

# System Modelling and Design

## Introduction to the B Method and B Toolkit

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# B Mathematical Toolkit

The mathematical toolkit of *B Method* (*B*) is based on

**set theory** simple set theory, consisting of aggregates having no ordering and no multiplicity. The only property possessed by a value and a set is membership of the set.

**logic** first-order predicate calculus. A predicate is a function from variables to Boolean. The first-order calculus allows quantification only over variables, not predicates for example.

**Numbers** Although B allows opaque types, essentially all numbers in a B development are eventually natural numbers, because real computers consist of binary numerals. B does not contain infinity and all implementable sets are finite. The set of natural numbers ( $\mathbb{N}$ ) is infinite and hence is not implementable.  $\mathbb{N}_1$  is  $\mathbb{N} - \{ 0 \}$

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# Set Theory

B uses sets to *model* other mathematical constructs such as: relations, functions, sequences.

The base for modelling with sets are

**powerset**  $P(S)$ , the powerset of the set  $S$ , is the set of all subsets of  $S$ .  $P(S)$  always contains the empty set.

$P_1(S)$  is the set of all *non-empty* subsets of  $S$ .

**product**  $X \times Y$ , the product of  $X$  and  $Y$ , is the set of ordered pairs with the first element from  $X$  and the second from  $Y$ .  $X \times Y = \{x, y \mid x \in X \wedge y \in Y\}$ .

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# Relations

A *relation* is a set of ordered pairs between the members of two sets.

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# Functions

	$\rightarrow$	set of partial functions
	$\rightarrow$	set of total functions
	$\rightarrow$	set of partial injection (one-to-one)
$X$	$\rightarrow$	set of total injection
	$\rightarrow$	set of partial surjection (onto)
	$\rightarrow$	set of total surjection
	$\rightarrow$	set of total bijection (one-to-one and onto)

# Some Terminology

The following terms will be used frequently:

**predicate** a predicate is a partial function from variables (state) to Boolean. The predicate is usually expressed as a closed expression, e.g. *amount* < *balance(customer)*.

**satisfies** we talk of some variables *satisfying* a predicate. This means that substituting the values of the variables into the predicate will make the predicate *true*.

**stronger and weaker** if  $P \Rightarrow Q$  we frequently say that, “*P* is *stronger than Q*”, although strictly we should say, “*P* is *at least as strong as Q*”. Similarly, we might say “*Q* is *weaker than P*”.

In the same vein we will talk of *strengthening* or *weakening* a predicate. Strengthening a predicate subsets the set of values that satisfy the predicate. Weakening a predicate superset the set of values that satisfy the predicate.

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All components of a B development will have a source form, used to specify machines and other input to the B-Toolkit, and a publication form used in documentation.

The notation for the source form will be `ASCII`. For example,

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account : ACCOUNT
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means the variable `account` is an element of the set `ACCOUNT`.

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state: consisting of a set of variables constrained by an *invariant*

operations: operations may change the state, *while maintaining the invariant*, and may return a sequence of results.

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For technical reasons that will not be explained now, machine variables in B must have at least two characters. Thus  $xx$  is a valid variable, while  $x$  is not.

**Warning:** this is likely to cause many mysterious problems in your first attempts to write B machines. The error messages of the B-Toolkit will not clearly identify the problem!

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# Object based

- Abstract machines are sometimes described as *object-based*, rather than *object-oriented*.
- You will notice that a machine can be compared with an *object*, that is, an instance of a *class*.
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# Substitutions

The foundation of B operations is a language called the *Generalised Substitution Language* or *GSL*. The GSL notation will not be described in this lecture. The elements of GSL are called *substitutions*, which have a role similar to statements or commands in a conventional programming language.

A substitution is a construct that, in some way, changes the state by substituting values into variables of the state.

The concept of the substitution is founded on the basic notion that the only way a state machine makes progress is by changing the value of the state.

We won't describe the GSL at this stage, but we will note that there are only 11 basis substitutions in the GSL.

Substitutions are given a formal semantics that in turn is expressed in in terms of substitution of values; thus the word “substitution” is a pun.

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# Abstract Machine Notation

*Abstract Machine Notation* (AMN) is the notation used to describe Abstract Machines.

AMN also incorporates a syntactic dressing up of the basic generalized substitution language (GSL).

