COMP2521 24T1
Abstract Data Types

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abstraction
abstract data types
stacks and queues
sets

Abstraction

is the process of
hiding or generalising
the details of an object or system
to focus on its high-level meaning or behaviour

Assembly languages abstract away machine code

```
0000000000000000 <fn>:
   0: 55                     push rbp
   1: 48 89 e5               mov  rbp, rsp
   4: 89 7d ec               mov  DWORD PTR [rbp-0x14], edi
   7: c7 45 fc 01 00 00 00   mov  DWORD PTR [rbp-0x04], 0x1
   e: c7 45 f8 01 00 00 00   mov  DWORD PTR [rbp-0x08], 0x1
  15: eb 0e                  jmp  25 <fn+0x25>
  17: 8b 45 fc               mov  eax, DWORD PTR [rbp-0x04]
  1a: 0f af 45 f8            imul eax, DWORD PTR [rbp-0x08]
  1e: 89 45 fc               mov  DWORD PTR [rbp-0x04], eax
  21: 83 45 f8 01            add  DWORD PTR [rbp-0x08], 0x1
  25: 8b 45 f8               mov  eax, DWORD PTR [rbp-0x08]
  28: 3b 45 ec               cmp  eax, DWORD PTR [rbp-0x14]
  2b: 7e ea                  jle  17 <fn+0x17>
  2d: 8b 45 fc               mov  eax, DWORD PTR [rbp-0x04]
  30: 5d                     pop  rbp
  31: c3                     ret
```

Abstraction
ADTs
Stacks
Queues
Sets

Modern programming languages abstract away assembly code

```
push rbp
mov  rbp, rsp
mov  DWORD PTR [rbp-0x14], edi
mov  DWORD PTR [rbp-0x04], 0x1
mov  DWORD PTR [rbp-0x08], 0x1
jmp  25 <fn+0x25>
mov  eax, DWORD PTR [rbp-0x04]
imul eax, DWORD PTR [rbp-0x08]
mov  DWORD PTR [rbp-0x04], eax
add  DWORD PTR [rbp-0x08], 0x1
mov  eax, DWORD PTR [rbp-0x08]
cmp  eax, DWORD PTR [rbp-0x14]
jle  17 <fn+0x17>
mov  eax, DWORD PTR [rbp-0x04]
pop  rbp
ret
```

```c
int fn(int n) {
    int res = 1;
    for (int i = 1; i <= n; i++) {
        res *= i;
    }
    return res;
}
```

A function abstracts away the details or steps of a computation

Abstraction

Examples

We drive a car by using a steering wheel and pedals

We operate a television through a remote control

We deposit and withdraw money to/from our bank account via an ATM

Abstraction

Why?

To use a system,
it should be enough to
understand what its components do
without knowing how...

Data Types

A data type is...

- a collection or grouping of values
  - could be atomic, e.g., `int`, `double`
  - could be composite/structured, e.g., arrays, structs
- a collection of operations on those values

Examples:

- `int`
  - operations: addition, multiplication, comparison
- array of `int`s
  - operations: index lookup, index assignment

Abstract Data Types

An abstract data type...

is a description of a data type
from the point of view of a user,
in terms of the operations on the data type and
the behaviour of these operations.

Importantly, an ADT does not specify
how the data type or operations should be implemented.

Example of an ADT: Stack

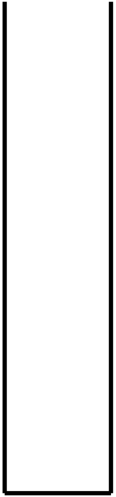
A stack is a linear collection of items with two main operations:

push
adds an item to the top of the stack

pop
removes the item at the top of the stack

User

Stack



Operations

push

adds an item to the top of the stack

pop

removes the item at the top of the stack

User

push 8

push 3

push 7

pop

pop

push 1

Stack

Operations

push

adds an item to the top of the stack

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removes the item at the top of the stack

Abstract Data Types

Example

User

push 8

push 3

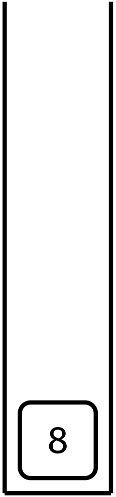
push 7

pop

pop

push 1

Stack



Operations

push

adds an item to the top of the stack

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removes the item at the top of the stack

Abstract Data Types

Example

User

push 8

push 3

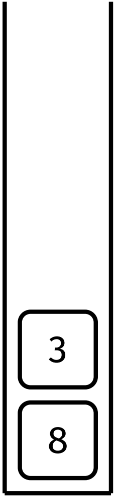
push 7

pop

pop

push 1

Stack



Operations

push

adds an item to the top of the stack

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removes the item at the top of the stack

Abstract Data Types

Example

User

push 8

push 3

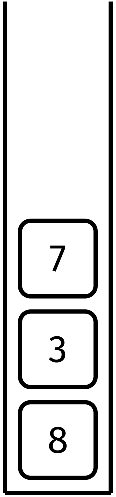
push 7

pop

pop

push 1

Stack



Operations

push

adds an item to the top of the stack

pop

removes the item at the top of the stack

Abstract Data Types
Example

User

push 8

push 3

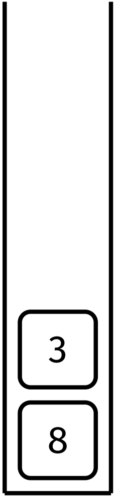
push 7

pop ⇒ 7

pop

push 1

Stack



Operations

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adds an item to the top of the stack

pop
removes the item at the top of the stack

Abstract Data Types
Example

User

push 8

push 3

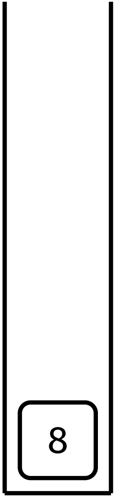
push 7

pop $\Rightarrow$ 7

pop $\Rightarrow$ 3

push 1

Stack



Operations

push

adds an item to the top of the stack

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removes the item at the top of the stack

Abstract Data Types

Example

User

push 8

push 3

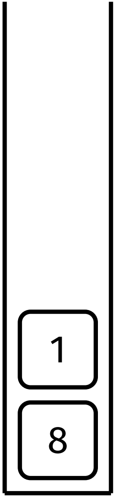
push 7

pop $\Rightarrow$ 7

pop $\Rightarrow$ 3

push 1

Stack



Operations

push

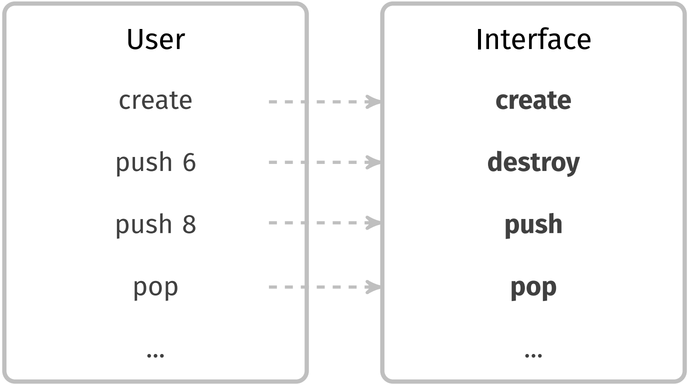
adds an item to the top of the stack

pop

removes the item at the top of the stack

The set of operations provided by an ADT is called the interface.

Users of an ADT only see and interact with the interface.



An ADT interface must:

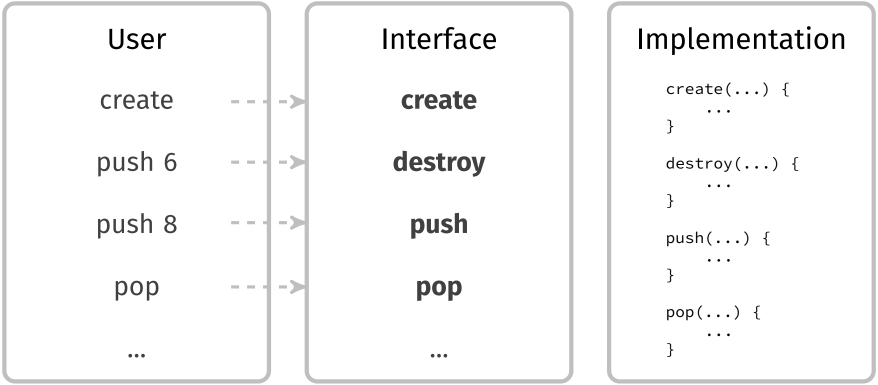
1. clearly describe the behaviour of each operation
2. describe the conditions under which each operation can be used

Example:

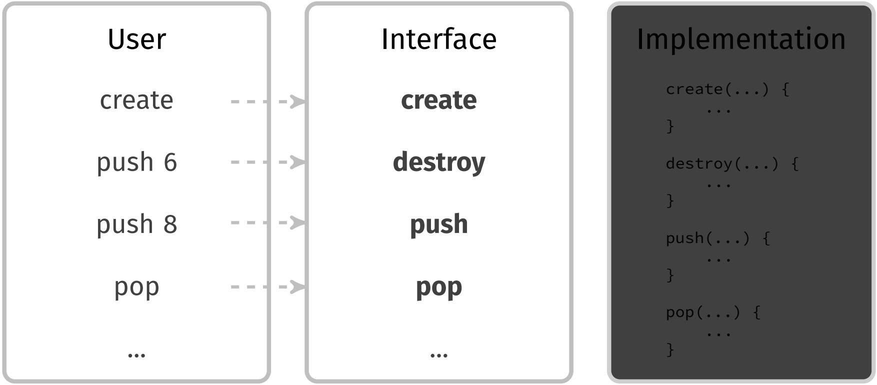
pop
removes the item at
the top of the stack

assumes that the stack is not empty

Builders of an ADT provide an implementation of its operations.



