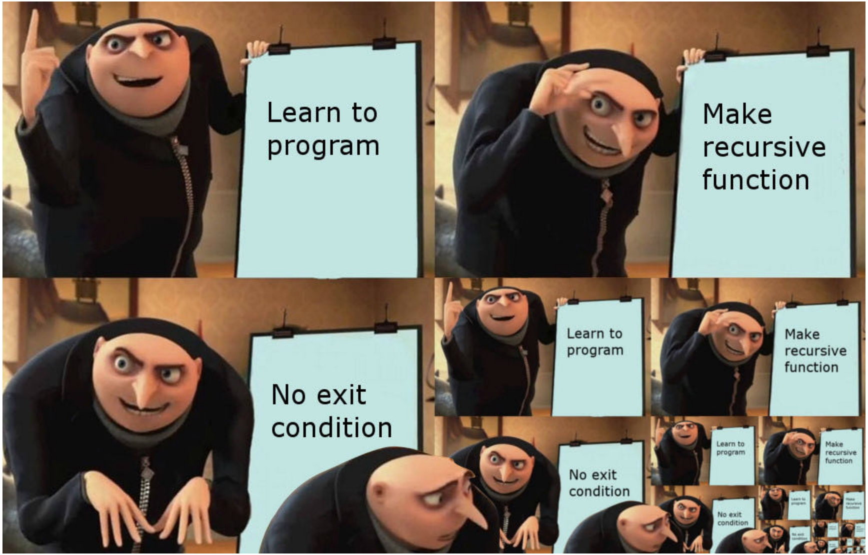
COMP2521 24T1
Recursion

Kevin Luxa

cs2521@cse.unsw.edu.au


Learn to program
Make recursive function
No exit condition
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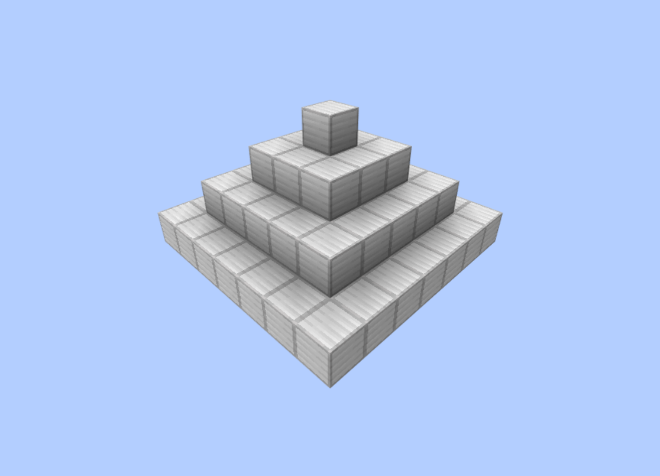
Definition

Recursion...
is a problem solving strategy where problems are solved via solving smaller or simpler instances of the same problem

