COMP1521 25T2

Week 5 Lecture 1

Bitwise Operators and Floating Point

Adapted from Angela Finlayson, Hammond Pearce,
Andrew Taylor and John Shepherd's slides

Assignment 1 is due Friday 6pm

Week 4 test: due Thursday 9pm (MIPS basics, control, arrays)

Week 6 Flexibility Week

Week 6 (next week) is flexibility week so nothing is due then!

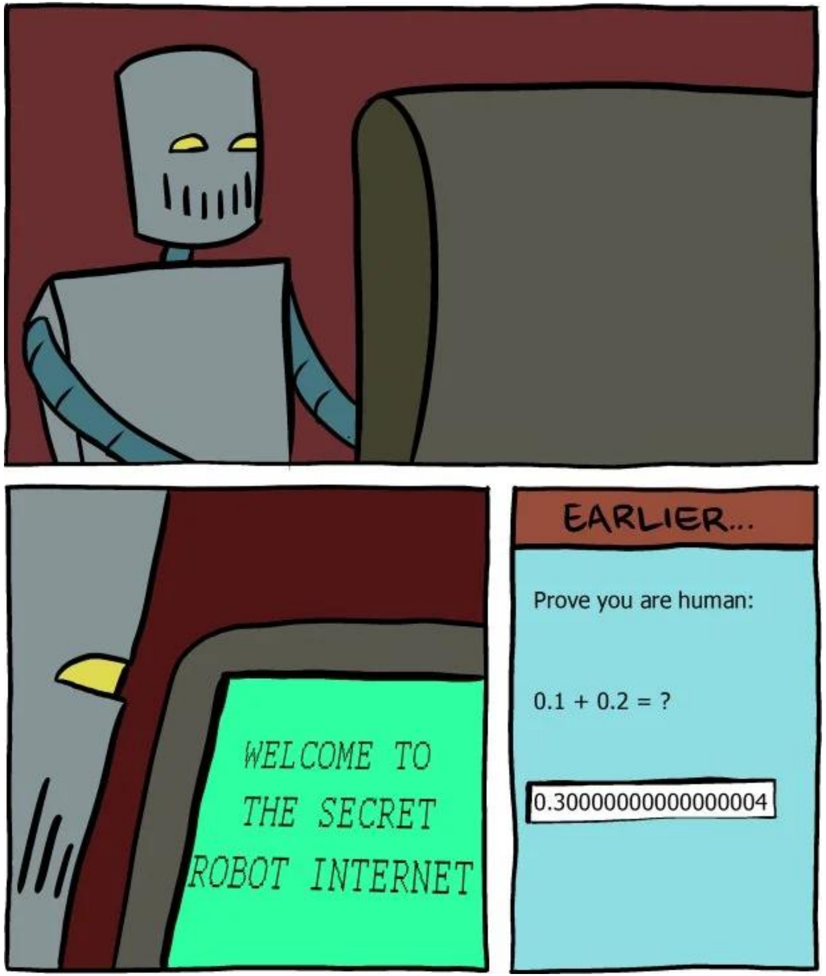
Week 5 lab: due Monday midday week 7

Week 5 test: due Thursday 9pm week 7 (MIPS strings)

Week 6 test: due Thursday 9pm week 7 (bitwise operators C)

Today’s Lecture

- Bitwise Operators
  - Recap
  - MIPS examples
  - C Coding examples
- Floating Point Representation



Recap Demo: bitwise.c

```
$ dcc bitwise.c print_bits.c -o bitwise
$ ./bitwise
Enter a: 23032
Enter b: 12345
Enter c: 3
     a = 0101100111111000 = 0x59f8 = 23032
     b = 0011000000111001 = 0x3039 = 12345
    ~a = 1010011000000111 = 0xa607 = 42503
 a & b = 0001000000111000 = 0x1038 = 4152
 a | b = 0111100111111001 = 0x79f9 = 31225
 a ^ b = 0110100111000001 = 0x69c1 = 27073
a >> c = 0000101100111111 = 0x0b3f = 2879
a << c = 1100111111000000 = 0xcfc0 = 53184
```

Recap Exercise 1

Given the following declarations:

```
// a signed 8-bit value
uint8_t x = 0x55;
uint8_t y = 0xAA;
```

What is the value of each of these expressions?

```
uint8_t a = x & y;
uint8_t b = x ^ y;
uint8_t c = x | y;
uint8_t d = ~x
```

```
uint8_t e = x >> 1;
uint8_t f = y >> 2;
uint8_t g = y << 2;
```

Recap Exercise 2:

How can we:

1) Check the bit at position 3 of an uint8_t?
2) Set the bit at position 3 of an uint8_t?
3) Unset the bit at position 3 of an unint8_t?

