COMP1521 25T1

Week 5 Lecture 1

Bitwise Operators and Floating Point

Adapted from Hammond Pearce,
Andrew Taylor and John Shepherd’s slides

Assignment 1 is due Friday 6pm

Week 4: test: due thursday 9pm (MIPS basics, control, arrays)

Week 6 Flexibility Week

Week 6 (next week)is flexibility week so nothing is due then!

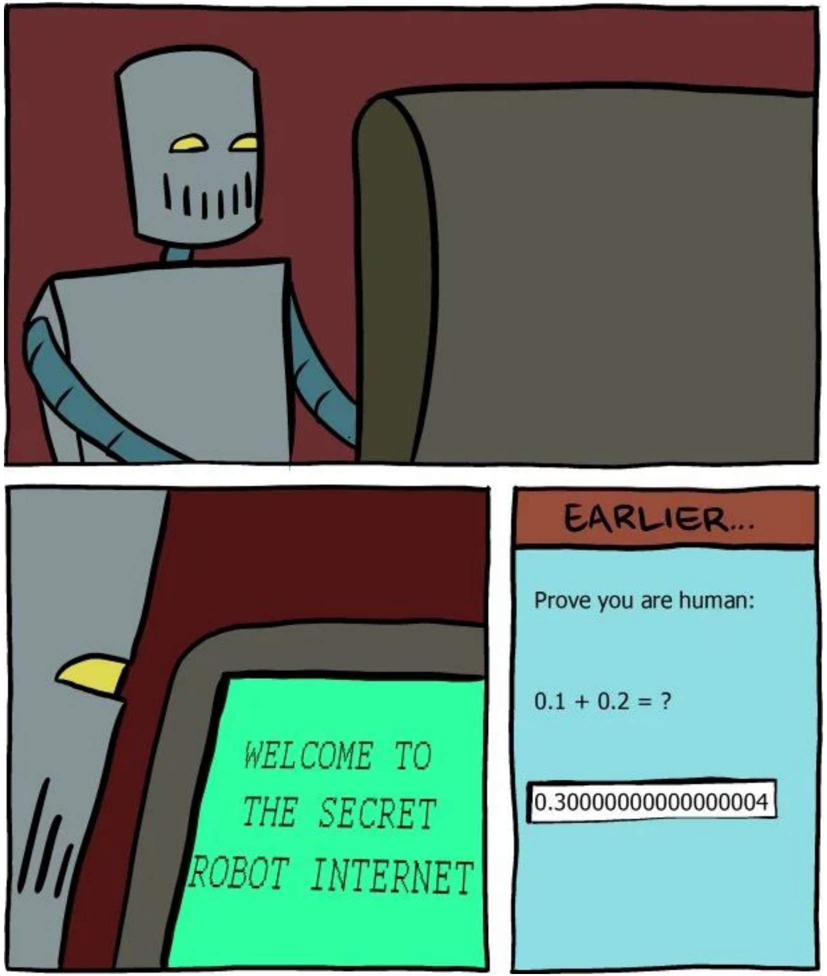
Week 5 lab: due monday midday week 7

Week 5 test: due thursday 9pm week 7 (MIPS strings)

Week 6 test: due thursday 9pm week 7 (bitwise operators C)

Today's Lecture

- Bitwise Operators
  - Recap
  - MIPS examples
  - C Coding examples
- Floating Point Representation



Recap Demo: bitwise.c

```
$ dcc bitwise.c print_bits.c -o bitwise
$ ./bitwise
Enter a: 23032
Enter b: 12345
Enter c: 3
     a = 0101100111111000 = 0x59f8 = 23032
     b = 0011000000111001 = 0x3039 = 12345
    ~a = 1010011000000111 = 0xa607 = 42503
 a & b = 0001000000111000 = 0x1038 = 4152
 a | b = 0111100111111001 = 0x79f9 = 31225
 a ^ b = 0110100111000001 = 0x69c1 = 27073
a >> c = 0000101100111111 = 0x0b3f = 2879
a << c = 1100111111000000 = 0xcfc0 = 53184
```