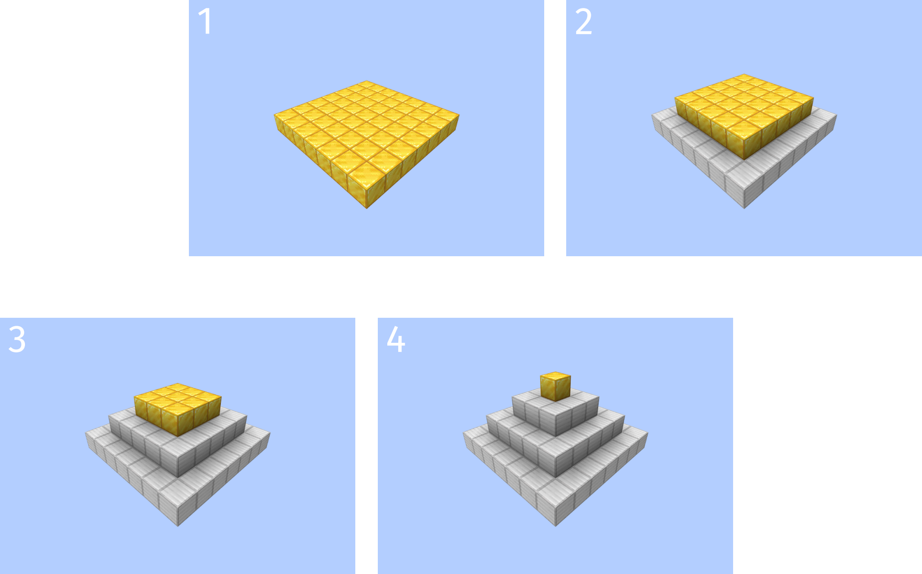
A recursive function calls itself

Example - Building a Pyramid



Example - Building a Pyramid

Iteratively



Example - Building a Pyramid

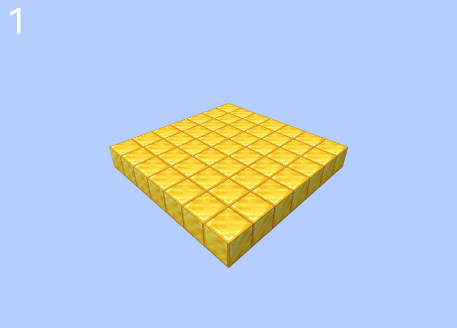
Iteratively

To build a pyramid of width n:

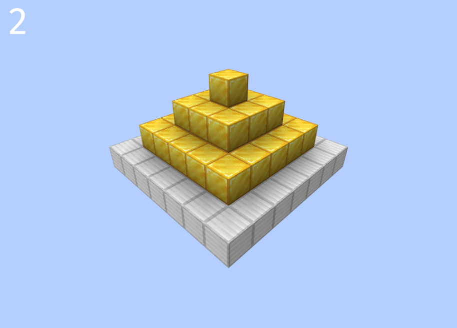
- For each width w from n down to 1 (decrementing by 2 each time):
  - Build a $w \times w$ layer of blocks on top

Example - Building a Pyramid

Recursively



Build a 7 x 7 layer of blocks



Build a pyramid of width 5 on top!

Example - Building a Pyramid

Recursively

To build a pyramid of width n:

1. Build an $n \times n$ layer
2. Then *build a pyramid of width* $n - 2$ on top

Example - Building a Pyramid

Recursively

To build a pyramid of width n:

1. Build an $n \times n$ layer
2. Then *build a pyramid of width* $n - 2$ on top

What's wrong with this method?

Example - Building a Pyramid

Recursively

To build a pyramid of width n:

1. If $n \leq 0$, do nothing
2. Otherwise:
    1. Build an $n \times n$ layer
    2. Then *build a pyramid of width* $n - 2$ on top

Example - Factorial

The factorial of n (where $n \geq 0$)
denoted by $n!$
is the product of all positive integers
less than or equal to n.

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$$

Example - Factorial

Iteratively

Iterative method:

```
int factorial(int n) {
    int res = 1;
    for (int i = 1; i <= n; i++) {
        res *= i;
    }
    return res;
}
```

Example - Factorial

Observation:

$$\begin{aligned} n! &= n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 \\ &= n \times (n-1)! \end{aligned}$$

For example:

$$\begin{aligned} 4! &= 4 \times 3 \times 2 \times 1 \\ &= 4 \times 3! \end{aligned}$$

Example - Factorial

Recursively

Recursive method:

```
int factorial(int n) {
    return n * factorial(n - 1);
}
```

Example - Factorial

Recursively

Recursive method:

```
int factorial(int n) {
    return n * factorial(n - 1);
}
```

What's wrong with this function?

Example - Factorial

Recursively

Recursive method:

```
int factorial(int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n - 1);
    }
}
```

Example - Factorial

Example:

```
factorial(3) = 3 * factorial(2)
             = 3 * (2 * factorial(1))
             = 3 * (2 * (1 * factorial(0)))
             = 3 * (2 * (1 * 1))
             = 3 * (2 * 1)
             = 3 * 2
             = 6
```

Example - Fibonacci

The Fibonacci sequence is a sequence where each number is the sum of the two previous numbers, and the first two numbers in the sequence are 0 and 1.

$$F_0 = 0$$
$$F_1 = 1$$
$$F_n = F_{n-1} + F_{n-2}$$

Example - Fibonacci

Recursive method:

```
int fib(int n) {
    if (n == 0) {
        return 0;
    } else if (n == 1) {
        return 1;
    } else {
        return fib(n - 1) + fib(n - 2);
    }
}
```

How Recursion Works

- A recursive function calls itself
- This is possible because there is a difference between a *function* and a *function call*
- Each function call creates a new mini-environment, called a *stack frame*, that holds all the local variables used by the function call