1) Check the bit at position n of an uint8_t?
2) Set the bit at position n of an uint8_t?
3) Unset the bit at position n of an unint8_t?

MIPS - Bit manipulation instructions

assembly	meaning	bit pattern
and r_d, r_s, r_t	r_d = r_s & r_t	000000ssssstttttddddd00000100100
or r_d, r_s, r_t	r_d = r_s | r_t	000000ssssstttttddddd00000100101
xor r_d, r_s, r_t	r_d = r_s ^ r_t	000000ssssstttttddddd00000100110
nor r_d, r_s, r_t	r_d = ~ (r_s | r_t)	000000ssssstttttddddd00000100111
andi r_t, r_s, I	r_t = r_s & I	001100ssssstttttIIIIIIIIIIIIIIII
ori r_t, r_s, I	r_t = r_s | I	001101ssssstttttIIIIIIIIIIIIIIII
xori r_t, r_s, I	r_t = r_s ^ I	001110ssssstttttIIIIIIIIIIIIIIII
not r_d, r_s	r_d = ~ r_s	pseudo-instruction

MIPS - Shift instructions

assembly	meaning	bit pattern
sllv r_d, r_t, r_s	$r_d = r_t \ll r_s$	000000ssssstttttddddd00000000100
srlv r_d, r_t, r_s	$r_d = r_t \gg r_s$	000000ssssstttttddddd00000000110
srav r_d, r_t, r_s	$r_d = r_t \gg r_s$	000000ssssstttttddddd00000000111
sll r_d, r_t, I	$r_d = r_t \ll$ I	00000000000tttttdddddIIIII000000
srl r_d, r_t, I	$r_d = r_t \gg$ I	00000000000tttttdddddIIIII000010
sra r_d, r_t, I	$r_d = r_t \gg$ I	00000000000tttttdddddIIIII000011

[1]

- `srl` and `srlv` shift zeroes into most-significant bit
  - This matches shift in C of unsigned values
- `sra` and `srav` propagate most-significant bit
  - This ensures the sign is maintained

MIPS Code Demos

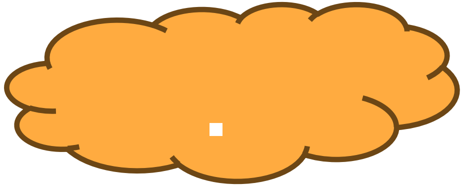
- odd_even.s

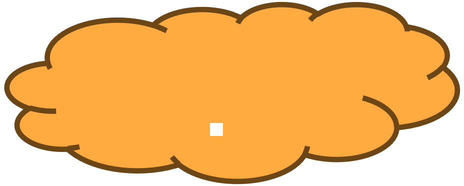
Code Demos

- xor.c
- pokemon.c
- set_low_bits.c
- bitset.c

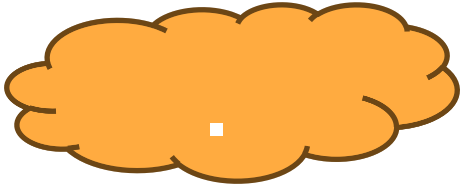
Demo: bitset.c

```
$ dcc bitset.c print_bits.c -o bitset
$ ./bitset

Set members can be 0-63, negative number to finish

Enter set a: 1 2 4 8 16 32 -1

Enter set b: 5 4 3 33 -1
a = 0000000000000000000000000000000100000000000000010000000100010110 = 0x100010116 =
4295033110
b = 0000000000000000000000000000001000000000000000000000000000111000 = 0x200000038 =
8589934648
a = {1,2,4,8,16,32}
b = {3,4,5,33}
a union b = {1,2,3,4,5,8,16,32,33}
a intersection b = {4}
cardinality(a) = 6
is_member(42, a) = 0
```