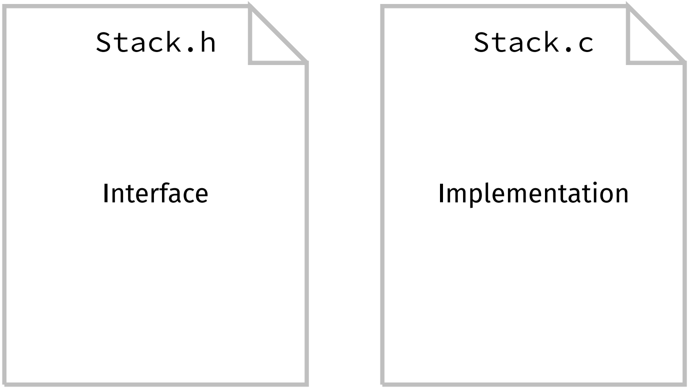
Users of an ADT **do not** see the implementation.



Abstract Data Types in C

In C, abstract data types are implemented using two files:

a .h file that contains the interface
a .c file that contains the implementation



Abstract Data Types in C

Interface — .h file

The interface includes:

- forward declaration of the struct for the concrete representation
  - via `typedef struct t *T`
  - **the struct is not defined in the interface**
- function prototypes for all operations
- clear description of operations
  - via comments
- a contract between the ADT and clients
  - documentation describes how an operation can be used
  - and what the expected result is *as long as the operation is used correctly*

Abstract Data Types in C

Interface — .h file

Stack.h

```c
typedef struct stack *Stack;

/** Creates a new empty stack */
Stack StackNew(void);

/** Frees memory allocated to the stack */
void StackFree(Stack s);

/** Adds an item to the top of the stack */
void StackPush(Stack s, int item);

/** Removes the item at the top of the stack
    Assumes that the stack is not empty */
int StackPop(Stack s);
```

Abstract Data Types in C

Implementation — `.c` file

The implementation includes:

- concrete definition of the data structures
  - definition of `struct t`
- function implementations for all operations

Abstract Data Types in C

Implementation — .c file

Stack.c

```
struct stack {
    ...
};

Stack StackNew(void) {
    ...
}

void StackFree(Stack s) {
    ...
}

void StackPush(Stack s, int item) {
    ...
}

int StackPop(Stack s) {
    ...
}
```

Abstract Data Types in C

User

A user of an ADT `#include`s the interface and uses the interface functions to interact with the ADT.

user.c

```c
#include "Stack.h"

int main(void) {
    Stack s = StackNew();
    StackPush(s, 6);
    StackPush(s, 8);
    int item = StackPop(s);
    ...
}
```

Abstract Data Types in C

Users of an ADT only see and interact with the interface —
they do not see the implementation!

user.c

```c
#include "Stack.h"

int main(void) {
	Stack s = StackNew();
	StackPush(s, 6);
	StackPush(s, 8);
	int item = StackPop(s);
	...
}
```

Stack.h

```c
typedef struct stack *Stack;

...
```

Stack.c

```c
struct stack {
	...
};
```

Abstract Data Types in C

Users of an ADT only see and interact with the interface — they do not see the implementation!

user.c

```c
#include "Stack.h"

int main(void) {
    Stack s = StackNew();

    // this is not valid!
    s->...
}
```

Stack.h

```c
typedef struct stack *Stack;

...
```

Stack.c

```c
struct stack {
    ...
};
```

This means users cannot access the concrete representation (struct) directly.

Naming conventions:

- ADTs are defined in files whose names start with an uppercase letter
  - For example, for a Stack ADT:
    - The interface is defined in `Stack.h`
    - The implementation is defined in `Stack.c`
- ADT interface function names are in `PascalCase` and begin with the name of the ADT

Creating/Using Abstract Data Types

1. Decide what operations you want to provide
   - Operations to create, query, manipulate
   - What are their inputs and outputs?
   - What are the conditions under which they can be used (if any)?
2. Provide the function signatures and documentation for these operations in a `.h` file
3. The "developer" builds a concrete implementation for the ADT in a `.c` file
4. The "user" `#include`s the interface in their program and uses the provided functions

Example - Bank Account

What operations can you perform on a simple bank account?

- Open an account
- Check balance
- Deposit money
- Withdraw money

Example - Bank Account

Interface (Account.h)


```
typedef struct account *Account;

/** Opens a new account with zero balance */
Account AccountOpen(void);

/** Closes an account */
void AccountClose(Account acc);

/** Returns account balance */
int AccountBalance(Account acc);

/** Withdraws money from account
    Returns true if enough balance, false otherwise
    Assumes amount is positive */
bool AccountWithdraw(Account acc, int amount);

/** Deposits money into account
    Assumes amount is positive */
void AccountDeposit(Account acc, int amount);
```

Example - Bank Account

User

```
int main(void) {
    Account acc = AccountOpen();
    printf("Balance: %d\n", AccountBalance(acc));

    AccountDeposit(acc, 50);
    printf("Balance: %d\n", AccountBalance(acc));

    AccountWithdraw(acc, 20);
    printf("Balance: %d\n", AccountBalance(acc));

    AccountWithdraw(acc, 40);
    printf("Balance: %d\n", AccountBalance(acc));

    AccountClose(acc);
}
```

Invalid usage of an ADT (breaking abstraction):

```
int main(void) {
    Account acc = AccountOpen();

    acc->balance = 1000000;

    // I'm a millionaire now, woohoo!
    printf("Balance: %d\n", AccountBalance(acc));

    AccountClose(acc);
}
```

Examples of ADTs

- Stack
- Queue
- Set
- Multiset
- Map
- Graph
- Priority Queue

Stacks

A stack is a collection of items,
such that the last item to enter
is the first item to leave:

Last In, First Out (LIFO)

(Think stacks of books, plates, etc.)

Stacks

A stack is a collection of items, such that the last item to enter is the first item to leave:

Last In, First Out (LIFO)

(Think stacks of books, plates, etc.)

- web browser history
- text editor undo/redo
- balanced bracket checking
- HTML tag matching
- RPN calculators
  (...and programming languages!)
- function calls

A stack supports the following operations:

push
add a new item to the top of the stack

pop
remove the topmost item from the stack

size
return the number of items on the stack

peek
get the topmost item on the stack without removing it

A Stack ADT can be used to check for balanced brackets.

Example of balanced brackets:

([{ }])

Examples of unbalanced brackets!

( ))) ((

([{ })]

([ ]) ([

Sample input: ([{ }])

char	stack	check
		-
(	(	-

Sample input: ([{ }])

char	stack	check
		-
(	(	-
[	([	-

Sample input: ([{ }])

char	stack	check
		-
(	(	-
[	([	-
{	([{	-

Stack ADTs

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
		-
(	(	-
[	([	-
{	([{	-
}	([	{ = }

Stack ADTs

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
		-
(	(	-
[	([	-
{	([{	-
}	([	{ = }
]	(	[=]

Stack ADTs

Example: Balancing Brackets

Sample input: ([{ }])

char	stack	check
		-
(	(	-
[	([	-
{	([{	-
}	([	{ = }
]	(	[=]
)		(=)

Sample input: ([{ }])

char	stack	check
		-
(	(	-
[	([	-
{	([{	-
}	([	$\{ = \}$
]	(	$[=]$
)		$(=)$
EOF		is empty

Sample input: ([{ })]

char	stack	check
		-
(	(	-

Sample input: ([{ })]

char	stack	check
		-
(	(	-
[	([	-

Sample input: ([{ })]

char	stack	check
		-
(	(	-
[	([	-
{	([{	-

Sample input: ([{ })]

char	stack	check
		-
(	(	-
[	([	-
{	([{	-
}	([	{ = }

Sample input: ([{ })]

char	stack	check
		-
(	(	-
[	([	-
{	([{	-
}	([	$\{ = \}$
)	(	$[\neq)$

Sample input: ([{ })]

char	stack	check
		-
(	(	-
[	([	-
{	([{	-
}	([	$\{ = \}$
)	(	$[\neq)$
		fail!

Stack ADT Interface

"Stack.h"

```
typedef struct stack *Stack;

/** Creates a new, empty Stack */
Stack StackNew(void);

/** Frees memory allocated for a Stack */
void StackFree(Stack s);

/** Adds an item to the top of a Stack */
void StackPush(Stack s, Item it);

/** Removes an item from the top of a Stack
    Assumes that the Stack is not empty */
Item StackPop(Stack s);

/** Gets the number of items in a Stack */
int StackSize(Stack s);

/** Gets the item at the top of a Stack
    Assumes that the Stack is not empty */
Item StackPeek(Stack s);
```

Stack ADT

How to implement a stack?

array

linked list

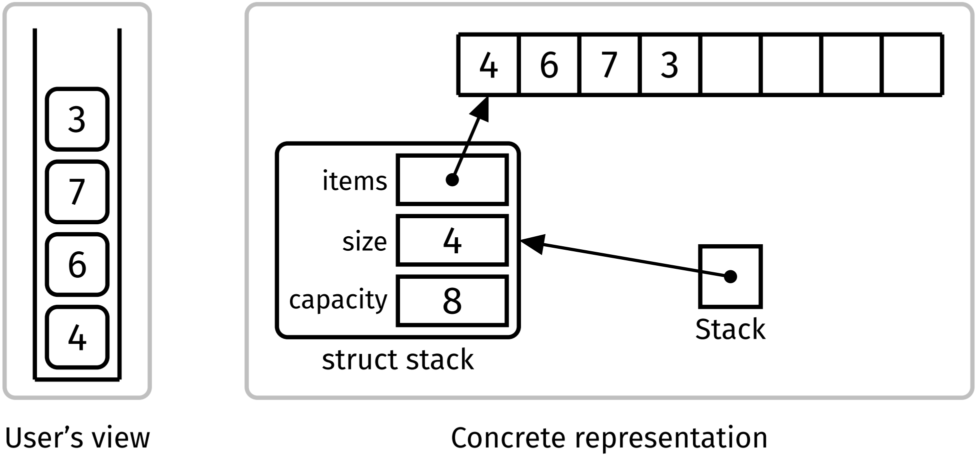
Dynamically allocate an array with an initial capacity

Fill the array sequentially — `s[0]`, `s[1]`, ...