How Recursion Works

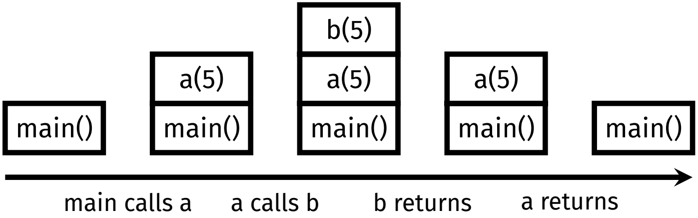
Consider this program (no recursion):

```c
int main(void) {
    a(5);
}

void a(int val) {
    b(val);
}

void b(int val) {
    printf("%d\n", val);
}
```

This is how the state of the stack changes:

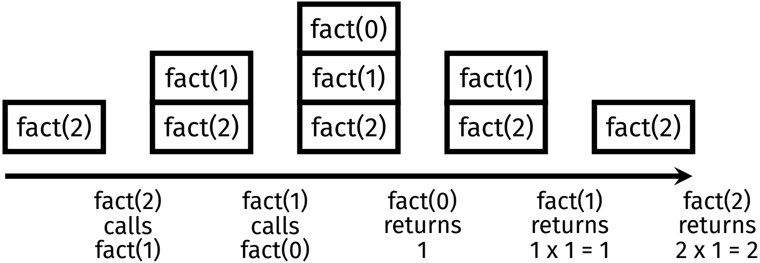


How Recursion Works

Now consider `factorial(2)`:

```c
int factorial(int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n - 1);
    }
}
```

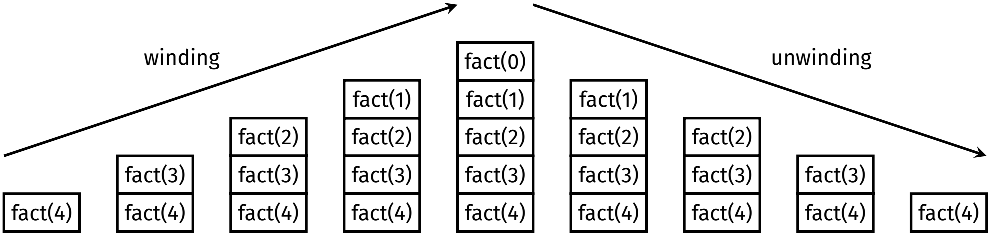
This is how the state of the stack changes:



How Recursion Works

When the stack is growing, that is called “winding”

When the stack is shrinking, that is called “unwinding”



How Recursion Works

Pre-order operations

Operations before the recursive call occur during winding.

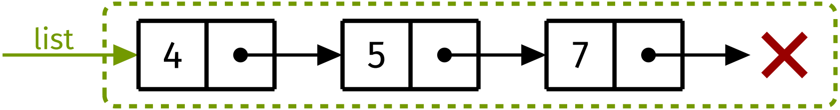
Post-order operations

Operations after the recursive call occur during unwinding.

Recursion on Linked Lists

Recall that recursion is
a problem solving strategy where problems are solved via
solving smaller or simpler instances of the same problem

How do we apply recursion to linked lists?



Recursion on Linked Lists

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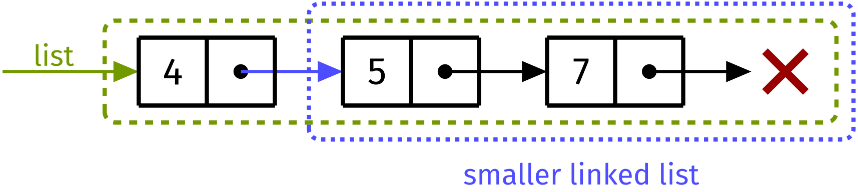
How do we apply recursion to linked lists?



Example - Summing a List

Example: summing values of a list

- Base case: empty list
  - Sum of an empty list is zero
- Non-empty lists
  - I can't solve the whole problem directly
  - But I do know the first value in the list
  - And if I can sum the rest of the list (smaller than whole list)
  - Then I can add the first value to the sum of the rest of the list, giving the sum of the whole list

Example - Summing a List

Example:

```
listSum([3, 1, 4]) = 3 + listSum([1, 4])
                   = 3 + (1 + listSum([4]))
                   = 3 + (1 + (4 + listSum([])))
                   = 3 + (1 + (4 + 0))
                   = 3 + (1 + 4)
                   = 3 + 5
                   = 8
```

Example - Summing a List

Recursive method:

```
struct node {
    int value;
    struct node *next;
};

int listSum(struct node *list) {
    if (list == NULL) {
        return 0;
    } else {
        return list->value + listSum(list->next);
    }
}
```

Example - List Append

Example: append a value to a list

```
struct node *listAppend(struct node *list, int value) {
    ...
}
```

`listAppend` should insert the given value at the end of the given list and return a pointer to the start of the updated list.