Exercise 1

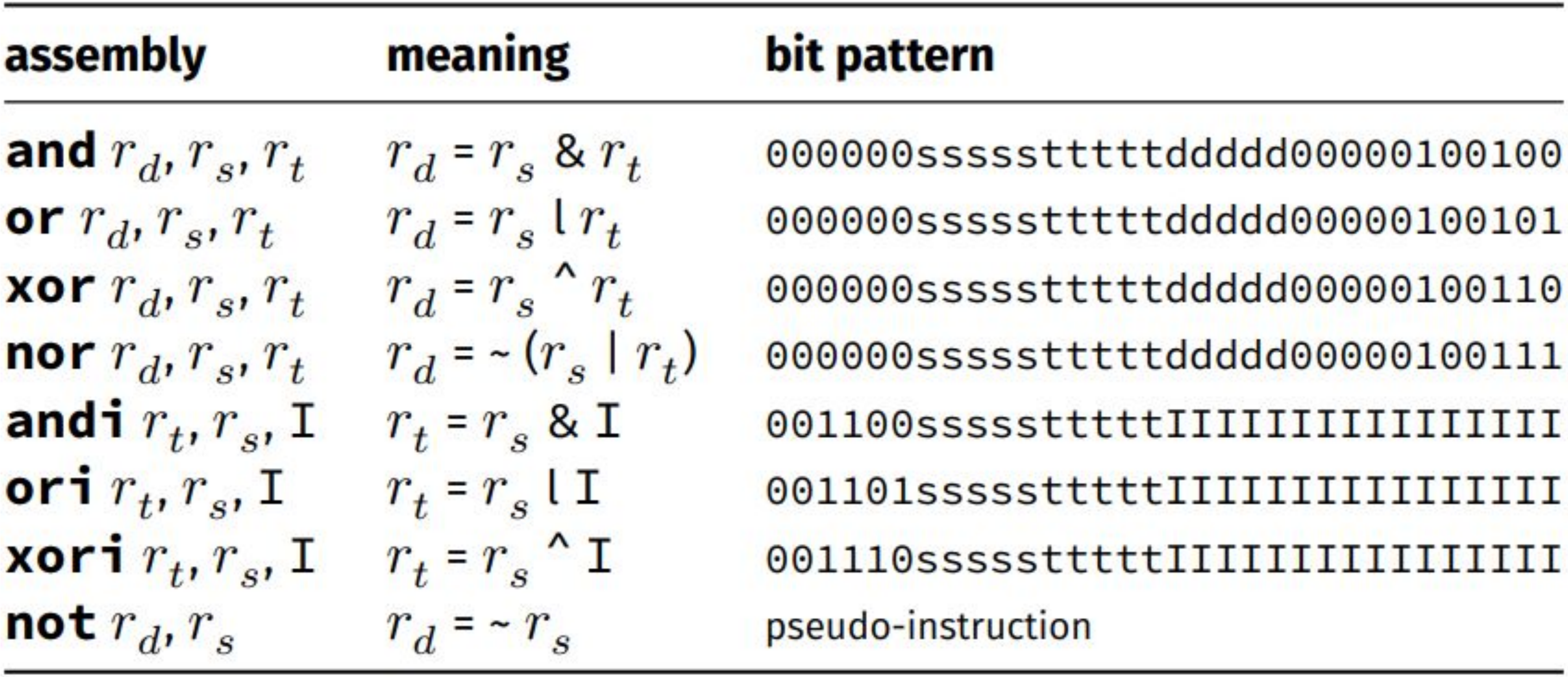
Given the following declarations:

```
// a signed 8-bit value
   uint8_t x = 0x55;
   uint8_t y = 0xAA;
```

What is the value of each of these expressions?