Maintain a counter of the number of items on the stack

Stack ADT

Array implementation



Example

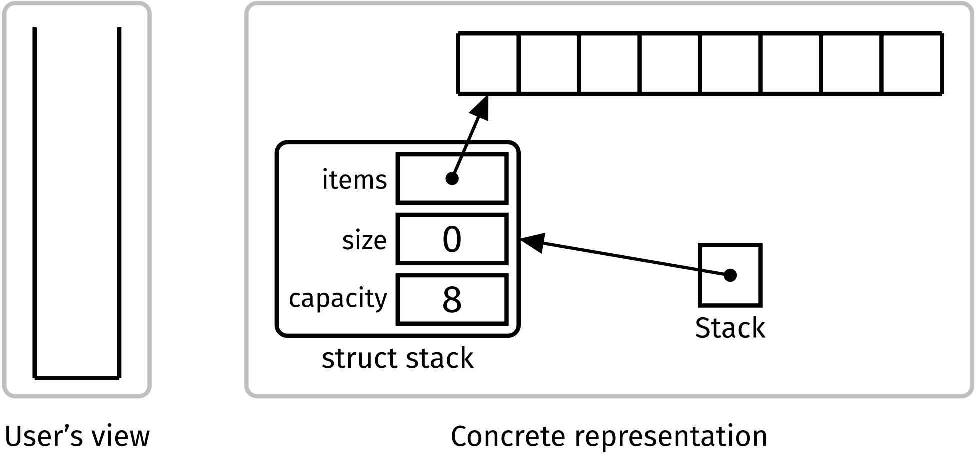
Perform the following operations:

$$\textsc{push}(9),\ \textsc{push}(2),\ \textsc{push}(6),\ \textsc{pop},\ \textsc{pop},\ \textsc{push}(8)$$

Stack ADT

Array implementation

PUSH(9) PUSH(2) PUSH(6) POP POP PUSH(8)



Stack ADT

Array implementation

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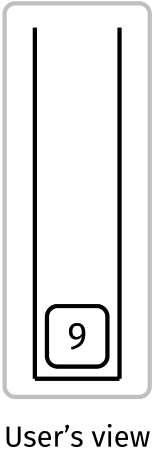

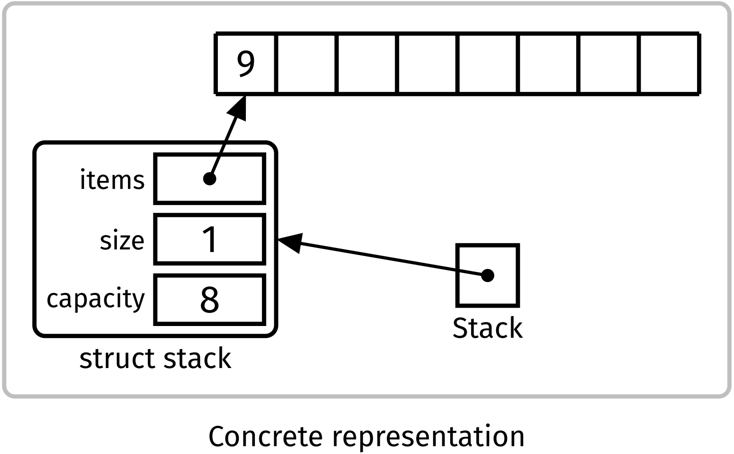

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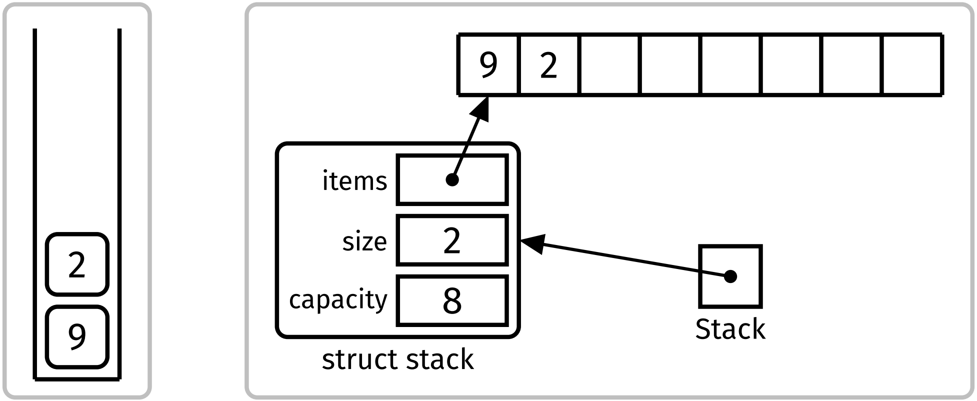


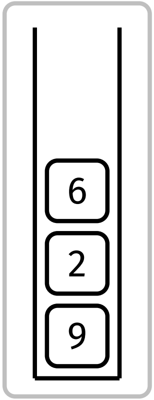
User's view

Concrete representation

Stack ADT

Array implementation

PUSH(9) PUSH(2) PUSH(6) POP POP PUSH(8)



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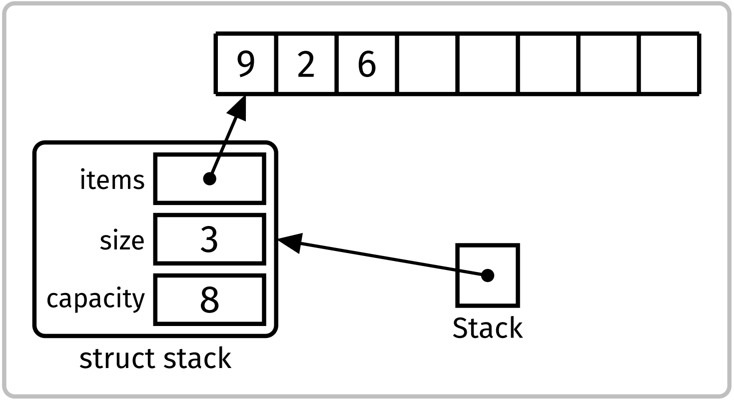
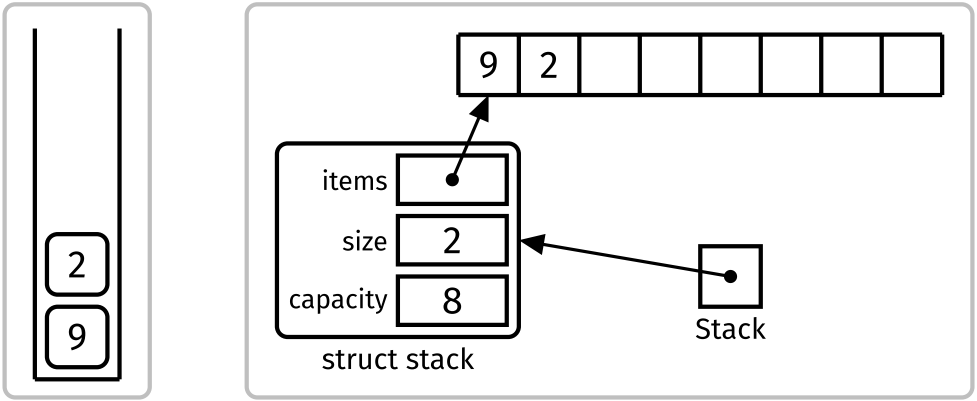


Concrete representation

PUSH(9) PUSH(2) PUSH(6) POP ⇒ 6 POP PUSH(8)



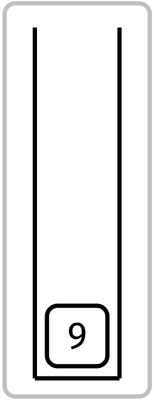
User's view

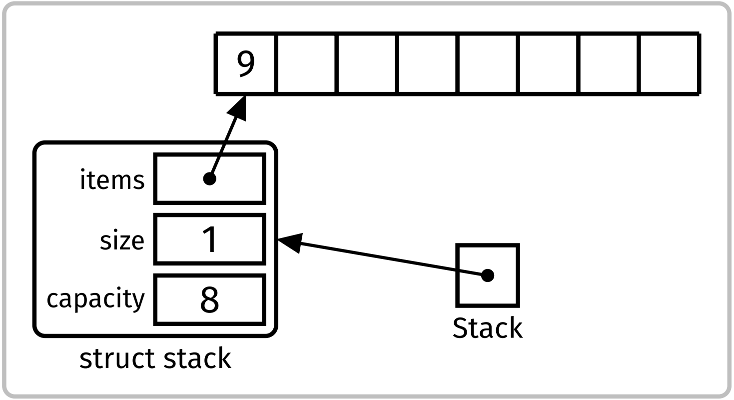
Concrete representation

Stack ADT

Array implementation

PUSH(9) PUSH(2) PUSH(6) POP ⇒ 6 POP ⇒ 2 PUSH(8)

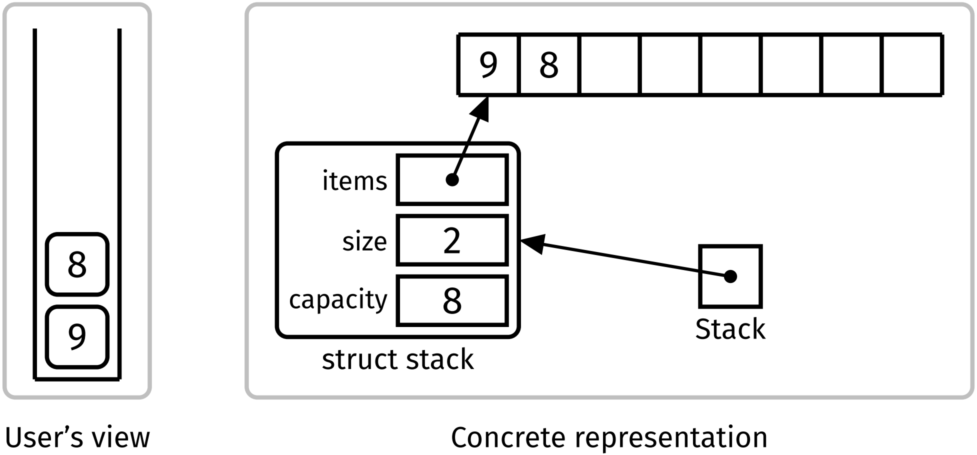




Stack ADT

Array implementation

PUSH(9) PUSH(2) PUSH(6) POP ⇒ 6 POP ⇒ 2 PUSH(8)