Floating point



IEEE 754 Floating Point Representation

- The industry standard
  - Used by almost all computers
- Crucial to understand when working with numeric computations
- Understand precision and accuracy limitations
  - Why using them for finance is unwise
  - Why sometimes
    - a + 1 == a
  - Why code like
    - if (x == y) for floats is not a good idea

Floating Point Numbers

- C has 3 floating point types
  - **float** ... typically 32-bit quantity (lower precision, narrower range)
  - **double** ... typically 64-bit quantity (higher precision, wider range)
  - **long double** … typically 128-bit quantity (but maybe only 80 bits used)
- Literal floating point values by default are **double**: 3.14159, 1.0/3, 1.0e-9
- Reminder: division of 2 ints gives an int e.g. 1 / 2 == 0

Code demo: float_output.c

Range of Floating Point Types

How do floating types have such a large range?

```
float         4 bytes  min=1.17549e-38   max=3.40282e+38
double        8 bytes  min=2.22507e-308  max=1.79769e+308
```

With the same number of bytes compare:

```
unsigned int  4 bytes  min=0   max= 4294967295 (4.29497e+09)
unsigned long 8 bytes  min=0   max= (1.84467e+19)
```

Code demo: floating_types.c

Fractions in different bases

The decimal fraction 0.75 means

- $7*10^{-1} + 5*10^{-2} = 0.7 + 0.05 = 0.75$
- or equivalently $75/10^2 = 75/100 = 0.75$

Similarly 0.11_2 means

- $1*2^{-1} + 1*2^{-2} = 0.5 + 0.25 = 0.75$
- or equivalently $3/2^2 = 3/4 = 0.75$

Similarly $0.C_{16}$ would means

- $12*16^{-1} = 0.75$
- or equivalently $12/16^1 = 3/4 = 0.75$

Note: We call the **.** a radix point rather than a decimal point when we are dealing with other bases.

Fractions in different bases

2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
8_{10}	4_{10}	2_{10}	1_{10}	0.5_{10}	0.25_{10}	0.125_{10}

Converting fractions to other bases

- The algorithm to convert a decimal fraction to another base is
  - Take the decimal (fractional) part of the number and multiply it by the base you are converting to.
  - The whole number part of the result becomes the next digit after the radix point in the converted number.
  - Repeat the process with the remaining fractional part.
  - Continue until the fractional part becomes zero or you have enough digits for the desired accuracy.

Note: This process does not always terminate because some fractions have repeating representations in certain bases.

Example: Converting Fractions

For example if we want to convert 0.3125 to base 2

- 0.3125 * 2 = **0**.625
- 0.625 * 2 = **1**.25
- 0.25 * 2 = **0**.5
- 0.5 * 2 = **1**.0

Therefore 0.3125 = 0.0101_2

Fixed Point Exercise 1:

Convert the following decimal values into binary

- 12.625
- 0.1

Code Demos

double_imprecision.c

Floating-Point Representation Issues

Representing floating point numbers with a fixed small number of bits means:

- Finite number of bit patterns
- Can only represent a finite subset of reals
  - almost all real values will have no exact representation
  - value of arithmetic operations may be real with no exact representation
- Must use **closest value** which can be exactly represented
  - this approximation introduces an error into our calculations
  - often, does not matter
  - sometimes ... can be disastrous
    - eg pacemakers, finance

Fixed-Point Representation

- A simple trick to represent fractional numbers as integers
  - Every value is multiplied by a particular constant and stored as an integer
    - e.g. if constant is 1000 then 56125 represents 56.125
    - we could not represent 3.141592
- Used on small embedded processors without floating point hardware
- Major limitation is range:
  - e.g. 16 bits used for integer part and 16 bits for fraction (equivalent to a scale factor of 2^{16})
    - minimum step $2^{-16} \approx 0.000015$
    - maximum $2^{15} \approx 32768$