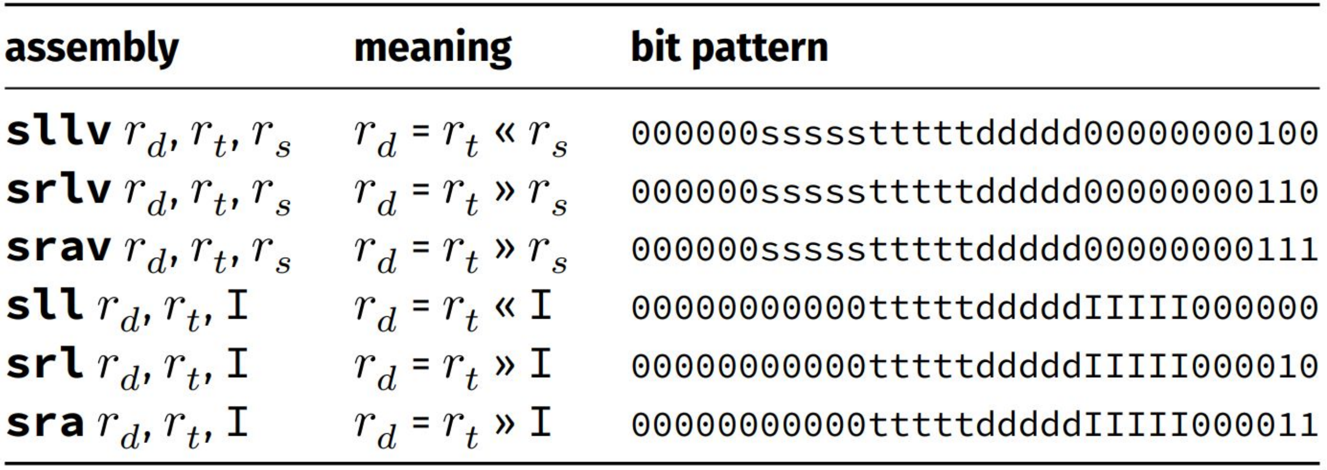
```
uint8_t a = x & y;          uint8_t e = x >> 1;
uint8_t b = x ^ y;          uint8_t f = y >> 2;
uint8_t c = x | y;          uint8_t g = y << 2;
uint8_t d = ~x
```

MIPS - Bit manipulation instructions



assembly	meaning	bit pattern
and r_d, r_s, r_t	$r_d = r_s \ \& \ r_t$	000000ssssstttttddddd00000100100
or r_d, r_s, r_t	$r_d = r_s \mid r_t$	000000ssssstttttddddd00000100101
xor r_d, r_s, r_t	$r_d = r_s \ \hat{} \ r_t$	000000ssssstttttddddd00000100110
nor r_d, r_s, r_t	$r_d = \sim(r_s \mid r_t)$	000000ssssstttttddddd00000100111
andi r_t, r_s, I	$r_t = r_s \ \& \ \mathrm{I}$	001100ssssstttttIIIIIIIIIIIIIIII
ori r_t, r_s, I	$r_t = r_s \mid \mathrm{I}$	001101ssssstttttIIIIIIIIIIIIIIII
xori r_t, r_s, I	$r_t = r_s \ \hat{} \ \mathrm{I}$	001110ssssstttttIIIIIIIIIIIIIIII
not r_d, r_s	$r_d = \sim r_s$	pseudo-instruction

MIPS - Shift instructions



assembly	meaning	bit pattern
sllv r_d, r_t, r_s	$r_d = r_t \ll r_s$	000000ssssstttttddddd00000000100
srlv r_d, r_t, r_s	$r_d = r_t \gg r_s$	000000ssssstttttddddd00000000110
srav r_d, r_t, r_s	$r_d = r_t \gg r_s$	000000ssssstttttddddd00000000111
sll r_d, r_t, I	$r_d = r_t \ll \mathrm{I}$	00000000000tttttdddddIIIII000000
srl r_d, r_t, I	$r_d = r_t \gg \mathrm{I}$	00000000000tttttdddddIIIII000010
sra r_d, r_t, I	$r_d = r_t \gg \mathrm{I}$	00000000000tttttdddddIIIII000011

- **`srl`** and **`srlv`** shift zeroes into most-significant bit
  - This matches shift in C of unsigned values
- **`sra`** and **`srav`** propagate most-significant bit
  - This ensures the sign is maintained

MIPS Code Demos

- odd_even.s
- mips_bits.s
- mips_negative_shifts.s

Code Demos

- xor.c
- pokemon.c
- set_low_bits0.c
- set_low_bits.c
- set_bits_in_range.c
- extract_bits_in_range.c
- bitset.c

Demo: pokemon.c

```
$ dcc pokemon.c print_bits.c -o pokemon
$ ./pokemon
0000010000000000 BUG_TYPE
0000000000010000 POISON_TYPE
1000000000000000 FAIRY_TYPE
1000010000010000 our_pokemon type (1)

Poisonous
1001010000000000 our_pokemon type (2)

Scary
```