Cost of push:

- Inserting item at index `size` is $O(1)$
- What if array is full?
  - If we double the size of the array with *realloc(3)* each time it is full, push will still be $O(1)$ on average

Cost of pop:

- Accessing item at index (`size - 1`) is $O(1)$

Stack ADT

Linked list implementation

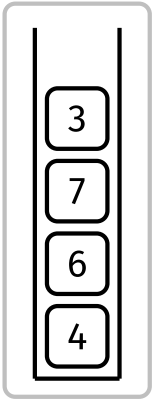
Store items in a linked list

To push an item, insert it at the beginning of the list

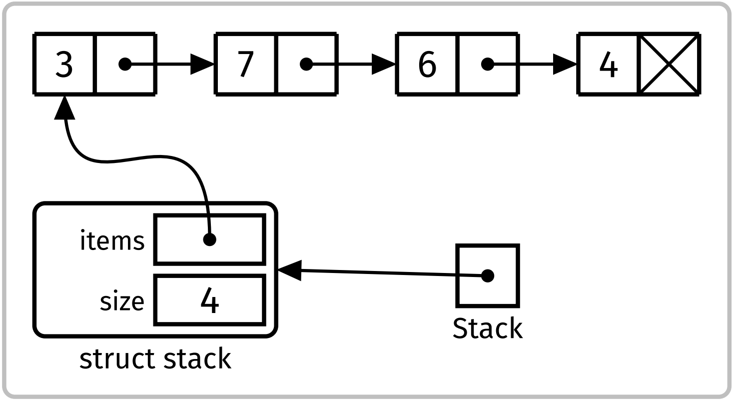
To pop an item, remove it from the beginning of the list

Stack ADT

Linked list implementation



User's view



Concrete representation

Example

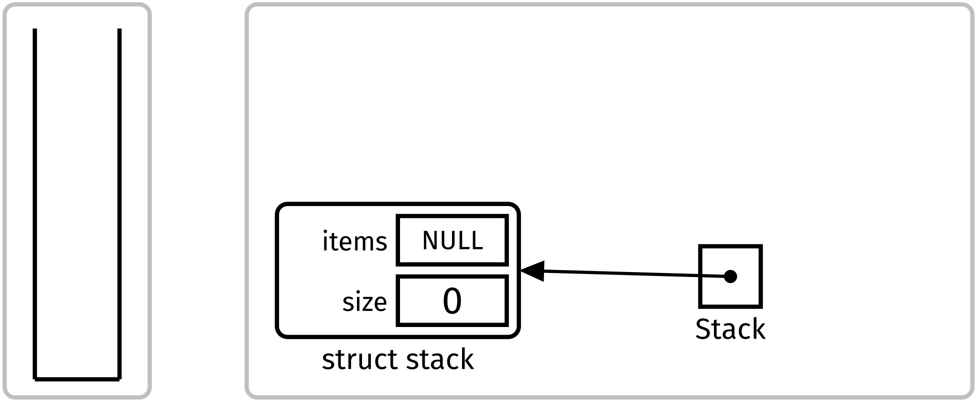
Perform the following operations:

$$\text{PUSH}(9),\ \text{PUSH}(2),\ \text{PUSH}(6),\ \text{POP},\ \text{POP},\ \text{PUSH}(8)$$

Stack ADT

Linked list implementation

PUSH(9) PUSH(2) PUSH(6) POP POP PUSH(8)



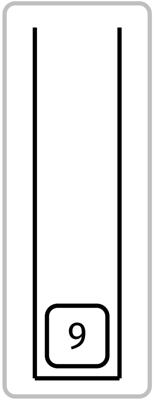
User's view

Concrete representation

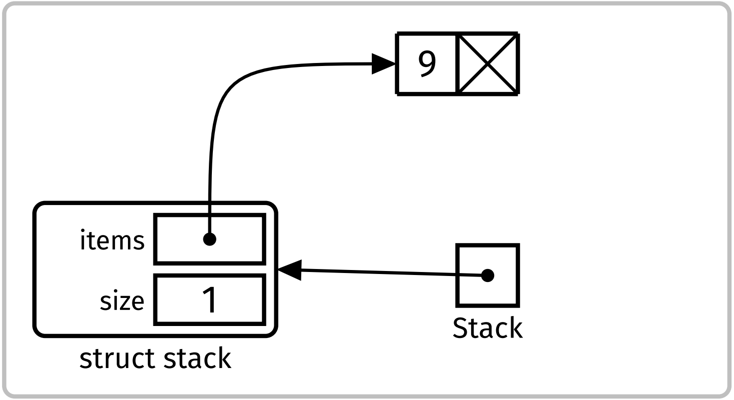
Stack ADT

Linked list implementation

PUSH(9) PUSH(2) PUSH(6) POP POP PUSH(8)

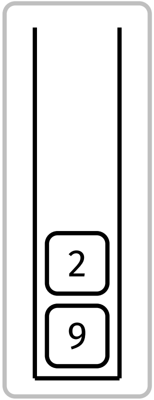


User's view

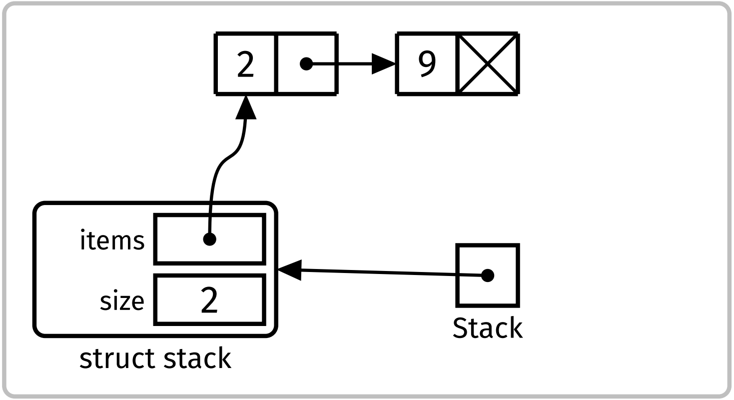


Concrete representation

PUSH(9) PUSH(2) PUSH(6) POP POP PUSH(8)



User's view

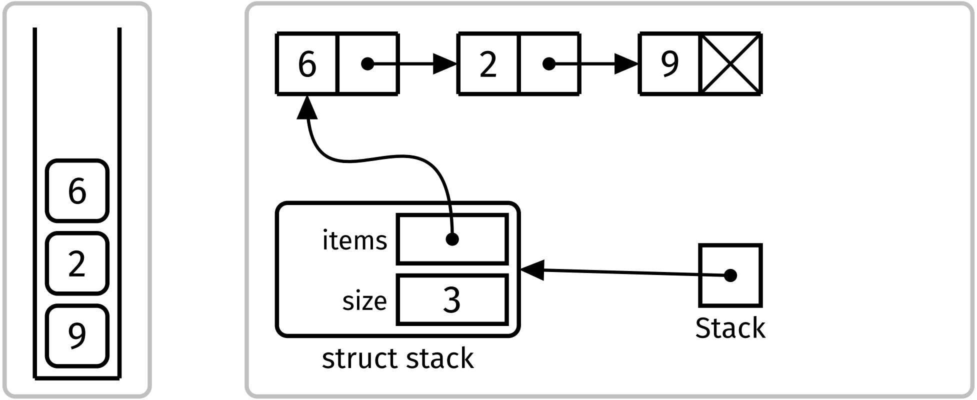


Concrete representation

Stack ADT

Linked list implementation

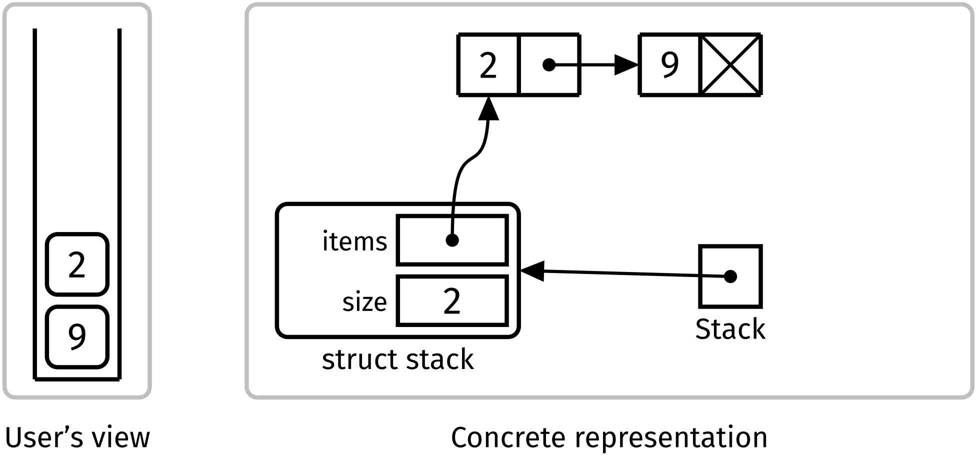
PUSH(9) PUSH(2) PUSH(6) POP POP PUSH(8)



User's view

Concrete representation

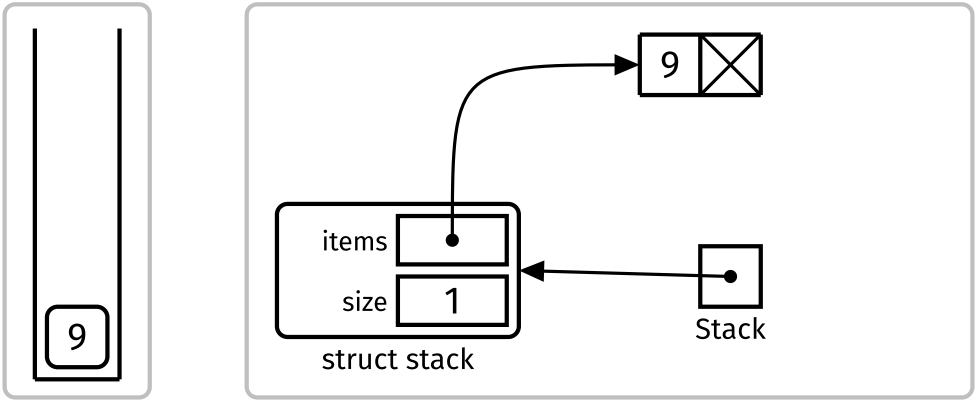
PUSH(9) PUSH(2) PUSH(6) POP ⇒ 6 POP PUSH(8)