IEEE Standard: Exponential Representation

Idea: use **scientific notation**

- e.g 6.0221515 $* 10^{23}$

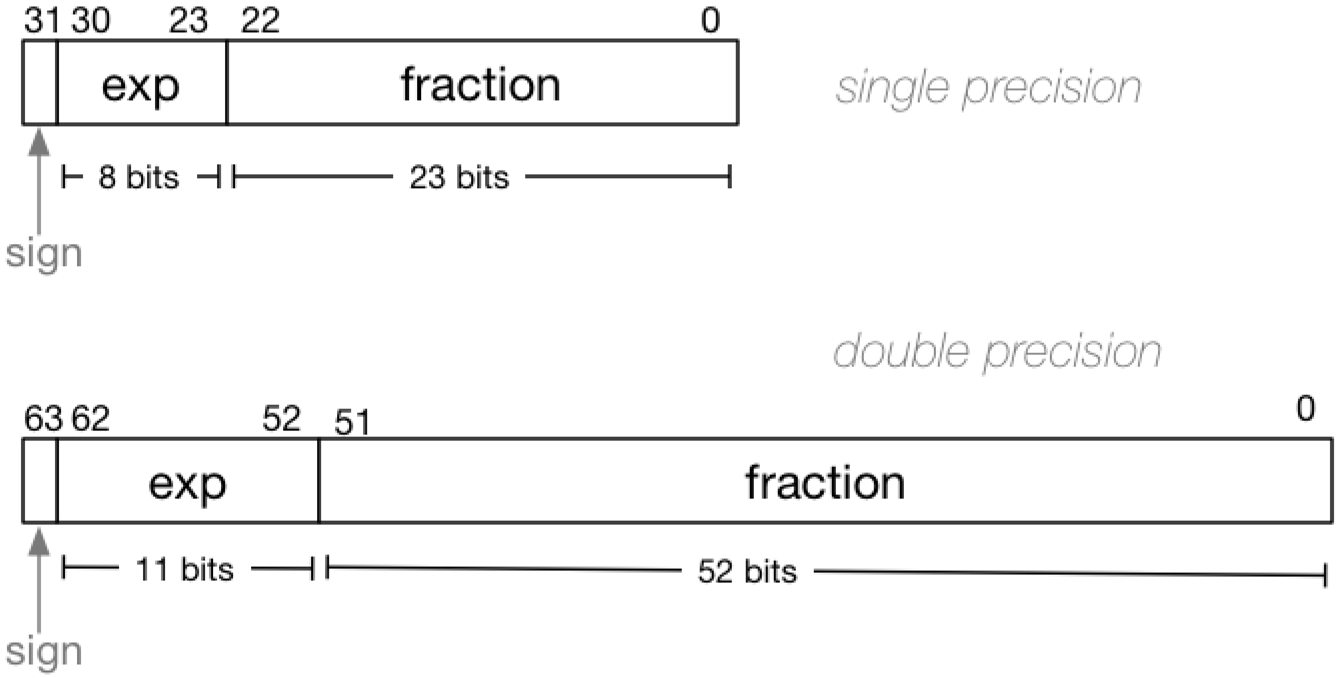
But in binary:

- 10.6875 = 1010.1011
  = 1.0101011 $* 2^{3}$

Allows a much bigger range of values to be represented than fixed point

- 8 bits for the exponent can represent numbers from 10^{-38} .. 10^{38}
- 11 bits for the exponent can represent numbers from 10^{-308} .. 10^{308}

IEEE 754 Standard



Note:
float in C is represented in this single precision format.
double in C is represented in this double precision format

Note: the fraction part is often called the mantissa

IEEE 754 Standard: Sign and Fraction

Sign bit: 0 for positive, 1 for negative

Fraction:

We don't want multiple representations of the same number so we **normalise** it

- Use representation with exactly 1 digit in front of the radix point
  - (i.e. 1.1001×2^3 rather than 1100.1×2^0 or 11.001×2^2)
- better to have only one representation (one bit pattern) representing a value
  - multiple representations would make arithmetic slower on CPU

Weird hack: the first bit must be a one we don't need to store it

- As long we have a special representation for zero
- To represent 1.1001×2^3 we would store 1001000000… for the fraction.