Demo: pokemon.c

```
#define FIRE_TYPE      0x0001
#define FIGHTING_TYPE  0x0002
#define WATER_TYPE     0x0004
#define FLYING_TYPE    0x0008
#define POISON_TYPE    0x0010
#define ELECTRIC_TYPE  0x0020
#define GROUND_TYPE    0x0040
#define PSYCHIC_TYPE   0x0080
#define ROCK_TYPE      0x0100
#define ICE_TYPE       0x0200
#define BUG_TYPE       0x0400
#define DRAGON_TYPE    0x0800
#define GHOST_TYPE     0x1000
#define DARK_TYPE      0x2000
#define STEEL_TYPE     0x4000
#define FAIRY_TYPE     0x8000
```

Demo: pokemon.c

```
$ dcc pokemon.c print_bits.c -o pokemon
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0000010000000000 BUG_TYPE
0000000000010000 POISON_TYPE
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1000010000010000 our_pokemon type (1)

Poisonous
1001010000000000 our_pokemon type (2)

Scary
```

Demo: set_low_bits.c

```
$ dcc set_low_bits.c print_bits.c -o set_low_bits
$ ./set_low_bits 3

The bottom 3 bits of 7 are ones:
00000000000000000000000000000111
$ ./set_low_bits 19

The bottom 19 bits of 524287 are ones:
00000000000001111111111111111111
$ ./set_low_bits 29

The bottom 29 bits of 536870911 are ones:
00011111111111111111111111111111
```

Demo: set_bit_range.c

```
$ dcc set_bit_range.c print_bits.c -o set_bit_range
$ ./set_bit_range 0 7

Bits 0 to 7 of 255 are ones:
00000000000000000000000011111111
$ ./set_bit_range 8 15

Bits 8 to 15 of 65280 are ones:
00000000000000001111111100000000
$ ./set_bit_range 8 23

Bits 8 to 23 of 16776960 are ones:
00000000111111111111111100000000
$ ./set_bit_range 1 30

Bits 1 to 30 of 2147483646 are ones:
01111111111111111111111111111110
```

Demo: extract_bit_range.c

```
$ dcc extract_bit_range.c print_bits.c -o extract_bit_range
$ ./extract_bit_range 4 7 42

Value 42 in binary is:
00000000000000000000000000101010

Bits 4 to 7 of 42 are:
0010
$ ./extract_bit_range 10 20 123456789

Value 123456789 in binary is:
00000111010110111100110100010101

Bits 10 to 20 of 123456789 are:
11011110011
```

Demo: bitset.c

```
$ dcc bitset.c print_bits.c -o bitset
$ ./bitset

Set members can be 0-63, negative number to finish

Enter set a: 1 2 4 8 16 32 -1

Enter set b: 5 4 3 33 -1
a = 0000000000000000000000000000000100000000000000010000000100010110 = 0x100010116 =
4295033110
b = 0000000000000000000000000000001000000000000000000000000000111000 = 0x200000038 =
8589934648
a = {1,2,4,8,16,32}
b = {3,4,5,33}
a union b = {1,2,3,4,5,8,16,32,33}
a intersection b = {4}
cardinality(a) = 6
is_member(42, a) = 0
```

IEEE 754 Floating Point Representation

- The industry standard
  - Used by almost all computers
- Crucial to understand when working with numeric computations
- Understand precision and accuracy limitations
  - Why using them for finance is unwise
  - Why sometimes
    - a + 1 == a
  - Why code like
    - if (x == y) is not a good idea

Floating Point Numbers

- C has 3 floating point types
  - **float** ... typically 32-bit quantity (lower precision, narrower range)
  - **double** ... typically 64-bit quantity (higher precision, wider range)
  - **long double** … typically 128-bit quantity (but maybe only 80 bits used)
- Literal floating point values by default are **double**: 3.14159, 1.0/3, 1.0e-9
- Reminder: division of 2 ints gives an int e.g. 1 / 2 == 0

Code demo: double_output.c

Range of Floating Point Types

How do floating types have such a large range?

```
float        4 bytes  min=1.17549e-38   max=3.40282e+38
double       8 bytes  min=2.22507e-308  max=1.79769e+308
```

With the same number of bytes compare:

```
unsigned int  4 bytes  min=0   max= 4294967295 (4.29497e+09)
unsigned long 8 bytes  min=0   max= (1.84467e+19)
```