Stack ADT

Linked list implementation

PUSH(9) PUSH(2) PUSH(6) POP ⇒ 6 POP ⇒ 2 PUSH(8)



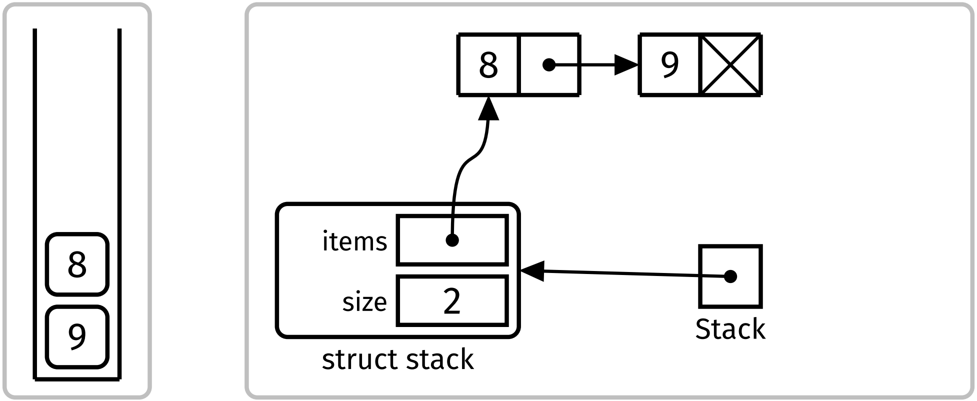
User's view

Concrete representation

Stack ADT

Linked list implementation

$\text{PUSH}(9)$ $\text{PUSH}(2)$ $\text{PUSH}(6)$ $\text{POP} \Rightarrow 6$ $\text{POP} \Rightarrow 2$ $\text{PUSH}(8)$



User's view

Concrete representation

Cost of push:

- Inserting at the beginning of a linked list is $O(1)$

Cost of pop:

- Removing from the beginning of a linked list is $O(1)$

Queues

A queue is a collection of items,
such that the first item to enter
is the first item to leave:

First In, First Out (FIFO)

(Think queues of people, etc.)

Queues

A queue is a collection of items,
such that the first item to enter
is the first item to leave:

First In, First Out (FIFO)

(Think queues of people, etc.)

- waiting lists
- call centres
- access to shared resources (e.g., printers)
- processes in a computer

A queue supports the following operations:

enqueue
add a new item to the end of the queue

dequeue
remove the item at the front of the queue

size
return the number of items in the queue

peek
get the frontmost item of the queue, without removing it

Queue ADT Interface

"Queue.h"


```c
typedef struct queue *Queue;

/** Create a new, empty Queue */
Queue QueueNew(void);

/** Free memory allocated to a Queue */
void QueueFree(Queue q);

/** Add an item to the end of a Queue */
void QueueEnqueue(Queue q, Item it);

/** Remove an item from the front of a Queue
    Assumes that the Queue is not empty */
Item QueueDequeue(Queue q);

/** Get the number of items in a Queue */
int QueueSize(Queue q);

/** Get the item at the front of a Queue
    Assumes that the Queue is not empty */
Item QueuePeek(Queue q);
```

How to implement a queue?

array

linked list (easier)

Queue ADT

Linked list implementation

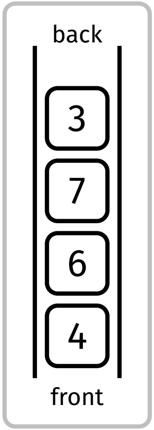
To enqueue an item, insert it at the end of the list

To dequeue an item, remove it from the beginning of the list

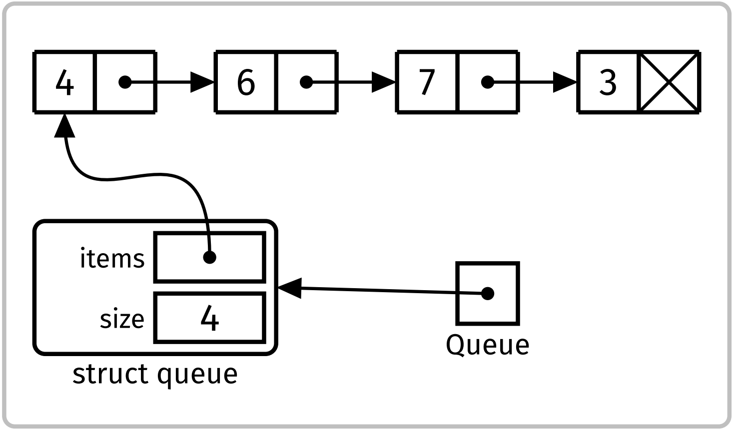
Queue ADT

Linked list implementation

What's the problem with this design?



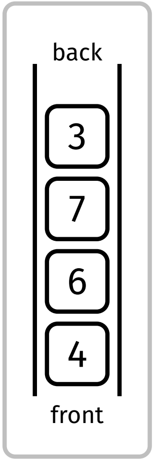
User's view



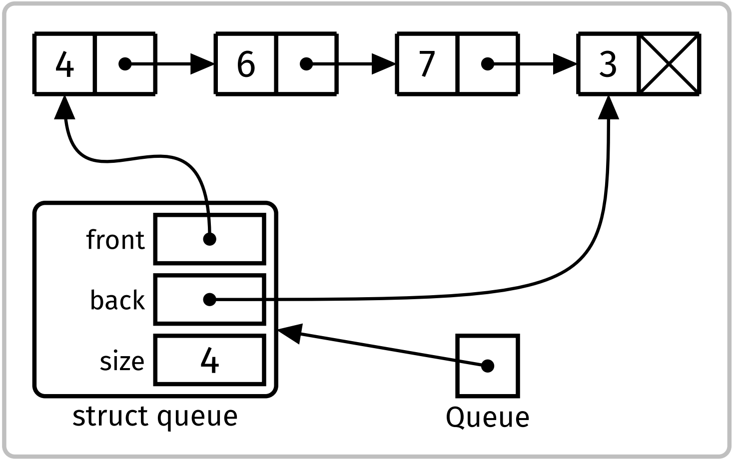
Concrete representation

Queue ADT
Linked list implementation

Improved design



User's view



Concrete representation

Example

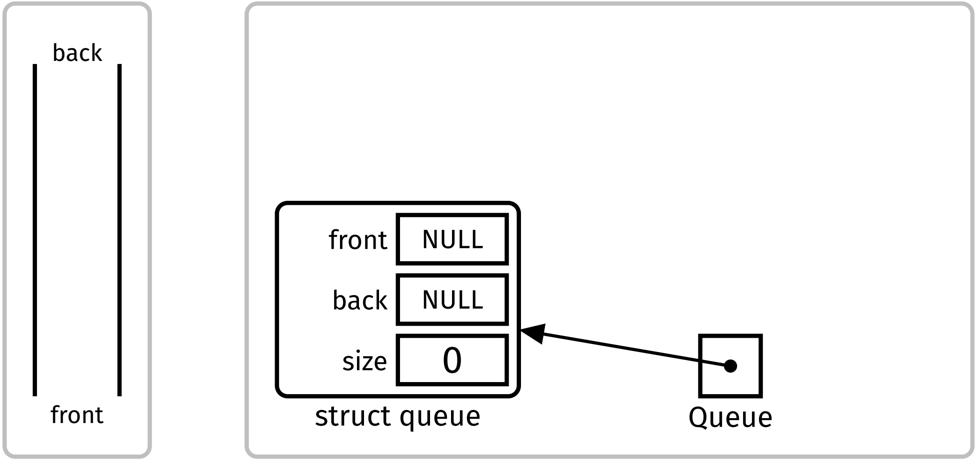
Perform the following operations:

$$\text{ENQ}(9),\ \text{ENQ}(2),\ \text{ENQ}(6),\ \text{DEQ},\ \text{DEQ},\ \text{ENQ}(8)$$

Queue ADT

Linked list implementation

ENQ(9) ENQ(2) ENQ(6) DEQ DEQ ENQ(8)



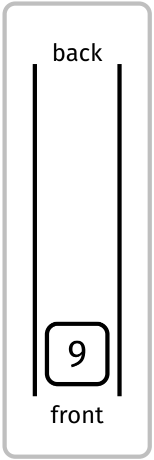
User's view

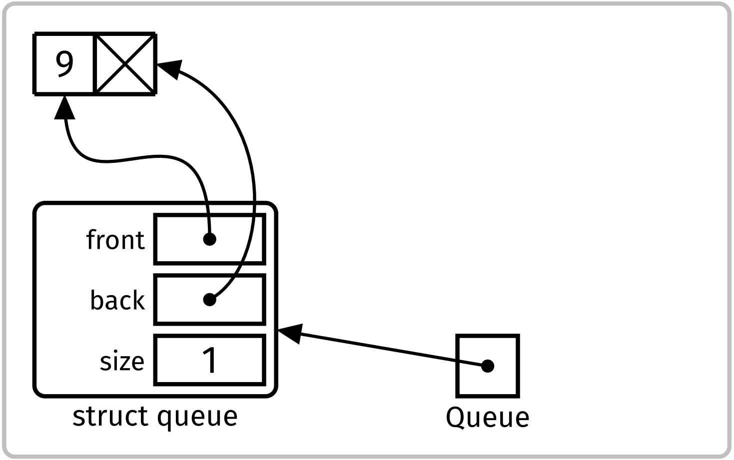
Concrete representation

Queue ADT

Linked list implementation

$\text{ENQ}(9)$ $\text{ENQ}(2)$ $\text{ENQ}(6)$ DEQ DEQ $\text{ENQ}(8)$





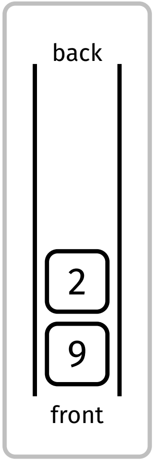
User's view

Concrete representation

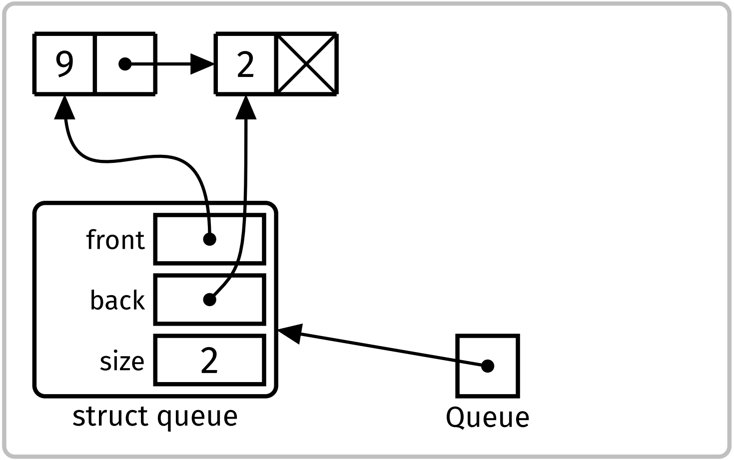
Queue ADT

Linked list implementation

ENQ(9) ENQ(2) ENQ(6) DEQ DEQ ENQ(8)