IEEE 754 Standard: Exponent

- represented relative to a bias value *B*
  - to represent exponent of x, we would store x+B
  - for floats the **bias** is 127
- e.g. we were representing 1.1001×2^3 we would store (3+127) = 130 = 10000010 for a float
- How bias is calculated:
  - Assume an 8-bit exponent, then bias $B = 2^{8-1}-1 = 127$
  - Valid bit patterns for exponent 00000001 .. 11111110 (1..254)
  - Exponent values we can represent -126 .. 127

IEEE 754 Example

150.75 = 10010110.11

// normalise fraction, compute exponent

= 1.001011011 × 2^7

// determine sign bit,

// map fraction to 24 bits, (don't store the leading 1)

// map exponent to 8 bits after adding on the bias of 127

= 01000011000101101100000000000000

where red is sign bit, green is exponent, blue is fraction

Note: B=127, e=2^7, so exponent = 127+7 = 134 = **10000110**

Check using explain_float_representation.c or Floating Point Calculator

Exercise 2: Floating Point Conversions

Question 1: Convert the decimal number 1 to a floating point number in IEEE 754 single-precision format.

Question 2: Convert the following IEEE 754 single-precision floating point numbers to decimal.

0 10000000 11000000000000000000000

1 01111110 10000000000000000000000

IEEE 754 Standard: Special Cases

Value	Exponent	Fraction	Example
0 (+ve or -ve)	all 0’s	all 0’s	
inf (∞ and -∞)	all 1’s	all 0’s	1.0/0
nan	all 1’s	Not all 0’s	0.0/0

IEEE 754 infinity.c

Representation of +- infinity : propagates sensibly through calculations

```
double x = 1.0/0.0;

printf("%lf\n", x); //prints inf

printf("%lf\n", -x); //prints -inf

printf("%lf\n", x - 1); // prints inf

printf("%lf\n", 2 * atan(x)); // prints 3.141593

printf("%d\n", 42 < x); // prints 1 (true)

printf("%d\n", x == INFINITY); // prints 1 (true)
```

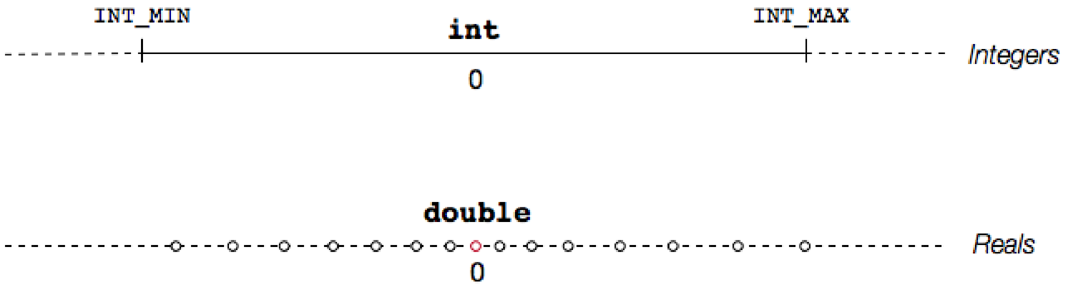
IEEE 754 nan.c

Representation for invalid results NaN (not a number)

- Ensures errors propagates sensibly through calculations

```
double x = 0.0/0.0;

printf("%lf\n", x); //prints nan

printf("%lf\n", x - 1); // prints nan

printf("%d\n", x == x); // prints 0 (false)

printf("%d\n", isnan(x)); // prints 1 (true)
```

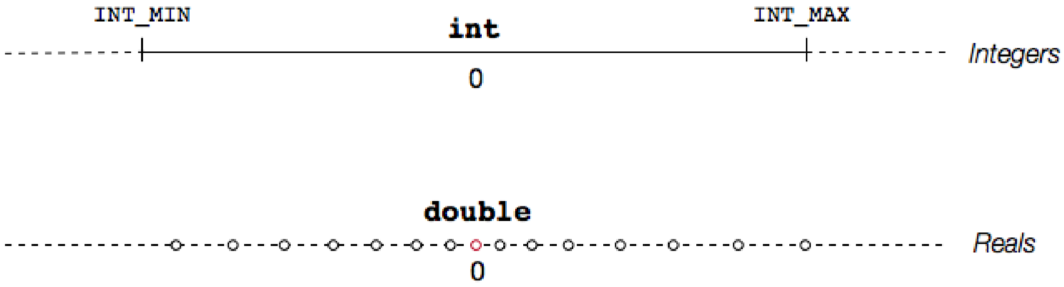
Distribution of Floating Point Numbers



integers ... subset (range) of the mathematical integers

- can represent all integer values in that subset
- each integer is 1 away from the next one and previous one
- all integers are represented accurately