Code demo: Floating_types.c

Fractions in different bases

The decimal fraction 0.75 means

- $7*10^{-1} + 5*10^{-2} = 0.7 + 0.05 = 0.75$
- or equivalently $75/10^2 = 75/100 = 0.75$

Similarly 0.11_2 means

- $1*2^{-1} + 1*2^{-2} = 0.5 + 0.25 = 0.75$
- or equivalently $3/2^2 = 3/4 = 0.75$

Similarly $0.C_{16}$ would means

- $12*16^{-1} = 0.75$
- or equivalently $12/16^1 = 3/4 = 0.75$

Note: We call the . a radix point rather than a decimal point when we are dealing with other bases.

Converting fractions to other bases

- The algorithm to convert a decimal fraction to another base is
  - Take the decimal (fractional) part of the number and multiply it by the base you are converting to.
  - The whole number part of the result becomes the next digit after the radix point in the converted number.
  - Repeat the process with the remaining fractional part.
  - Continue until the fractional part becomes zero or you have enough digits for the desired accuracy.

Note: This process does not always terminate because some fractions have repeating representations in certain bases.

Example: Converting Fractions

For example if we want to convert 0.3125 to base 2

- 0.3125 * 2 = **0**.625
- 0.625 * 2 = **1**.25
- 0.25 * 2 = **0**.5
- 0.5 * 2 = **1**.0

Therefore 0.3125 = 0.0101_2

Floating Point Exercise 1:

Convert the following decimal values into binary

- 12.625
- 0.1

Code Demos

double_imprecision.c

Floating Point Representation Issues

Representing floating point numbers with a fixed small number of bits means:

- a finite number of bit patterns
- can only represent a finite subset of reals
    - almost all real values will have no exact representation
    - value of arithmetic operations may be real with no exact representation
- we must use **closest value** which can be exactly represented
    - this approximation introduces an error into our calculations
    - often, does not matter
    - sometimes ... can be disastrous
        - eg pacemakers, finance

Fixed Point Representation

- A simple trick to represent fractional numbers as integers
  - every value is multiplied by a particular constant and stored as an integer
    - e.g. if constant is 1000 then 56125 represents 56.125
    - we could not represent 3.141592
- Used on small embedded processors without floating point hardware
- Major limitation is range:
  - 16 bits used for integer part and 16 bits for fraction (equivalent to a scale factor of 2^{16})
    - minimum $2^{-16} \approx 0.000015$
    - maximum $2^{15} \approx 32768$

IEEE Standard: Exponential Representation

Idea: use **scientific notation**

- e.g 6.0221515 * 10^{23}

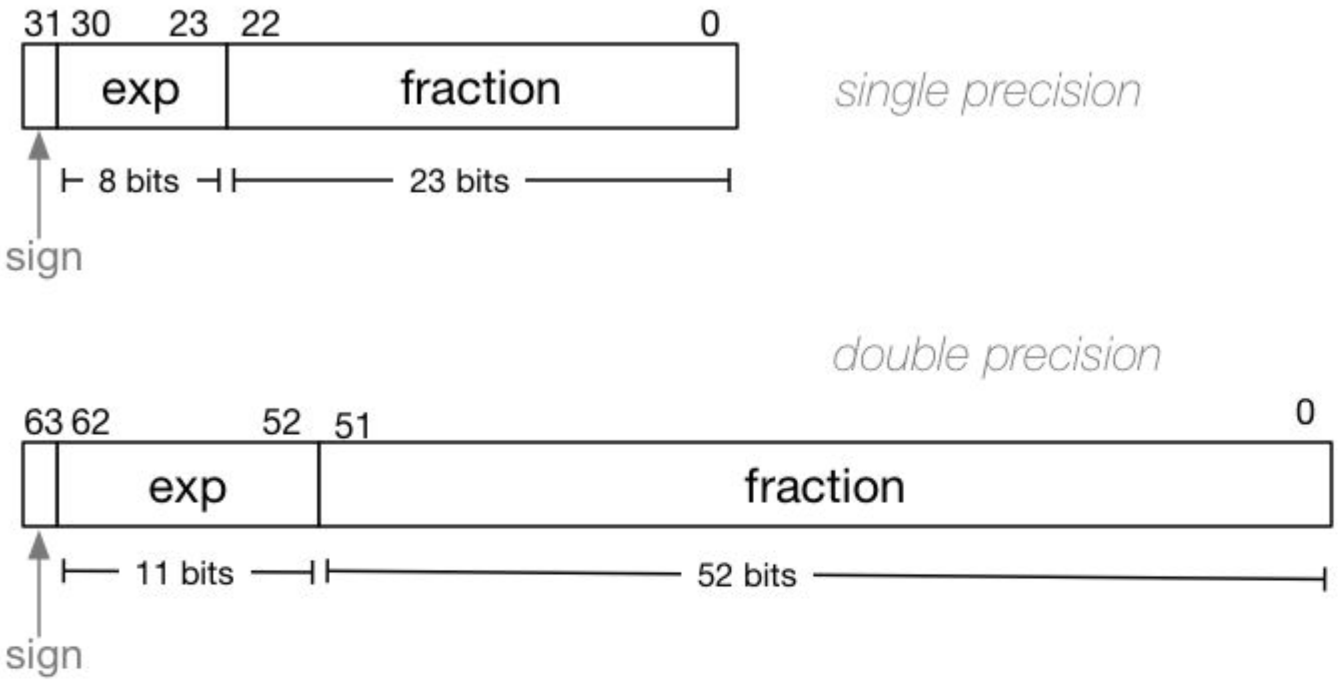
But in binary:

- 10.6875 = 1010.1011
  = 1.0101011 * 2^3

Allows a much bigger range of values to be represented than fixed point

- 8 bits for the exponent can represent numbers from 10^{-38} .. 10^{38}
- 11 bits for the exponent can represent numbers from 10^{-308} .. 10^{308}

IEEE 754 Standard



Note:
float in C is represented in this single precision format.
double in C is represented in this double precision format

Note: the fraction part is often called the mantissa

IEEE 754 Standard: Sign and Fraction

Sign bit: 0 for positive, 1 for negative

Fraction:

We don’t want multiple representations of the same number so we **normalise** it

- Use representation with exactly 1 digit in front of the radix point
  - (i.e. 1.1001×2^3 rather than 1100.1×2^0 or 11.001×2^2)
- better to have only one representation (one bit pattern) representing a value
  - multiple representations would make arithmetic slower on CPU

Weird hack: the first bit must be a one we don't need to store it

- as we long we have a special representation for zero
- To represent 1.1001×2^3 we would store 1001000000… for the fraction.

IEEE 754 Standard: Exponent

- represented relative to a bias value *B*
  - to represent exponent of x, we would store x+B
  - for floats the **bias** is 127
- e.g. we were representing 1.1001×2^3 we would store (3+127) = 130 = 10000010 for a float
- How bias is calculated:
  - assume an 8-bit exponent, then bias $B = 2^{8-1}-1 = 127$
  - valid bit patterns for exponent 00000001 .. 11111110 (1..254)
  - exponent values we can represent -126 .. 127

IEEE 754 Example

150.75 = 10010110.11

// normalise fraction, compute exponent

= 1.001011011 × 2^7

// determine sign bit,

// map fraction to 24 bits, (don’t store the leading 1)

// map exponent to 8 bits after adding on the bias of 127

= 01000011000101101100000000000000

where red is sign bit, green is exponent, blue is fraction

Note: B=127, e=2^7, so exponent = 127+7 = 134 = **10000110**

Check using explain_float_representation.c or Floating Point Calculator

Exercise 2: Floating Point Conversions

Question 1: Convert the decimal numbers 1 to a floating point number in IEEE 754 single-precision format.

Question 2: Convert the following IEEE 754 single-precision floating point numbers to decimal.

