COMP1917: Computing 1

16. Binary Search Trees

Reading: Moffat, Section 10.3, 10.5

Binary Search Trees - Motivation

- a Linked List is a one-dimensional recursive structure – each node has one pointer to the next node
  - Problem: finding, deleting or inserting items takes a long time because of the need to linearly search for the item of interest
- a Binary Tree is constructed from nodes, where each node contains:
  - a “left” pointer (which could be `NULL`)
  - a “right” pointer (which could be `NULL`)
  - a “data” value
- this leads to a multi-branching recursive structure which can speed up finding, deleting and inserting of items considerably

Binary Tree Structure

We define a self-referential structure similar to a Linked List, but with two pointers, to the “left” and “right” branch:

```
typedef struct tnode Tnode;

struct tnode {
    int    data;
    Tnode *left;
    Tnode *right;
};
```

Binary Trees

- a “root” pointer points to the topmost node in the tree
- the left and right pointers recursively point to smaller “subtrees” on either side
- a `NULL` pointer represents a binary tree with no elements – an empty tree (or subtree)

Binary Search Tree

- a Binary Search Tree is a binary tree in which the items are ordered from left to right across the tree.
- To ensure the items remain ordered, new items must be inserted according to these rules:
  - if the new item has a data value less than the data value at this node, it is recursively inserted into the left subtree
  - if the new item has a data value greater than the data value at this node, it is recursively inserted into the right subtree

Building a Binary Search Tree

The shape of the tree depends on the order in which the nodes are inserted:

```
 10,5,15,2,8,12     5,2,10,8,15,12     8,12,5,15,2,10


       10                 5                  8
      /  \               / \                / \
     5    15            2   10             5   12
    / \  /                  / \           /   / \
   2   8 12                8   15        2   10  15
                               /
                              12
```

Question: what if the order is 2, 5, 8, 10, 12, 15 ?

Binary Search Tree Operations

```
Tnode * makeTnode( int data ); // create new node

Tnode * findTnode( int data, Tnode *root );

Tnode * insertTnode( Tnode *new_node, Tnode *root );

void printTree( Tnode *root ); // print all items
void freeTree ( Tnode *root ); // free entire tree

int treeSize   ( Tnode *root ); // number of items
int treeHeight( Tnode *root ); // max depth of an item
```


Making a New Node

```
/*
   Create a new Tnode with the specified data value.
*/
Tnode * makeTnode( int data )
{
   Tnode *new_node =(Tnode *)malloc( sizeof( Tnode ));
   if( new_node == NULL ) {
      fprintf(stderr,"Error: memory allocation failed.\n");
      exit( 1 );
   }
   new_node->data  = data;
   new_node->left  = NULL;
   new_node->right = NULL;
   return( new_node );
}
```


Finding a Node in the Tree

```
/*
   Search tree to find a node with specified data value.
*/
Tnode * findTnode( int data, Tnode *root )
{
   Tnode *node = root; // start at the root of the tree

   // keep searching until data found, or exit from tree
   while(( node != NULL )&&( node->data != data )) {
      if( data < node->data )
         node = node->left;  // branch left or right,
      else                   // depending on data value
         node = node->right;
   }
   return( node );
}
```


Recursive version of `findTnode()`

```
/*
   Search tree to find a node with the specified
   data value, and return a pointer to this node.
   If no such node exists, return NULL.
*/
Tnode * findTnode( int data, Tnode *root )
{
   if(( root == NULL )||( root->data == data )) {
      return( root );      // found node, or exited tree
   }
   else if( data < root->data ) { // search left subtree
      return( findTnode( data, root->left  ));
   }
   else {                         // search right subtree
      return( findTnode( data, root->right ));
   }
}
```


Insert New Node into Binary Search Tree

```
Tnode * insertTnode( Tnode *new_node, Tnode *root )
{
   Tnode *child = root, *parent = NULL;
   while( child != NULL ) {  // find parent for new node
       parent = child;
       if( new_node->data < parent->data )
           child = parent->left;
       else
           child = parent->right;
   }
   if( parent == NULL )              // tree was empty
       root = new_node;
   else if( new_node->data < parent->data )
       parent->left  = new_node; // insert to the left
   else
       parent->right = new_node; // insert to the right
   return( root );
}
```

Recursive version of `insertTnode()`

```
Tnode * insertTnode( Tnode *new_node, Tnode *root )
{
   if( root == NULL ) { // we have reached the insertion point
      root = new_node;
   }
   else if( new_node->data < root->data ) {
          // insert new node into (and update) left subtree
      root->left  = insertTnode(new_node, root->left);
   }
   else { // insert new node into (and update) right subtree
      root->right = insertTnode(new_node, root->right);
   }
   return( root );
}
```

Printing a Binary Search Tree

```
/*
   Print all items of the tree in order
*/
void printTree( Tnode *root )
{
  if( root != NULL ) {
     printTree( root->left  ); // recursively print smaller items
     printf("%c",root->data ); //             print current item
     printTree( root->right ); // recursively print larger items
  }
}
```

Freeing all items from a Tree

```
/*
   Recursively free all the items from a Binary Tree
*/
void freeTree( Tnode *root )
{
   if( root != NULL ) {
      freeTree( root->left );
      freeTree( root->right );
      free( root );
   }
}
```

Computing the Size of a Tree

```
/*
   Compute the size of a Binary Tree
  (the number of items stored in the tree)
*/
int treeSize( Tnode *root )
{
   if( root == NULL ) {
      return( 0 );
   }
   else {
      return( 1 + treeSize( root->left  )
                + treeSize( root->right ));
   }
}
```

Computing the Height of a Tree

```
int treeHeight( Tnode *root )
{
   int leftHeight, rightHeight;

   if( root == NULL ) {
      return( 0 );
   }
   else {
      leftHeight  = treeHeight( root->left );
      rightHeight = treeHeight( root->right );

      if( leftHeight > rightHeight )
         return( 1 + leftHeight );
      else
         return( 1 + rightHeight );
   }
}
```


Questions

- what is the maximum number of items that could be stored in a BST with height `H` ?
- if the number of items in a BST is `N`, what is the minimum height such a tree could have?
- how would you delete an item from a BST (so that the BST structure is preserved)?