User's view

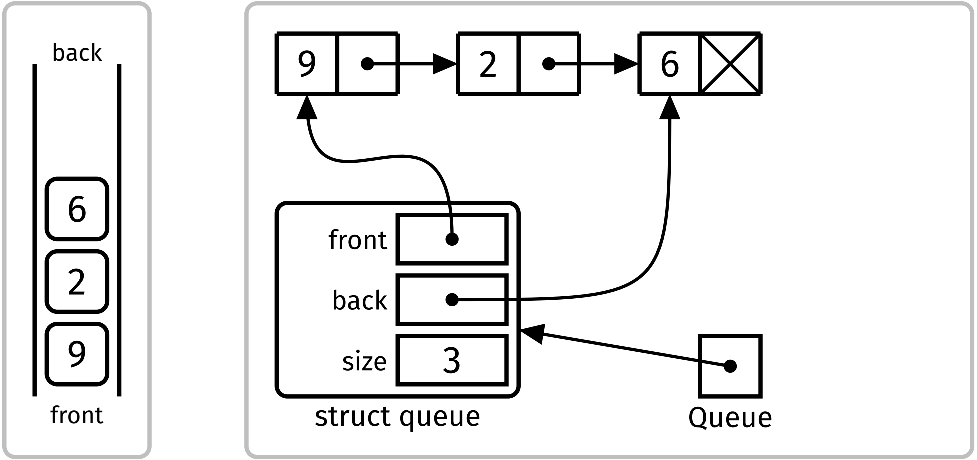


Concrete representation

Queue ADT

Linked list implementation

ENQ(9) ENQ(2) ENQ(6) DEQ DEQ ENQ(8)



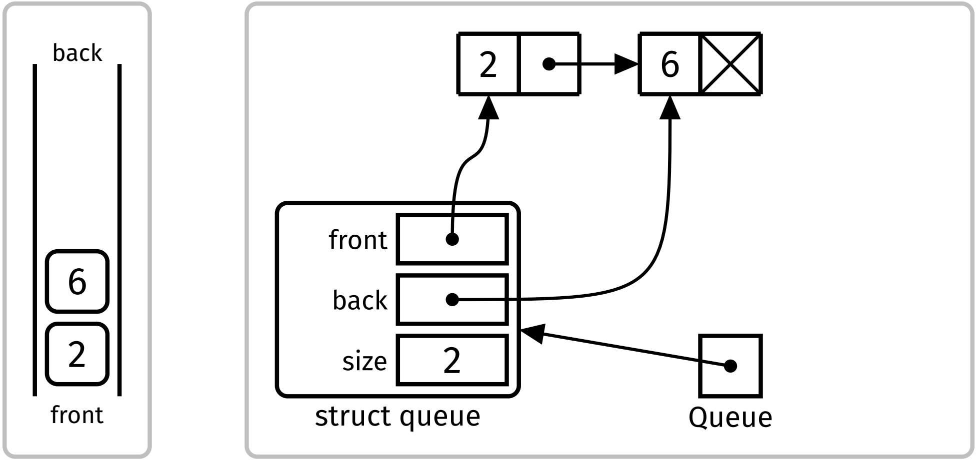
User's view

Concrete representation

Queue ADT

Linked list implementation

ENQ(9) ENQ(2) ENQ(6) DEQ ⇒ 9 DEQ ENQ(8)



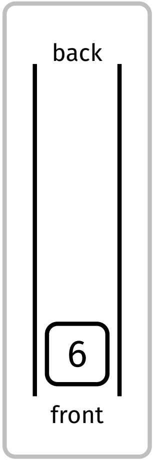
User's view

Concrete representation

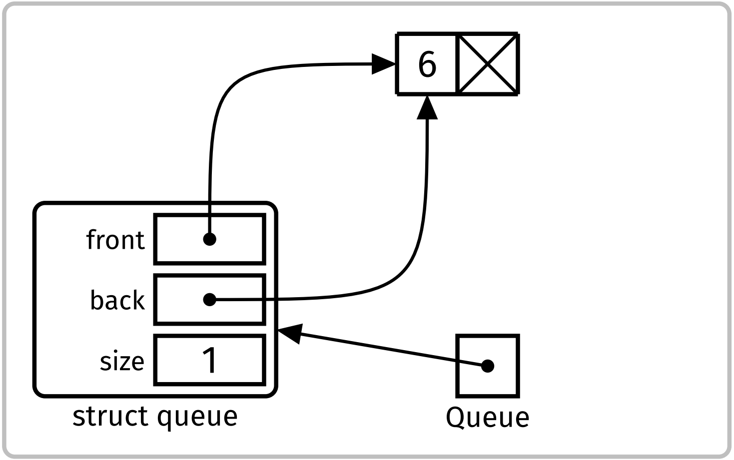
Queue ADT

Linked list implementation

$\text{ENQ}(9)$ $\text{ENQ}(2)$ $\text{ENQ}(6)$ $\text{DEQ} \Rightarrow 9$ $\text{DEQ} \Rightarrow 2$ $\text{ENQ}(8)$



User's view

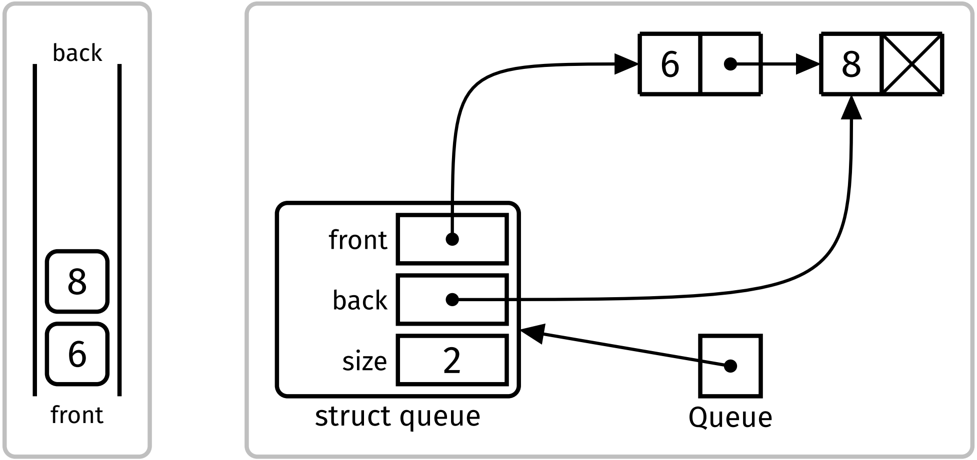


Concrete representation

Queue ADT

Linked list implementation

$\text{ENQ}(9)$ $\text{ENQ}(2)$ $\text{ENQ}(6)$ $\text{DEQ} \Rightarrow 9$ $\text{DEQ} \Rightarrow 2$ $\text{ENQ}(8)$



User's view

Concrete representation

Queue ADT

Linked list implementation - Analysis

Cost of enqueue:

- Inserting at the end of the linked list is $O(1)$

Cost of dequeue:

- Removing from the beginning of the linked list is $O(1)$

Queue ADT

Array implementation

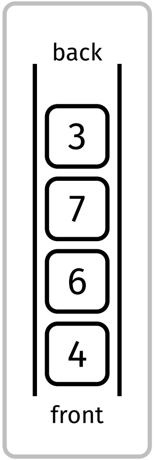
Dynamically allocate an array with an initial capacity

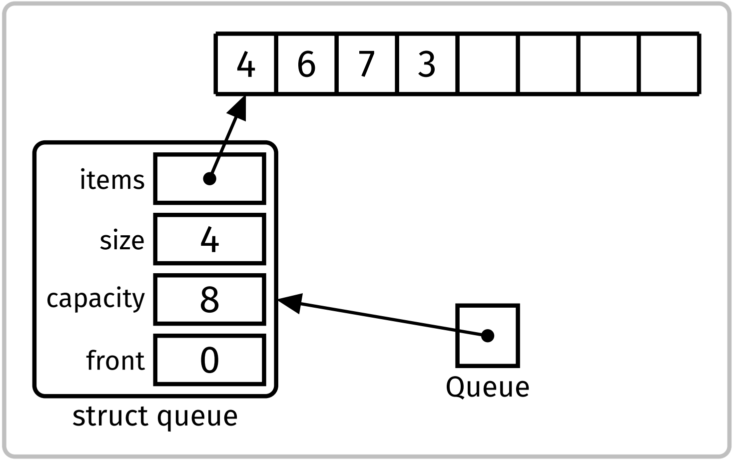
Maintain an index to the front of the queue