0 10000000 11000000000000000000000

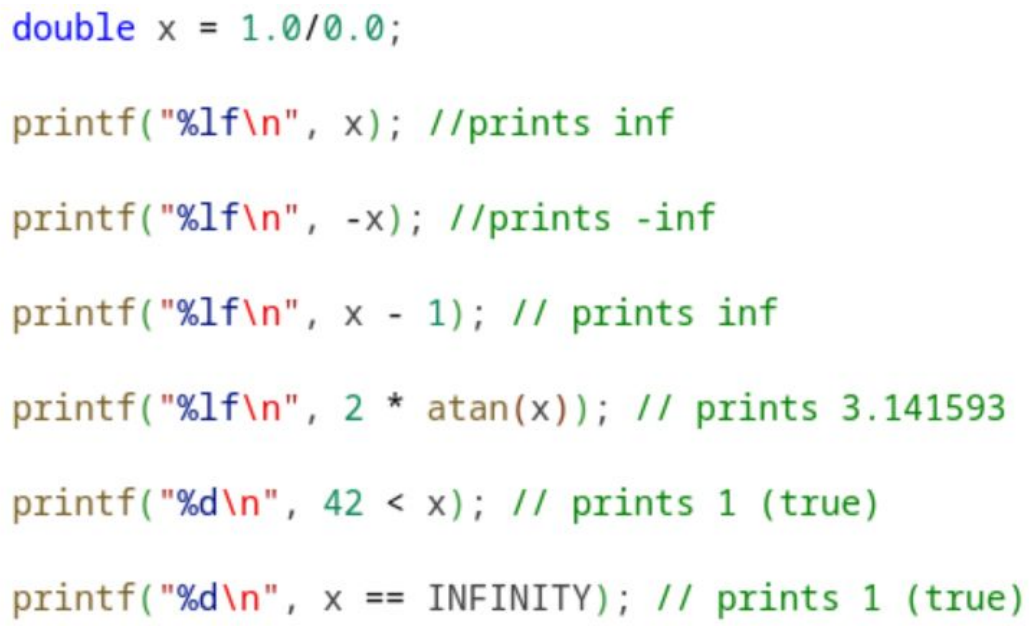
1 01111110 10000000000000000000000

IEEE 754 Standard: Special Cases

Value	**Exponent**	**Fraction**	**Example**
0 (+ve or -ve)	all 0’s	all 0’s	
inf (∞ and -∞)	all 1’s	all 0’s	1.0/0
nan	all 1’s	Not all 0’s	0.0/0

IEEE 754 infinity.c

Representation of +- infinity : propagates sensibly through calculations

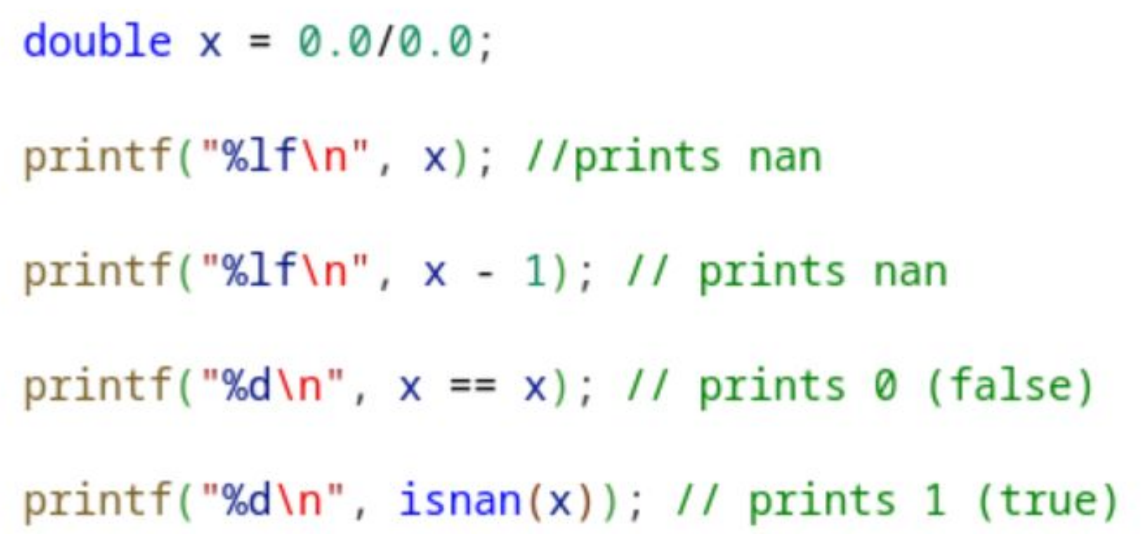


```
double x = 1.0/0.0;

printf("%lf\n", x); //prints inf

printf("%lf\n", -x); //prints -inf

printf("%lf\n", x - 1); // prints inf

printf("%lf\n", 2 * atan(x)); // prints 3.141593

printf("%d\n", 42 < x); // prints 1 (true)

printf("%d\n", x == INFINITY); // prints 1 (true)
```