AMN gives B an appearance and a feel of a programming language, although the level of abstraction is not changed by this syntactic sugaring.

We will use only a few AMN constructs here.

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# The B-Toolkit

The B-Toolkit is a configuration management tool that provides the following facilities:

## introduction of new machines

- animation of specifications
- automatic & interactive proof
- markup of machines
- generation of code
- execution of generated code
- automatic remakes
- hypertext displays of machines

- syntax and type analysis
- generation of proof obligations
- introduction of user theories
- maintenance of documents
- generation of interfaces
- generation of base machines
- browsing of designs & specifications
- online help

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# The B-Toolkit

The B-Toolkit is a configuration management tool that provides the following facilities:

introduction of new machines	syntax and type analysis
animation of specifications	generation of proof obligations
automatic & interactive proof	introduction of user theories
markup of machines	maintenance of documents
generation of code	generation of interfaces
execution of generated code	generation of base machines
automatic remakes	browsing of designs & specifications
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# The B-Toolkit interface

The interface of the B-Toolkit is very compact, but has a large number of configurations.

**Menu bar** the top line contains menus that control the functions of the toolkit.

**Environments** Below the menu bar is a set of environments: **Main**, **Provers**, etc that present different views on the development process.

**Machine panel** below the **Environments** is a panel that contains the names of machines or other constructs. This panel contains colour coded buttons that provide access to one of the functions of the toolkit.

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# Introducing a new machine

To introduce a new machine you would select  
**Introduce/New/Machine** in the **Main** environment of the B-Toolkit.

Having introduced the machine, a template will appear in your editor.  
The machine should be “filled in” and saved.

Then the machine should be committed and analyzed, by selecting  
the **cmt** (commit) and **anl** (analyze) buttons in the **Main** environment.

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# A Simple Model I

As a first simple model we will take a simple coffee club, but we will do it in two steps.

First we will model a “piggy bank” into which we can feed money and also take money out using the following operations:

**Feedbank**(*amount*)

feed *amount* cents to the piggybank.

**RobBank**(*amount*)

Rob the piggybank of *amount* cents.

*money*  $\leftarrow$  **CashLeft**

Query the piggybank to obtain the amount of *money* left in the



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# A Simple Model II

In order to model the operations we will use a variable *piggybank* whose value is a natural number, representing the contents of the piggybank in cents.

Let's step through the specification of a machine that “owns” and manages the piggy bank.

# A Simple Model II

In order to model the operations we will use a variable *piggybank* whose value is a natural number, representing the contents of the piggybank in cents.

Let's step through the specification of a machine that “owns” and manages the piggy bank.

# PiggyBank0.mch I

**MACHINE** *PiggyBank0*

**VARIABLES** *piggybank*

**INVARIANT** *piggybank*  $\in \mathbb{N}$

**INITIALISATION** *piggybank*  $:= 0$

**OPERATIONS**

# PiggyBank0.mch II

```
FeedBank ( amount )  $\hat{=}$   
  PRE amount  $\in \mathbb{N}$  THEN  
    piggybank := piggybank + amount  
  END ;
```

```
RobBank ( amount )  $\hat{=}$   
  PRE amount  $\in \mathbb{N}$  THEN  
    piggybank := piggybank - amount  
  END ;
```

```
money  $\longleftarrow$  CashLeft  $\hat{=}$   
  BEGIN  
    money := piggybank  
  END
```

```
END
```

# Machine Structure

MACHINE	name	set and numeric parameters
CONSTRAINTS	predicate	
INCLUDES/SEES/USES	machine	parameters
SETS	names	
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PROPERTIES	predicate	
VARIABLES	names	
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In general, the clauses of a machine can appear in any order, although machines are stored and marked up according to a canonic structure.

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Note the hierarchy of constraints (clauses consisting of a *predicate* in the machine structure)

*constraints* constrains the machine *parameters*

*properties* constrains the *sets* and *constants*

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# Machine Parameters

Machine parameters enable the specification of generic machines.

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The form of an operation is

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An *operation-signature* has the form

- $\boxed{\textit{name}(\textit{args})}$  for an operation that only makes a state substitution, or
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# Invariant and Preconditions

The invariant of a machine is an expression of the properties that the state has to satisfy for the operations to correctly model the required behaviour.

The invariant expresses what might be called *safety* or *integrity* conditions.