Maintain a counter of the number of items in the queue

Queue ADT

Array implementation





User's view

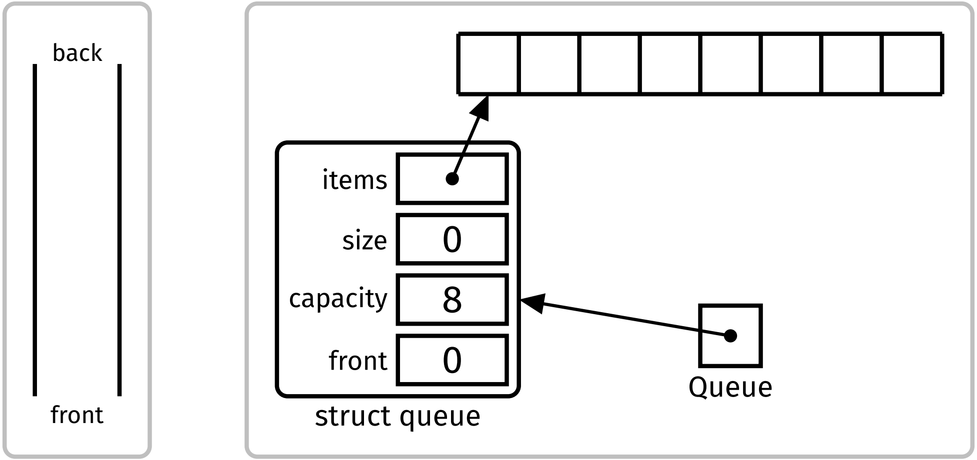
Concrete representation

Example

Perform the following operations:

$\text{ENQ}(9)$, $\text{ENQ}(2)$, $\text{ENQ}(6)$, DEQ, DEQ, $\text{ENQ}(8)$

ENQ(9) ENQ(2) ENQ(6) DEQ DEQ ENQ(8)

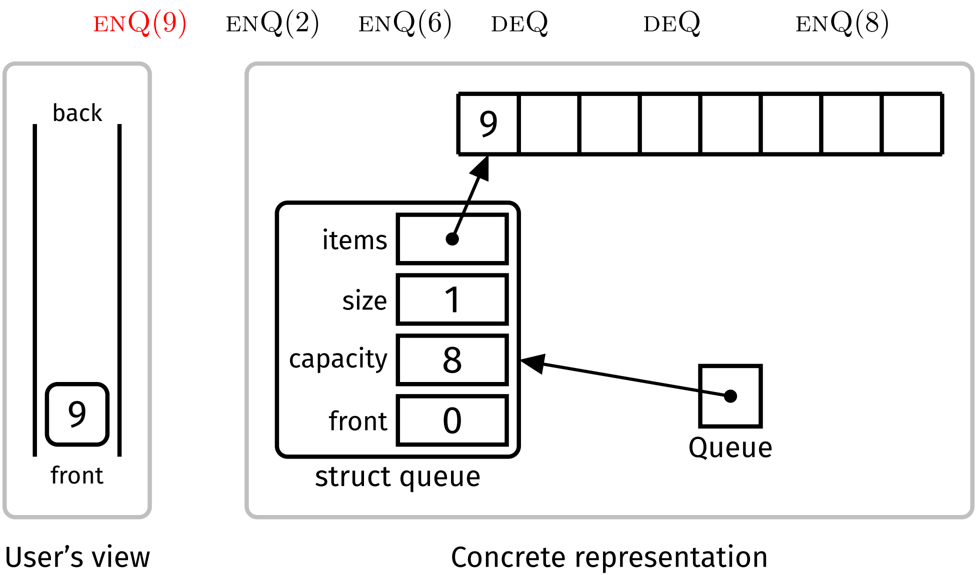


User's view

Concrete representation

Queue ADT

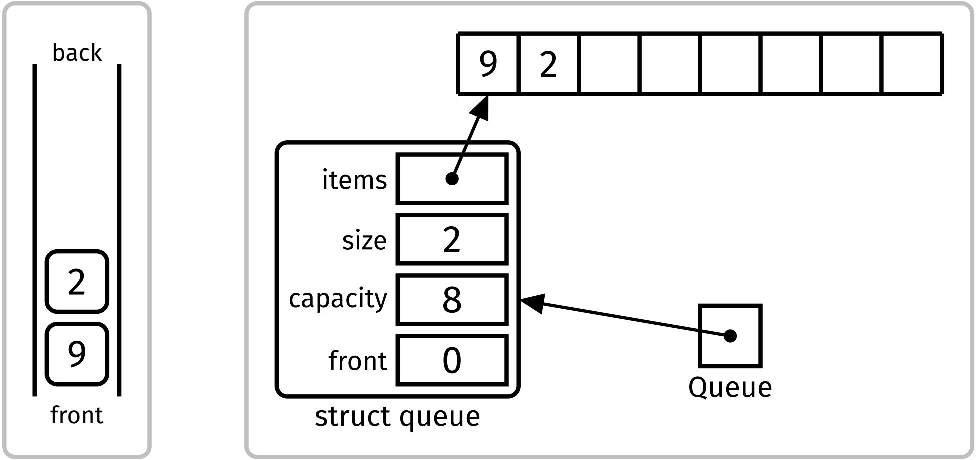
Array implementation



Queue ADT

Array implementation

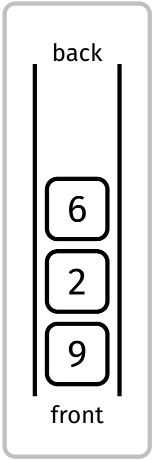
ENQ(9) ENQ(2) ENQ(6) DEQ DEQ ENQ(8)



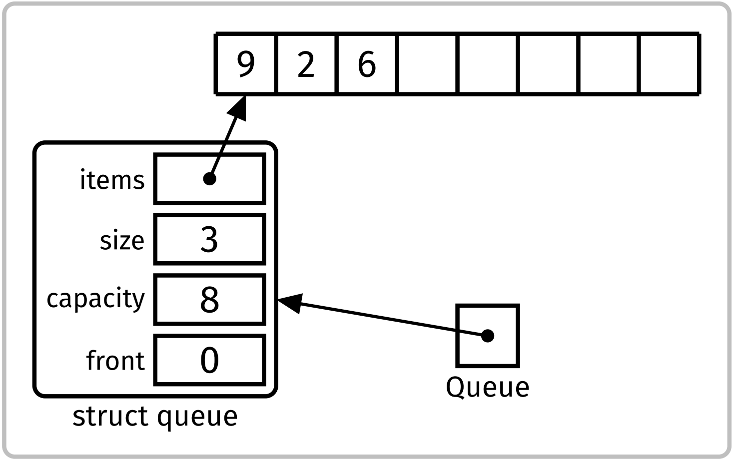
User's view

Concrete representation

ENQ(9) ENQ(2) ENQ(6) DEQ DEQ ENQ(8)



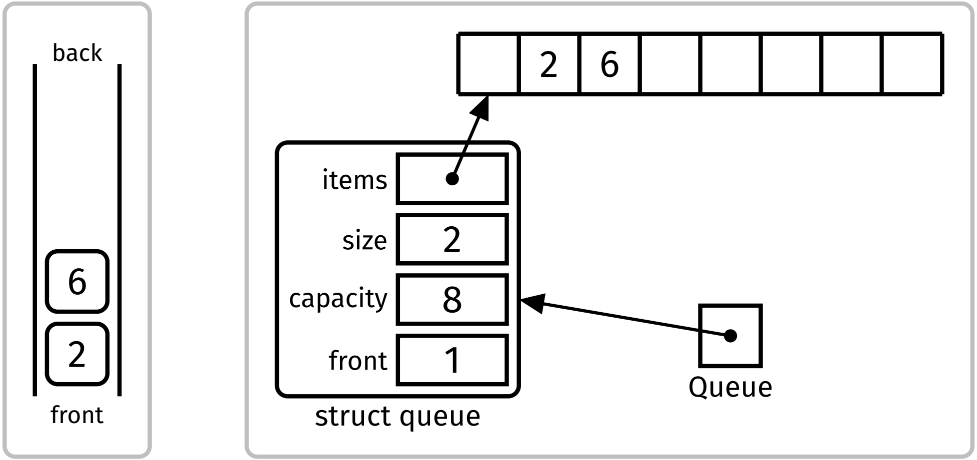
User's view



Concrete representation

Queue ADT
Array implementation

ENQ(9) ENQ(2) ENQ(6) DEQ ⇒ 9 DEQ ENQ(8)



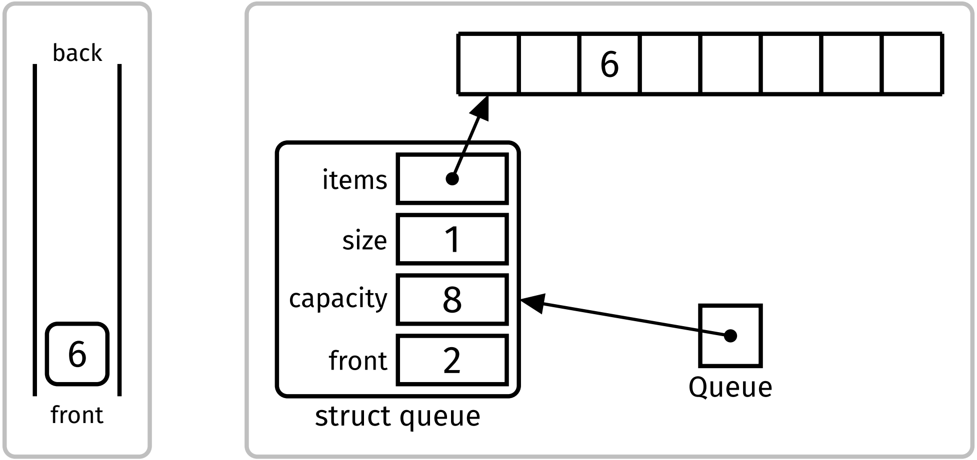
User's view

Concrete representation

Queue ADT

Array implementation

$\text{ENQ}(9)$ $\text{ENQ}(2)$ $\text{ENQ}(6)$ $\text{DEQ} \Rightarrow 9$ $\text{DEQ} \Rightarrow 2$ $\text{ENQ}(8)$