Distribution of Floating Point Numbers



floating point ... subset of the mathematical real numbers

- Floating point numbers not evenly distributed
  - Numbers closer to 0 have higher precision which is good
  - Representations get further apart as values get bigger
  - This works well for most calculations but can cause weird bugs

Distribution of Floating Point Numbers

A 64-bit **double** uses 52 bits for the fraction (mantissa).

- Between 2^n and 2^{n+1} there are 2^{52} doubles evenly spaced
  - e.g. in the interval 2^{-42} and 2^{-43} there are 2^{52} doubles
  - and in the interval between 1 and 2 there are 2^{52} doubles
  - and in the interval between 2^{42} and 2^{43} there are 2^{52} doubles

- near 0.001 - doubles are about 0.00000000000000000002 apart
- near 1000 - doubles are about 0.0000000000002 apart
- near 1000000000000000 - doubles are about 0.25 apart
- **above 2^{53} - doubles are more than 1 apart**

Code Demos

double_disaster.c

double_catastrophe.c

explain_float_representation.c

What did we learn today?

- Bitwise Operators
  - Recap
  - MIPS examples
  - C Coding examples
- Floating Point Representation

Next Lecture: File Systems

Reach Out

Content Related Questions:
[Forum]

Admin related Questions email:
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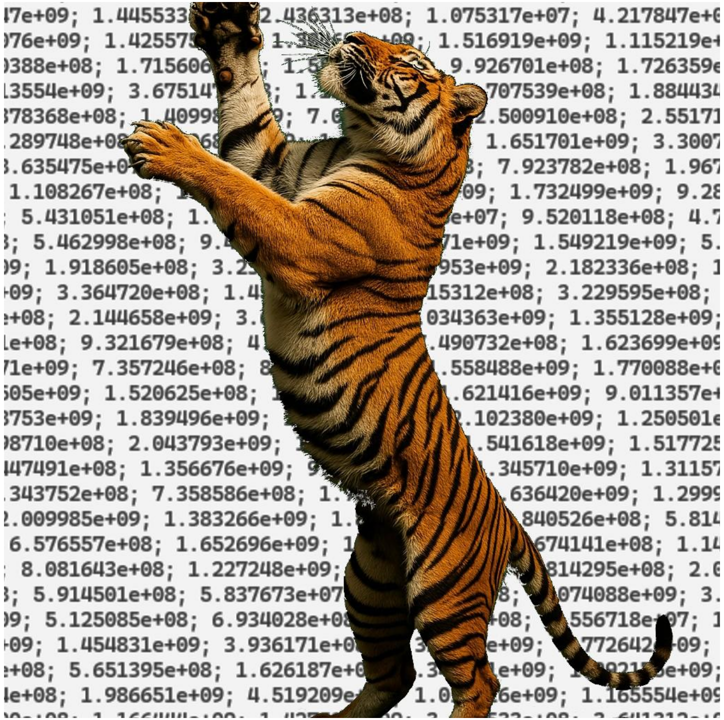

Student Support | I Need Help With…

My Feelings and Mental Health
Managing Low Mood, Unusual Feelings & Depression

- **Mental Health Connect** — student.unsw.edu.au/**counselling** Telehealth
- **Mind HUB** — student.unsw.edu.au/**mind-hub** Online Self-Help Resources
- **In Australia Call Afterhours UNSW Mental Health Support Line** — 1300 787 026 5pm-9am
- **Outside Australia Afterhours 24-hour Medibank Hotline** — +61 (2) 8905 0307

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Reporting Sexual Assault/Harassment

- **Equity Diversity and Inclusion (EDI)** — edi.unsw.edu.au/**sexual-misconduct**

Educational Adjustments
To Manage my Studies and Disability / Health Condition

- **Equitable Learning Service (ELS)** — student.unsw.edu.au/**els**

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