The initial state must satisfy the invariant, and it is an obligation that each operation *maintains* the invariant: it is guaranteed that the invariant is true before an operation is invoked and it is the duty of the operation to ensure that the invariant is true after the operation.

The precondition of an operation should capture all combinations of state and operation arguments before an operation that are required to ensure that the invariant is satisfied after the operation.

It is important that the invariant is as strong as necessary, and the precondition is as weak as possible, but no weaker than necessary.



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# Trivial preconditions

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Alternatively, generate the proof obligations and try to discharge them.

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# Proof obligation generation and proof

Having analyzed a machine, you should routinely generate the proof obligations by selecting the **pog** (proof obligation generator) button in the **Main** environment.

Then move to the **Provers** environment, select the **prv** (provers) button for the machine, and select **AutoProver**. If there are unproved obligations then you should either try to discharge the proof obligation using the **BToolProver**, or at least inspect the obligation to see if it is true.

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# Viewing the proof obligations

Select the **Provers** environment and select the **ppf** (prettyprint proof) button for the machine of interest.

Select the proof obligations from the list.

Select the **Documents** environment, and notice that there is a green **.prf** construct for the chosen machine.

Mark-up the proof obligations by selecting the **dmu** (document markup) button; the view by selecting the **shw** (show) button.

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# Adding a non-trivial precondition

An attempt to discharge the outstanding proof obligation for the operation `RobBank` will leave  $amount \leq piggybank$  unprovable.

This occurs because the machine invariant says that  $piggybank \in \mathbb{N}$ , that is  $0 \leq piggybank$  both before and after an operation.

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# Towards understanding preconditions

Run the following experiment:

- 1 run the animator on **PiggyBank** with RobBank having a trivial precondition;
- 2 run the animator on **PiggyBank** with RobBank having the non-trivial precondition.

In each case:

- 1 enable display invariants — the default is not displays
- 2 run:
  - FeedBank(5)
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Describe the results. Notice very carefully that failure of the precondition **does not stop** the operation from going ahead.

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- 1 run the animator on **PiggyBank** with `RobBank` having a trivial precondition;
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In each case:

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# Total and Partial operations: preconditions

Operations without non-trivial preconditions are *total* operations: that is the operation may be invoked in any state of the machine, and for any value of the arguments of the operation. Such operations are also called *robust*.

Operations with non-trivial preconditions are *partial* operations: that is the operation may not be defined outside of the precondition. Such operations are also called *fragile*.

A precondition is an *assumption*, it is not a condition that is going to be tested by the implementer of the operation.

It is the obligation of the invoker of the operation to ensure that the precondition holds. The precondition is the part of the contract that applies to the client of the operation.

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# Modelling a Coffee Club

We will now model a coffee club with the following facilities for members:

**Joining** a person can join the club. For the purpose of this simple exercise we identify each member by an element of the set `NAME`. Of course we want all members to be distinct.

**Contributing** members can contribute money to the club. This is used to increase the credit of the member, which in turn is used to pay for cups of coffee.

**Buy coffee** a member can buy a cup of coffee. The price of a cup of coffee is deducted from the members credit.

**Credit** a member can obtain their current credit balance.

The above behaviour is modelled by the machine `CoffeeClub`, initially named `CoffeeClub0`.

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# A CoffeeClub machine I

**MACHINE** *CoffeeClub0* ( *NAME* )  
**INCLUDES** *PiggyBank*  
**PROMOTES** *RobBank* , *CashLeft*  
**CONSTANTS** *coffee*  
**PROPERTIES** *coffee* = 120  
**VARIABLES** *finances*  
**INVARIANT** *finances*  $\in$  *NAME*  $\leftrightarrow \mathbb{N}$   
**INITIALISATION** *finances* := { }

## OPERATIONS



# A CoffeeClub machine II

**NewMember** ( *member* )  $\hat{=}$

**PRE** *member*  $\in$  *NAME*

**THEN**

*finances* ( *member* )  $:= 0$

**END ;**

**Contribute** ( *member* , *amount* )  $\hat{=}$

**PRE** *member*  $\in$  *NAME*  $\wedge$  *amount*  $\in \mathbb{N}$

**THEN**

*finances* ( *member* )  $:=$  *finances* ( *member* ) + *amount* ||

*FeedBank* ( *amount* )