Example - List Append

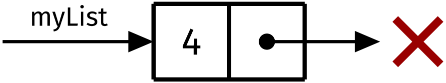
What’s wrong with this solution?

```c
struct node *listAppend(struct node *list, int value) {
    if (list == NULL) {
        return newNode(value);
    } else {
        listAppend(list->next, value);
        return list;
    }
}
```

Example - List Append


```c
struct node *listAppend(struct node *list, int value) {
    if (list == NULL) {
        return newNode(value);
    } else {
        listAppend(list->next, value);
        return list;
    }
}
```

Consider this list...



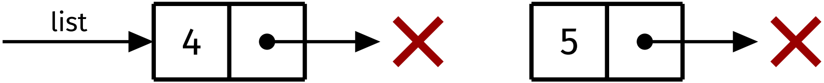
...and this function call:

```c
listAppend(myList, 5);
```

Example - List Append

```c
1  struct node *listAppend(struct node *list, int value) {
2      if (list == NULL) {
3          return newNode(value);
4      } else {
5          listAppend(list->next, value);
6          return list;
7      }
8  }
```

The recursive call on line 5 creates a new node and returns it...



...but this new node is not attached to the list!
The node containing 4 still points to NULL.

Example - List Append

Correct solution:

```
struct node *listAppend(struct node *list, int value) {
    if (list == NULL) {
        return newNode(value);
    } else {
        list->next = listAppend(list->next, value);
        return list;
    }
}
```

Example - List Append

Why does this work?

```
list->next = listAppend(list->next, value);
```

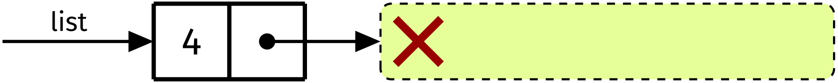
Consider the following list:

Two cases to consider:
(1) The rest of the list is empty
(2) The rest of the list is not empty

Example - List Append

```
list->next = listAppend(list->next, value);
```

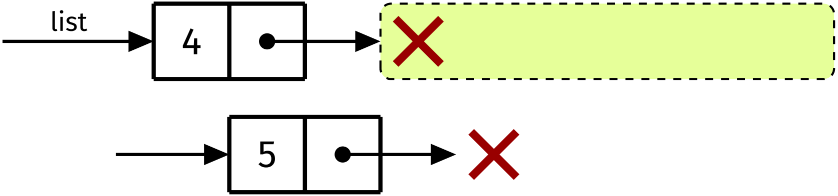
Case 1: The rest of the list is empty



Example - List Append

```
list->next = listAppend(list->next, value);
```

Case 1: The rest of the list is empty

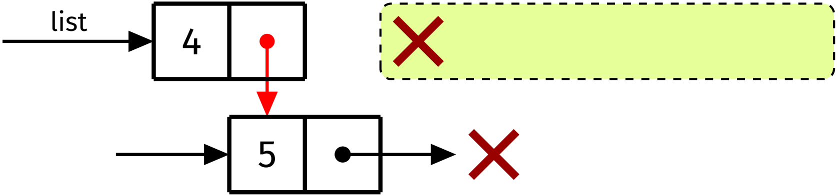


In this case, `listAppend(list->next, value)` will return a new node

Example - List Append


```
list->next = listAppend(list->next, value);
```

Case 1: The rest of the list is empty



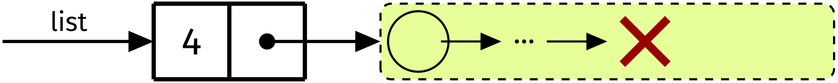
In this case, `listAppend(list->next, value)` will return a new node

`list->next = ...` causes `list->next` to point to this new node

Example - List Append

```
list->next = listAppend(list->next, value);
```

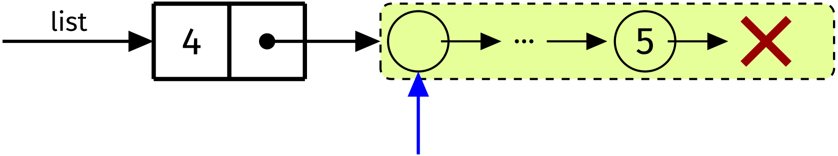
Case 2: The rest of the list is **not** empty