IEEE 754 nan.c

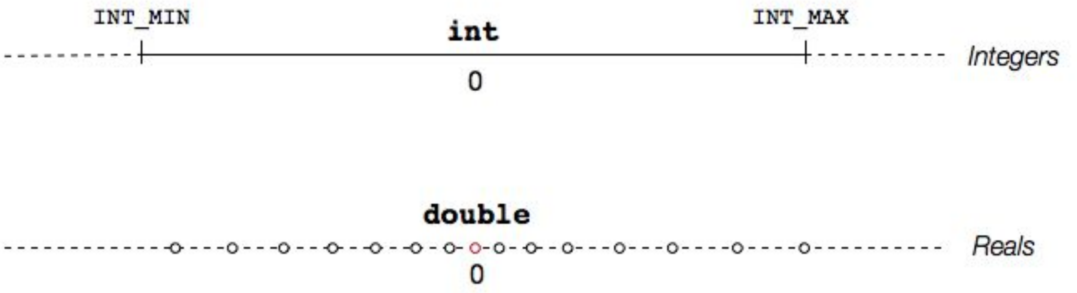
Representation for invalid results NaN (not a number)

- ensures errors propagates sensibly through calculations



```
double x = 0.0/0.0;

printf("%lf\n", x); //prints nan

printf("%lf\n", x - 1); // prints nan

printf("%d\n", x == x); // prints 0 (false)

printf("%d\n", isnan(x)); // prints 1 (true)
```

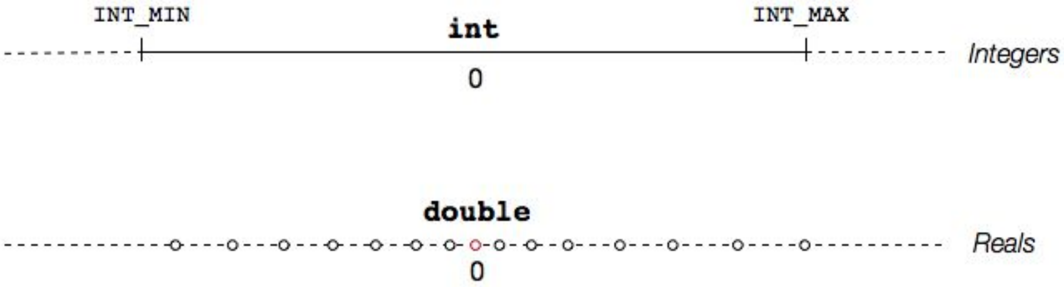
Distribution of Floating Point Numbers



integers ... subset (range) of the mathematical integers

- can represent all integer values in that subset
- each integer is 1 away from the next one and previous one
- all integers are represented accurately

Distribution of Floating Point Numbers



floating point ... subset of the mathematical real numbers

- floating point numbers not evenly distributed
  - numbers closer to 0 have higher precision which is good
  - representations get further apart as values get bigger
  - this works well for most calculations but can cause weird bugs

Distribution of Floating Point Numbers

A 64-bit **double** uses 52 bits for the fraction (mantissa).

- Between 2^n and 2^{n+1} there are 2^{52} doubles evenly spaced
  - e.g. in the interval 2^{-42} and 2^{-43} there are 2^{52} doubles
  - and in the interval between 1 and 2 there are 2^{52} doubles
  - and in the interval between 2^{42} and 2^{43} there are 2^{52} doubles

- near 0.001 - doubles are about 0.0000000000000000002 apart
- near 1000 - doubles are about 0.0000000000002 apart
- near 1000000000000000 - doubles are about 0.25 apart
- **above 2^{53} - doubles are more than 1 apart**

Code Demos

double_disaster.c

double_catastrophe.c

explain_float_representation.c

What did we learn today?

- Bitwise Operators
    - Recap
    - MIPS examples
    - C Coding examples
- Floating Point Representation

Next Lecture: File Systems

Feedback Please!

Your feedback is valuable!

If you have any feedback from today's lecture, please follow the link below or use the QR Code.

Please remember to keep your feedback constructive, so I can action it and improve your learning experience.

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