**END ;**

# A CoffeeClub machine III

**BuyCoffee** ( *member* )  $\hat{=}$

**PRE** *member*  $\in$  *NAME*

**THEN**

*finances* ( *member* )  $:=$  *finances* ( *member* )  $-$  *coffee*

**END ;**

*credit*  $\leftarrow$  **Credit** ( *member* )  $\hat{=}$

**PRE** *member*  $\in$  *NAME*

**THEN** *credit*  $:=$  *finances* ( *member* )

**END**

**END**

# Aspects of CoffeeClub0

Aspects of this machine are:

- The NAME set is represented by a machine parameter.
- The PiggyBank machine is *included* into this machine. This embeds the state of PiggyBank into this machine, and gives CoffeeClub access to the operations of PiggyBank.
- The operations RobBank and CashLeft are *promoted* to the interface of CoffeeClub.
- A constant `coffee` is used for the cost of a cup of coffee.
- The state of the machine consists of a variable `finances`, which is a partial function from NAME to N.
- Three operations NewMember, Contribute, BuyCoffee and Credit are used to model the required behaviour.

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# Some notes on machine inclusion

Included machine state: the included machine's state is “added” to the state of the including machine.

Referencing included state: the variables in the state of the included machine may be referenced by the including machine.

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# Problems with CoffeeClub

The specification given by this machine is not adequate. It is easy to show that the operations can break the invariant.

Generating the proof obligations and attempting to discharge them will illustrate some of the problems. Run the AutoProver on the proof obligations and examine any undischarged proof obligations.

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# Identifying and fixing the problems I

The problems are enumerated below:

**NewMember** this operation has an undesirable functional property: if an existing member—or a new member with the same name as an existing member—with credit runs this operation then their finances are set to 0! The specification alerts the user to this undesirable effect by adding a precondition  $member \notin \text{dom}(finances)$ , that is, the prospective member is not an existing member.

**Contribute** the function *finances* is partial, so the expression used to update the member's finances:

$$finances(member) := finances(member) + amount$$

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# Identifying and fixing the problems II

**BuyCoffee** In order to buy a coffee, two things are required

- 1 the person must be a member, otherwise *finances(member)* will be undefined;
- 2 a member must have enough finance to cover the price of a cup of coffee. If this is not the case then *finances(member) – coffee* will not be a natural number, breaking the invariant.

So a precondition of:

$$\begin{aligned} & \text{member} \in \text{dom}(\text{finances}) \wedge \\ & \text{finances}(\text{member}) \geq \text{coffee} \end{aligned}$$

is required.

**Credit** *finances(member)* assumes *member*  $\in$  *dom(finances)*, so this needs to be added to the precondition.

The following versions of **PiggyBank** and **CoffeeClub** have appropriately strengthened preconditions.

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# PiggyBank.mch I

**MACHINE** *PiggyBank*

**VARIABLES** *piggybank*

**INVARIANT** *piggybank*  $\in \mathbb{N}$

**INITIALISATION** *piggybank*  $:= 0$

**OPERATIONS**

# PiggyBank.mch II

**FeedBank** ( *amount* )  $\hat{=}$

**PRE** *amount*  $\in \mathbb{N}$  **THEN**

*piggybank* := *piggybank* + *amount*

**END ;**

**RobBank** ( *amount* )  $\hat{=}$

**PRE** *amount*  $\in \mathbb{N} \wedge \textit{amount} \leq \textit{piggybank}$  **THEN**

*piggybank* := *piggybank* - *amount*

**END ;**

*money*  $\longleftarrow$  **CashLeft**  $\hat{=}$

**BEGIN**

*money* := *piggybank*

**END**

**END**

# CoffeeClub.mch I

**MACHINE** *CoffeeClub* ( *NAME* )  
**INCLUDES** *PiggyBank*  
**PROMOTES** *RobBank* , *CashLeft*  
**CONSTANTS** *coffee*  
**PROPERTIES** *coffee* = 120  
**VARIABLES** *finances*  
**INVARIANT** *finances*  $\in$  *NAME*  $\rightarrow \mathbb{N}$   
**INITIALISATION** *finances* := {}