Example - List Append

```
list->next = listAppend(list->next, value);
```

Case 2: The rest of the list is **not** empty

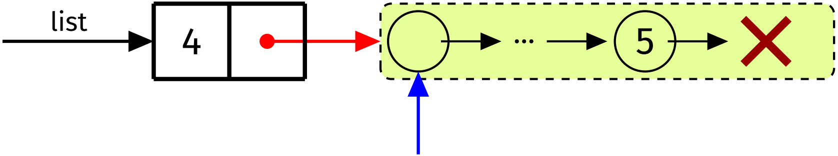


In this case, `listAppend(...)` will append the value to the rest of the list and return a pointer to the (start of the) rest of the list

Example - List Append

```
list->next = listAppend(list->next, value);
```

Case 2: The rest of the list is **not** empty



In this case, `listAppend(...)` will append the value to the rest of the list and return a pointer to the (start of the) rest of the list

`list->next = ...` causes `list->next` to point to the start of the rest of the list (which it was already pointing to)

How to Write a Recursive Function

1. Consider whether using recursion is appropriate
   - Can the solution be expressed in terms of a smaller instance of the same problem?
2. Identify the base case(s)
3. Identify the subproblem(s)
   - **Assume** that the function works for the subproblem(s)
     - Like in mathematical induction!
4. Think about how to relate the original problem to the subproblem(s)

Exercises

Exercise 1:

- Given a linked list, print the items in the list in reverse.

Exercise 2:

- Given a linked list and an index, return the value at that index. Index 0 corresponds to the first value, index 1 the second value, and so on.

Exercise 3:

- Given a linked list and a value, delete the first instance of the value from the list (if it exists), and return the updated list.

Recursive Helper Functions

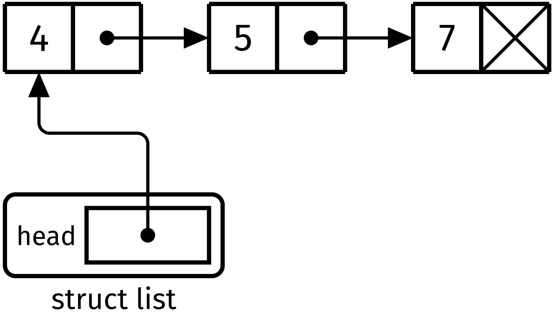
Sometimes, recursive solutions require recursive helper functions

- Data structure uses a “wrapper” struct
- Recursive function needs to take in extra information (e.g., state)

Recursive Helper Functions

Wrapper structs

Wrapper struct for a linked list:




```c
struct node {
    int value;
    struct node *next;
};

struct list {
    struct node *head;
};
```

Recursive Helper Functions

Wrapper structs

Example: Implement this function:

```
void listAppend(struct list *list, int value);
```

```c
void listAppend(struct list *list, int value);
```

We can't recurse with this function because our recursive function needs to take in a `struct node` pointer.

Solution: Use a recursive helper function!

Recursive Helper Functions

Wrapper structs

```
void listAppend(struct list *list, int value) {
    list->head = doListAppend(list->head, value);
}

struct node *doListAppend(struct node *node, int value) {
    if (node == NULL) {
        return newNode(value);
    } else {
        node->next = doListAppend(node->next, value);
        return node;
    }
}
```

Our convention for naming recursive helper functions is to prepend “do” to the name of the original function.

Recursive Helper Functions

Passing extra information

Problem:

- Print a linked list in a numbered list format, starting from 1.

```
void printNumberedList(struct node *list);
```

Example:

- Suppose the input list contains the following elements: [11, 9, 2023]
- We expect the following output:

```
1. 11
2. 9
3. 2023
```

Recursive Helper Functions

Passing extra information

We need to keep track of the current number.

Solution:

- Use a recursive helper function that takes in an extra integer

```
void printNumberedList(struct node *list) {
    doPrintNumberedList(list, 1);
}

void doPrintNumberedList(struct node *list, int num) {
    if (list == NULL) return;

    print("%d. %d\n", num, list->value);
    doPrintNumberedList(list->next, num + 1);
}
```

Recursion vs. Iteration

- If there is a simple iterative solution, a recursive solution will generally be slower
    - Due to a stack frame needing to be created for each function call
- A recursive solution will generally use more memory than an iterative solution

Feedback

https://forms.office.com/r/5c0fb4tvMb