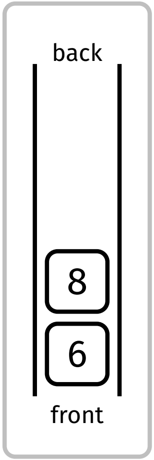
User's view

Concrete representation

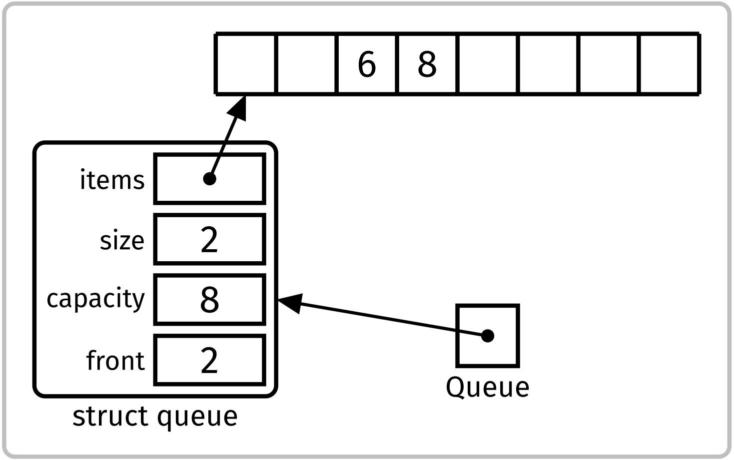
Queue ADT

Array implementation

$\text{ENQ}(9)$ $\text{ENQ}(2)$ $\text{ENQ}(6)$ $\text{DEQ} \Rightarrow 9$ $\text{DEQ} \Rightarrow 2$ $\text{ENQ}(8)$



User's view



Concrete representation

Queue ADT

Array implementation - Analysis

Cost of enqueue:

- Dequeue involves calculating insertion index and inserting item at that index $\Rightarrow O(1)$

Cost of dequeue:

- Dequeue involves accessing item at index `front` $\Rightarrow O(1)$

Sets

A set is an unordered collection of distinct elements.

In this lecture we are concerned with sets of integers.

Basic set operations:

- Create an empty set
- Insert an item into the set
- Delete an item from the set
- Check if an item is in the set
- Get the size of the set
- Display the set

Set

Interface

```c
#include <stdbool.h>

typedef struct set *Set;

/** Creates a new empty set */
Set SetNew(void);

/** Free memory used by set */
void SetFree(Set set);

/** Inserts an item into the set */
void SetInsert(Set set, int item);

/** Deletes an item from the set */
void SetDelete(Set set, int item);

/** Checks if an item is in the set */
bool SetContains(Set set, int item);

/** Returns the size of the set */
int SetSize(Set set);

/** Displays the set */
void SetShow(Set set);
```

Counting and displaying distinct numbers:

```
#include <stdio.h>

#include "Set.h"

int main(void) {
    Set s = SetNew();

    int val;
    while (scanf("%d", &val) == 1) {
        SetInsert(s, val);
    }

    printf("Number of distinct values: %d\n", SetSize(s));
    printf("Values: ");
    SetShow(s);

    SetFree(s);
}
```

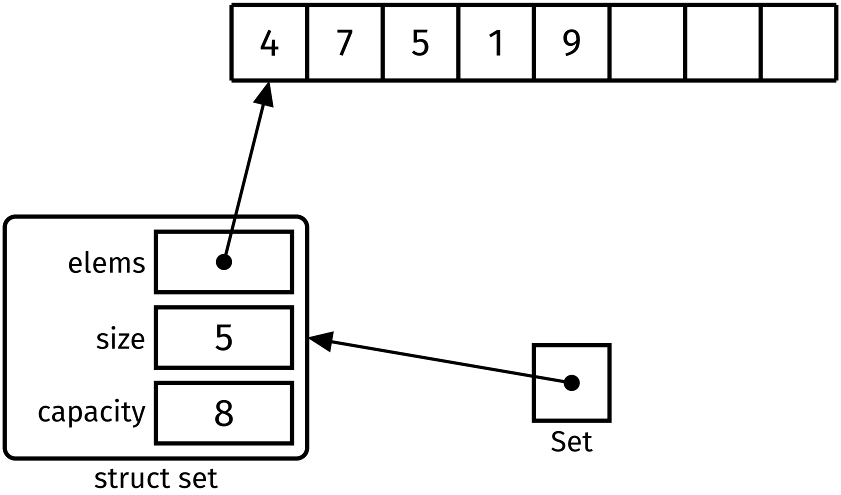
Set Implementation

Different ways to implement a set:

- Unordered array
- Ordered array
- Ordered linked list

Set Implementation

Unordered array



How do we check if an element exists?

- Perform linear scan of array $\Rightarrow O(n)$

```
bool SetContains(Set s, int elem) {
    for (int i = 0; i < s->size; i++) {
        if (s->elems[i] == elem) {
            return true;
        }
    }

    return false;
}
```

How do we insert an element?

- If the element doesn't exist, insert it after the last element

```
void SetInsert(Set s, int elem) {
    if (SetContains(s, elem)) {
        return;
    }

    if (s->size == s->capacity) {
        // error message
    }

    s->elems[s->size] = elem;
    s->size++;
}
```

Time complexity: $O(n)$

- SetContains is $O(n)$ and inserting after the last element is $O(1)$

How do we delete an element?

- If the element exists, overwrite it with the last element

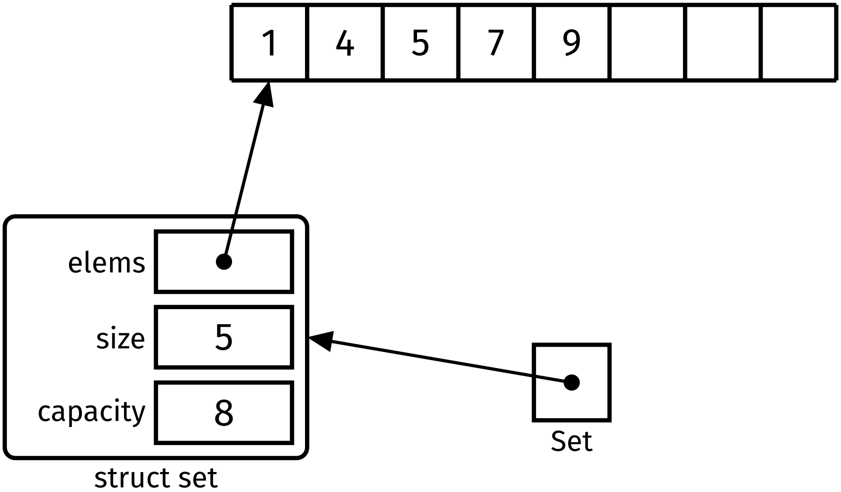
```
void SetDelete(Set s, int elem) {
    for (int i = 0; i < s->size; i++) {
        if (s->elems[i] == elem) {
            s->elems[i] = s->elems[s->size - 1];
            s->size--;
            return;
        }
    }
}
```

Time complexity: $O(n)$

- Finding the element is $O(n)$, overwriting it with the last element is $O(1)$

Set Implementation

Ordered array



How do we check if an element exists?

- Perform binary search $\Rightarrow O(\log n)$

```
bool SetContains(Set s, int elem) {
    int lo = 0;
    int hi = s->size - 1;

    while (lo <= hi) {
        int mid = (lo + hi) / 2;
        if (elem < s->elems[mid]) {
            hi = mid - 1;
        } else if (elem > s->elems[mid]) {
            lo = mid + 1;
        } else {
            return true;
        }
    }

    return false;
}
```

How do we insert an element?

- Use binary search to find the index of the smallest element which is *greater than or equal to* the given element
- If this element *is* the given element, then it already exists, so no need to do anything
- Otherwise, insert the element at that index and shift everything greater than it up

Time complexity of insertion?

- Binary search lets us find the insertion point in $O(\log n)$ time
- ...but we still have to potentially shift up to n elements, which is $O(n)$

How do we delete an element?

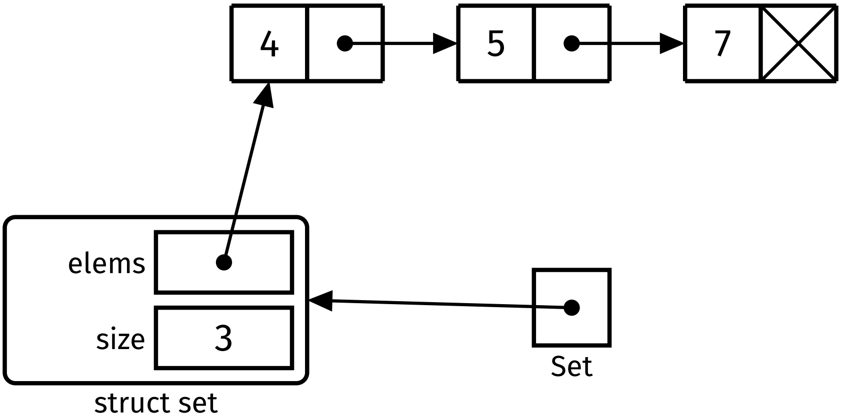
- Use binary search to find the element
- If the element exists, shift everything greater than it down

Time complexity?

- Binary search lets us find the element in $O(\log n)$ time
- ...but we still have to potentially shift up to n elements, which is $O(n)$

Set Implementation

Ordered linked list



How do we check if an element exists?

- Traverse the list $\Rightarrow O(n)$

```
bool SetContains(Set s, int elem) {
    for (struct node *curr = s->elems; curr != NULL; curr = curr->next) {
        if (curr->elem == elem) {
            return true;
        }
    }

    return false;
}
```

We always have to traverse the list from the start. Therefore...

- Insertion and deletion are also $O(n)$

However, this analysis hides a crucial advantage of linked lists:

- Finding the insertion/deletion point is $O(n)$
- But inserting/deleting a node is $O(1)$, as no shifting is required

Set ADT Summary

Data Structure	Contains	Insert	Delete
Unordered array	$O(n)$	$O(n)$	$O(n)$
Ordered array	$O(\log n)$	$O(n)$	$O(n)$
Ordered linked list	$O(n)$	$O(n)$	$O(n)$

Feedback

https://forms.office.com/r/5c0fb4tvMb