## OPERATIONS

# CoffeeClub.mch II

**NewMember** ( *member* )  $\hat{=}$

**PRE**  $member \in NAME \wedge member \notin \text{dom} ( finances )$

**THEN**

$finances ( member ) := 0$

**END ;**

**Contribute** ( *member* , *amount* )  $\hat{=}$

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**THEN**

$finances ( member ) := finances ( member ) + amount \parallel$

$FeedBank ( amount )$

**END ;**



# CoffeeClub.mch III

**BuyCoffee** ( *member* )  $\hat{=}$

**PRE**  $member \in NAME \wedge member \in \text{dom} ( finances ) \wedge$   
 $finances ( member ) \geq coffee$

**THEN**

$finances ( member ) := finances ( member ) - coffee$

**END ;**

$credit \leftarrow$  **Credit** ( *member* )  $\hat{=}$

**PRE**  $member \in NAME \wedge member \in \text{dom} ( finances )$

**THEN**  $credit := finances ( member )$

**END**

**END**

# Specifying a Robust machine

Most of the operations of the **CoffeeClub** machine are fragile, that is the operations have non-trivial preconditions. This means that there are combinations of state and operations arguments for which the operation will fail.

Such operations are not safe to use in an application programmer interface (API) or user interface (UI).

We will build an API machine, **CoffeeClubAPI**, with robust versions of the operations of **CoffeeClub**. These operations will use guards that discharge the precondition of the fragile operation ensuring that it is safe to invoke the fragile operation.

Each operation returns a response that reports whether the operation was successful, or why the precondition failed.

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Such operations are not safe to use in an application programmer interface (API) or user interface (UI).

We will build an API machine, **CoffeeClubAPI**, with robust versions of the operations of **CoffeeClub**. These operations will use guards that discharge the precondition of the fragile operation ensuring that it is safe to invoke the fragile operation.

Each operation returns a response that reports whether the operation was successful, or why the precondition failed.

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# CoffeeClubAPI.mch I

**MACHINE** *CoffeeClubAPI* ( *NAME* )

**INCLUDES** *CoffeeClub* ( *NAME* )

**SETS** *RESPONSE* = { *OK* ,

*existing\_member* ,

*not\_a\_member* ,

*not\_enough\_finance* ,

*not\_enough\_in\_bank* }

## OPERATIONS

# CoffeeClubAPI.mch II

```
response  $\leftarrow$  NewMemberAPI ( member )  $\hat{=}$   
  PRE member  $\in$  NAME THEN  
    IF member  $\in$  dom ( finances )  
    THEN response := existing_member  
    ELSE  
      response := OK || NewMember ( member )  
    END  
  END ;  
response  $\leftarrow$  ContributeAPI ( member , amount )  $\hat{=}$   
  PRE member  $\in$  NAME  $\wedge$  amount  $\in \mathbb{N}$  THEN  
    IF member  $\notin$  dom ( finances ) THEN  
      response := not_a_member  
    ELSE response := OK || Contribute ( member , amount )  
    END  
  END ;
```

## CoffeeClubAPI.mch III

```

response  $\leftarrow$  BuyCoffeeAPI ( member )  $\hat{=}$ 
  PRE member  $\in$  NAME THEN
    SELECT member  $\notin$  dom ( finances ) THEN
      response := not_a_member
    WHEN finances ( member ) < coffee THEN
      response := not_enough_finance
    ELSE response := OK || BuyCoffee ( member )
    END
  END ;
response , credit  $\leftarrow$  CreditAPI ( member )  $\hat{=}$ 
  PRE member  $\in$  NAME THEN
    IF member  $\notin$  dom ( finances ) THEN
      response := not_a_member || credit : $\in$   $\mathbb{N}$ 
    ELSE response := OK || credit  $\leftarrow$  Credit ( member )
    END
  END ;

```



# CoffeeClubAPI.mch IV

```
response  $\leftarrow$  RobBankAPI ( amount )  $\hat{=}$   
  PRE amount  $\in \mathbb{N}$  THEN  
    IF piggybank < amount THEN  
      response := not_enough_in_bank  
    ELSE response := OK || RobBank ( amount )  
    END  
  END ;  
money  $\leftarrow$  CashLeftAPI  $\hat{=}$  money  $\leftarrow$  CashLeft  
END
```

# A Question of Identity

The CoffeeClub, in addition to being a very simple model, also exhibits a serious deficiency:

It uses names for the identity of members.

This is clearly inadequate. For example we have a restriction that two people with the same name cannot belong to the club.

In all *real* systems we need to allocate unique identifiers for each member of—for each component of—a system.

Subsequent system models will demonstrate this.

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