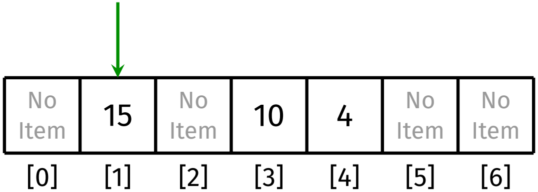
Suppose $h(k) = k \;\%\; 7$ and $h_2(k) = k \;\%\; 3 + 1$

[0]	[1]	[2]	[3]	[4]	[5]	[6]
No Item	15	No Item	10	4	No Item	No Item

Example: Insert 22

Suppose $h(k) = k \ \% \ 7$ and $h_2(k) = k \ \% \ 3 + 1$

$h(22) = 22 \ \% \ 7 = 1 \Rightarrow$ collision!



Double Hashing

Example: Insert 22

Suppose $h(k) = k \ \% \ 7$ and $h_2(k) = k \ \% \ 3 + 1$

$$h(22) = 22 \ \% \ 7 = 1 \Rightarrow \text{collision!}$$

$$h_2(22) = 22 \ \% \ 3 + 1 = 2$$

No Item	15	No Item	10	4	No Item	No Item
[0]	[1]	[2]	[3]	[4]	[5]	[6]

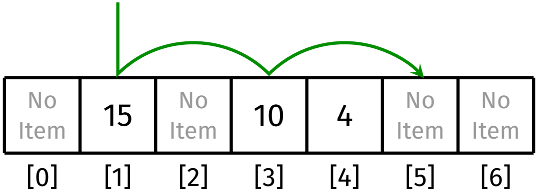
Double Hashing

Example: Insert 22

Suppose $h(k) = k \;\%\; 7$ and $h_2(k) = k \;\%\; 3 + 1$

$$h(22) = 22 \;\%\; 7 = 1 \Rightarrow \text{collision!}$$

$$h_2(22) = 22 \;\%\; 3 + 1 = 2$$



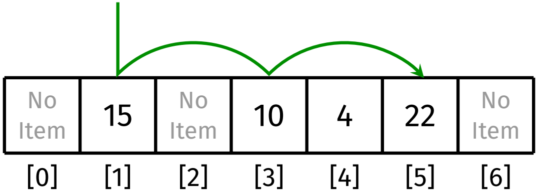
Double Hashing

Example: Insert 22

Suppose $h(k) = k \ \% \ 7$ and $h_2(k) = k \ \% \ 3 + 1$

$h(22) = 22 \ \% \ 7 = 1 \Rightarrow$ collision!

$h_2(22) = 22 \ \% \ 3 + 1 = 2$



Double Hashing

Example

Given a hash table with 11 slots that uses double hashing,
with primary hash function $h(k) = k \% 11$
and secondary hash function $h_2(k) = k \% 5 + 1$,
insert the following keys:

5 20 16 1 42 15

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]

Double Hashing

Example

Given a hash table with 11 slots that uses double hashing,
with primary hash function $h(k) = k \,\%\, 11$
and secondary hash function $h_2(k) = k \,\%\, 5 + 1$,
insert the following keys:

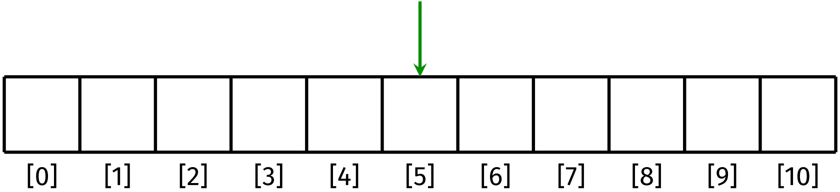
5 20 16 1 42 15

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]

Given a hash table with 11 slots that uses double hashing, with primary hash function $h(k) = k \,\%\, 11$ and secondary hash function $h_2(k) = k \,\%\, 5 + 1$, insert the following keys:

5 20 16 1 42 15

$$h(5) = 5 \,\%\, 11 = 5$$



Given a hash table with 11 slots that uses double hashing, with primary hash function $h(k) = k \% 11$ and secondary hash function $h_2(k) = k \% 5 + 1$, insert the following keys:

5 20 16 1 42 15

$$h(5) = 5 \% 11 = 5$$

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
					5					

Double Hashing

Example

Given a hash table with 11 slots that uses double hashing, with primary hash function $h(k) = k \% 11$ and secondary hash function $h_2(k) = k \% 5 + 1$, insert the following keys:

5 20 16 1 42 15

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
					5					

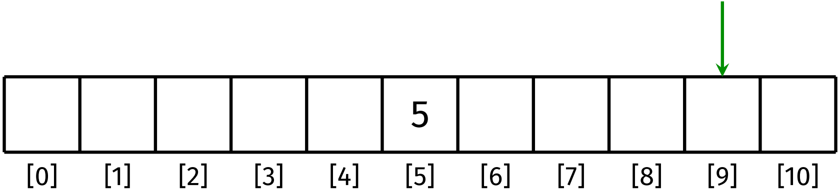
Double Hashing

Example

Given a hash table with 11 slots that uses double hashing, with primary hash function $h(k) = k \ \% \ 11$ and secondary hash function $h_2(k) = k \ \% \ 5 + 1$, insert the following keys:

5 20 16 1 42 15

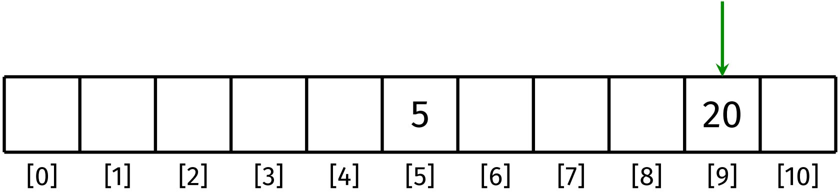
$$h(20) = 20 \ \% \ 11 = 9$$



Given a hash table with 11 slots that uses double hashing, with primary hash function $h(k) = k \% 11$ and secondary hash function $h_2(k) = k \% 5 + 1$, insert the following keys:

5 20 16 1 42 15

$$h(20) = 20 \% 11 = 9$$



Double Hashing

Example

Given a hash table with 11 slots that uses double hashing,
with primary hash function $h(k) = k \ \% \ 11$
and secondary hash function $h_2(k) = k \ \% \ 5 + 1$,
insert the following keys:

5 20 16 1 42 15

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
					5				20	

Double Hashing

Example

Given a hash table with 11 slots that uses double hashing, with primary hash function $h(k) = k \ \% \ 11$ and secondary hash function $h_2(k) = k \ \% \ 5 + 1$, insert the following keys:

5 20 16 1 42 15

$h(16) = 16 \ \% \ 11 = 5 \Rightarrow$ collision!

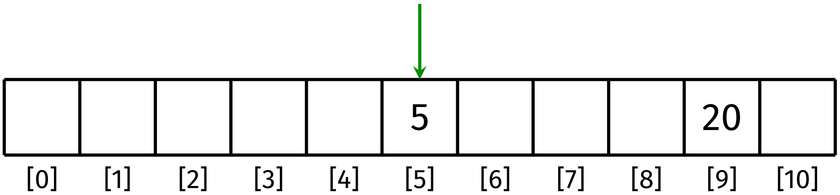
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
					5				20	

Given a hash table with 11 slots that uses double hashing,
with primary hash function $h(k) = k \% 11$
and secondary hash function $h_2(k) = k \% 5 + 1$,
insert the following keys:

5 20 16 1 42 15

$$h(16) = 16 \% 11 = 5 \Rightarrow \text{collision!}$$

$$h_2(16) = 16 \% 5 + 1 = 2$$



Double Hashing

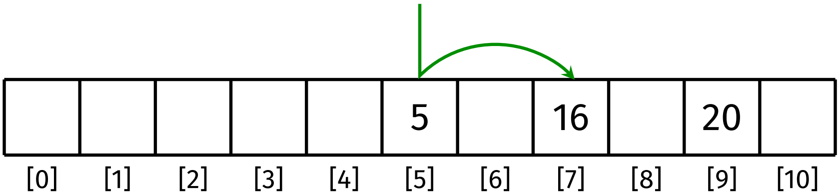
Example

Given a hash table with 11 slots that uses double hashing,
with primary hash function $h(k) = k \% 11$
and secondary hash function $h_2(k) = k \% 5 + 1$,
insert the following keys:

5 20 16 1 42 15

$$h(16) = 16 \% 11 = 5 \Rightarrow \text{collision!}$$

$$h_2(16) = 16 \% 5 + 1 = 2$$



Double Hashing

Example

Given a hash table with 11 slots that uses double hashing,
with primary hash function $h(k) = k \% 11$
and secondary hash function $h_2(k) = k \% 5 + 1$,
insert the following keys:

5 20 16 1 42 15

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
					5		16		20	

Double Hashing

Example

Given a hash table with 11 slots that uses double hashing,
with primary hash function $h(k) = k \ \% \ 11$
and secondary hash function $h_2(k) = k \ \% \ 5 + 1$,
insert the following keys:

5 20 16 1 42 15

$$h(1) = 1 \ \% \ 11 = 1$$

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
					5		16		20	

Double Hashing

Example

Given a hash table with 11 slots that uses double hashing,
with primary hash function $h(k) = k \% 11$
and secondary hash function $h_2(k) = k \% 5 + 1$,
insert the following keys:

5 20 16 1 42 15

$$h(1) = 1 \% 11 = 1$$

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
	1				5		16		20	

Double Hashing

Example

Given a hash table with 11 slots that uses double hashing,
with primary hash function $h(k) = k \% 11$
and secondary hash function $h_2(k) = k \% 5 + 1$,
insert the following keys:

5 20 16 1 42 15

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
	1				5		16		20	

Double Hashing

Example

Given a hash table with 11 slots that uses double hashing,
with primary hash function $h(k) = k \;\%\; 11$
and secondary hash function $h_2(k) = k \;\%\; 5 + 1$,
insert the following keys:

5 20 16 1 42 15

$h(42) = 42 \;\%\; 11 = 9 \Rightarrow$ collision!

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
	1				5		16		20	

Double Hashing

Example

Given a hash table with 11 slots that uses double hashing,
with primary hash function $h(k) = k \% 11$
and secondary hash function $h_2(k) = k \% 5 + 1$,
insert the following keys:

5 20 16 1 42 15

$$h(42) = 42 \% 11 = 9 \Rightarrow$$ collision!

$$h_2(42) = 42 \% 5 + 1 = 3$$

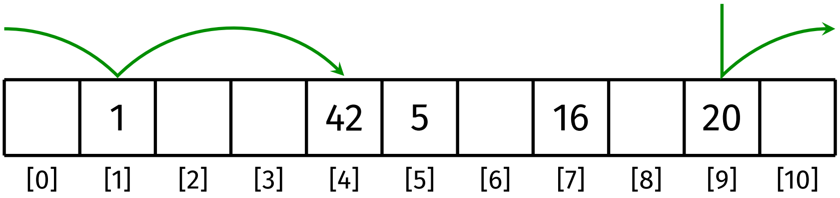
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
	1				5		16		20	

Given a hash table with 11 slots that uses double hashing, with primary hash function $h(k) = k \% 11$ and secondary hash function $h_2(k) = k \% 5 + 1$, insert the following keys:

5 20 16 1 42 15

$$h(42) = 42 \% 11 = 9 \Rightarrow$$ collision!

$$h_2(42) = 42 \% 5 + 1 = 3$$



Given a hash table with 11 slots that uses double hashing,
with primary hash function $h(k) = k \% 11$
and secondary hash function $h_2(k) = k \% 5 + 1$,
insert the following keys:

5 20 16 1 42 15

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
	1			42	5		16		20	

Double Hashing

Example

Given a hash table with 11 slots that uses double hashing,
with primary hash function $h(k) = k \% 11$
and secondary hash function $h_2(k) = k \% 5 + 1$,
insert the following keys:

5 20 16 1 42 15

$h(15) = 15 \% 11 = 4 \Rightarrow$ collision!

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
	1			42	5		16		20	

Double Hashing

Example

Given a hash table with 11 slots that uses double hashing, with primary hash function $h(k) = k \;\%\; 11$ and secondary hash function $h_2(k) = k \;\%\; 5 + 1$, insert the following keys:

5 20 16 1 42 15

$$h(15) = 15 \;\%\; 11 = 4 \Rightarrow \text{collision!}$$

$$h_2(15) = 15 \;\%\; 5 + 1 = 1$$

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
	1			42	5		16		20	

Double Hashing

Example

Given a hash table with 11 slots that uses double hashing, with primary hash function $h(k) = k \% 11$ and secondary hash function $h_2(k) = k \% 5 + 1$, insert the following keys:

5 20 16 1 42 15

$$h(15) = 15 \% 11 = 4 \Rightarrow$$ collision!

$$h_2(15) = 15 \% 5 + 1 = 1$$

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
	1			42	5	15	16		20	

Double Hashing

Example

Given a hash table with 11 slots that uses double hashing, with primary hash function $h(k) = k \% 11$ and secondary hash function $h_2(k) = k \% 5 + 1$, insert the following keys:

5 20 16 1 42 15

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
	1			42	5	15	16		20	

Assuming integer keys and values:

```
struct hashTable {
    struct slot *slots;
    int numSlots;
    int numItems;
    int hash2Mod;
};

struct slot {
    int key;
    int value;
    bool empty;
};
```

Double Hashing

Constructor

```
HashTable HashTableNew(void) {
    HashTable ht = malloc(sizeof(*ht));
    ht->slots = malloc(INITIAL_CAPACITY * sizeof(struct slot));
    for (int i = 0; i < ht->numSlots; i++) {
        ht->slots[i].empty = true;
    }

    ht->numSlots = INITIAL_CAPACITY;
    ht->numItems = 0;
    ht->hash2Mod = findSuitableMod(INITIAL_CAPACITY);
    return ht;
}
```

Double Hashing

Insert - Implementation


```
void HashTableInsert(HashTable ht, int key, int value) {
    if (/* load factor exceeds threshold */) {
        // resize
    }

    int i = hash(key, ht->numSlots);
    int inc = hash2(key, ht->hash2Mod);

    for (int j = 0; j < ht->numSlots; j++) {
        if (ht->slots[i].empty) {
            ht->slots[i].key = key;
            ht->slots[i].value = value;
            ht->slots[i].empty = false;
            ht->numItems++;
            return;
        }
        if (ht->slots[i].key == key) {
            ht->slots[i].value = value;
            return;
        }

        i = (i + inc) % ht->numSlots;
    }
}
```

Double Hashing

Lookup - Implementation

```
int HashTableGet(HashTable ht, int key) {
    int i = hash(key, ht->numSlots);
    int inc = hash2(key, ht->hash2Mod);

    for (int j = 0; j < ht->numSlots; j++) {
        if (ht->slots[i].empty) break;
        if (ht->slots[i].key == key) {
            return ht->slots[i].value;
        }

        i = (i + inc) % ht->numSlots;
    }

    error;
}
```

Double Hashing

Deletion

How to delete an item?

Backshift method is harder to implement due to large increments

Tombstone method (lazy deletion) still works

Analysis of lookup:

- Hash function is $O(1)$
- Subsequent cost depends on probe path length
  - Affected by load factor $\alpha = M/N$
  - Average cost for successful search $= \frac{1}{\alpha} \ln\left(\frac{1}{1-\alpha}\right)$
  - Average cost for unsuccessful search $= \frac{1}{1-\alpha}$

Example costs (assuming large hash table):

load factor (α)	0.50	0.67	0.75	0.90
search hit	1.4	1.6	1.8	2.6
search miss	1.5	2.0	3.0	5.5

Can be significantly better than linear probing

- Especially if table is heavily loaded

Collision resolution approaches:

- Separate chaining: Easy to implement, allows $\alpha > 1$
- Linear probing: Fast if $\alpha \ll 1$, complex deletion
- Double hashing: Avoids clustering issues with linear probing

All approaches can be used to achieve $O(1)$ performance on average, assuming

- good hash function
- table is appropriately resized if load factor exceeds threshold

Design Issues

- How to resize a hash table?
- How to avoid two calls when performing lookup?

Design Issues

How do we resize a hash table?

- Hash function depends on the number of slots
  - Items may not belong at the same index after resizing
- So all items must be re-inserted
- How much to resize by?
  - Good strategy is to roughly double the number of slots every resizing

Design Issues

How to avoid two calls when performing lookup?

- `HashTableGet` assumes the given key exists, and generates an error if it doesn't
- So to look up an item which we don't know exists, we must perform two calls:
  - One call to `HashTableContains` to check for existence of key
  - One call to `HashTableGet` to get the value
- Idea: Provide another function that allows user to specify a default value to return if key does not exist

```
int HashTableGetOrDefault(HashTable ht, int key, int defaultValue);
```

Feedback

https://forms.office.com/r/5c0fb4tvMb