COMP1521 23T3 — Integers

https://www.cse.unsw.edu.au/~cs1521/23T3/

10 types of students

There are only 10 types of students …

- those that understand binary
- those that don't understand binary

Decimal Representation

- Can interpret decimal number `4705` as:
  $4 \times 10^3 + 7 \times 10^2 + 0 \times 10^1 + 5 \times 10^0$
- The *base* or *radix* is 10 … digits `0` – `9`
- Place values:

…	1000	100	10	1
…	10^3	10^2	10^1	10^0

- Write number as 4705_{10}
  - Note use of subscript to denote base

Representation in Other Bases

- base 10 is an arbitrary choice
- can use any base
- e.g. could use base 7
- Place values:

...	343	49	7	1
...	7^3	7^2	7^1	7^0

- Write number as 1216_7 and interpret as:
  $1 \times 7^3 + 2 \times 7^2 + 1 \times 7^1 + 6 \times 7^0 == 454_{10}$

Binary Representation

- Modern computing uses binary numbers
  - because digital devices can easily produce high or low level voltages which can represent 1 or 0.
- The *base* or *radix* is 2
  Digits `0` and `1`
- Place values:

$\cdots$	8	4	2	1
$\cdots$	2^3	2^2	2^1	2^0

- Write number as 1011_2 and interpret as:
  $1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 == 11_{10}$

Hexadecimal Representation

- Binary numbers hard for humans to read — too many digits!
- Conversion to decimal awkward and hides bit values
- Solution: write numbers in hexadecimal!
- The *base* or *radix* is 16 ... digits `0`, `1`, `2`, `3`, `4`, `5`, `6`, `7`, `8`, `9`, `A`, `B`, `C`, `D`, `E`, `F`
- Place values:

...	4096	256	16	1
...	16^3	16^2	16^1	16^0

- Write number as $3AF1_{16}$ and interpret as:
  $3 \times 16^3 + 10 \times 16^2 + 15 \times 16^1 + 1 \times 16^0 == 15089_{10}$
- in C, **`0x`** prefix denotes hexadecimal, e.g. **`0x3AF1`**

Octal & Binary C constants

- Octal (based 8) representation used to be popular for binary numbers
- Similar advantages to hexadecimal
- in C a leading **`0`** denotes octal, e.g. **`07563`**
- binary constants were only recently added to C - some C compilers will not recognize them

```
printf("%d", 0x2A);     // prints 42
printf("%d", 052);      // prints 42
printf("%d", 0b101010); // might compile and print 42
```


Binary Constants

In hexadecimal, each digit represents 4 bits

```
    0100 1000 1111 1010 1011 1100 1001 0111
0x     4    8    F    A    B    C    9    7
```

In octal, each digit represents 3 bits

```
    01 001 000 111 110 101 011 110 010 010 111
0    1   1   0   7   6   5   3   6   2   2   7
```

In binary, each digit represents 1 bit

```
    0b01001000111110101011110010010111
```


Binary to Hexadecimal

- Example: Convert 1011111000101001_2 to Hex:
- Example: Convert 10111101011100_2 to Hex:

Hexadecimal to Binary

- Reverse the previous process …
- Convert each hex digit into equivalent 4-bit binary representation
- Example: Convert $AD5_{16}$ to Binary:

Representing Negative Integers

- modern computers almost always use two's complement to represent integers
- positive integers and zero represented in obvious way
- negative integers represented in clever way to make arithmetic in silicon fast/simpler
- for an n-bit binary number the representation of $-b$ is $2^n - b$
- e.g. in 8-bit two's complement -5 is represented as $2^8 - 5$ == 11111011_2

Code example: printing all 8 bit twos complement bit patterns

- Some simple code to examine all 8 bit twos complement bit patterns.

```
for (int i = -128; i < 128; i++) {
    printf("%4d ", i);
    print_bits(i, 8);
    printf("\n");
}
```

source code for 8_bit_twos_complement.c

```
$ dcc 8_bit_twos_complement.c print_bits.c -o 8_bit_twos_complement
```

source code for print_bits.c source code for print_bits.h

Code example: printing all 8 bit twos complement bit patterns

```
$ ./8_bit_twos_complement
-128 10000000
-127 10000001
-126 10000010
...
  -3 11111101
  -2 11111110
  -1 11111111
   0 00000000
   1 00000001
   2 00000010
   3 00000011
...
 125 01111101
 126 01111110
 127 01111111
```


Code example: printing bits of int

```c
int a = 0;
printf("Enter an int: ");
scanf("%d", &a);
// sizeof returns number of bytes, a byte has 8 bits
int n_bits = 8 * sizeof a;
print_bits(a, n_bits);
printf("\n");
```

source code for print_bits_of_int.c

```
$ dcc print_bits_of_int.c print_bits.c -o print_bits_of_int
$ ./print_bits_of_int
Enter an int: 42
00000000000000000000000000101010
$ ./print_bits_of_int
Enter an int: -42
11111111111111111111111111010110
```


Code example: printing bits of int

```
$ ./print_bits_of_int
Enter an int: 0
00000000000000000000000000000000
$ ./print_bits_of_int
Enter an int: 1
00000000000000000000000000000001
$ ./print_bits_of_int
Enter an int: -1
11111111111111111111111111111111
$ ./print_bits_of_int
Enter an int: 2147483647
01111111111111111111111111111111
$ ./print_bits_of_int
Enter an int: -2147483648
10000000000000000000000000000000
$
```

Bits in Bytes in Words

- Many hardware operations works with bytes: 1 byte == 8 bits
- C's **`sizeof`** gives you number of bytes used for variable or type
- **`sizeof`** *variable* - returns number of bytes to store ***variable***
- **`sizeof (`***type***`)`** - returns number of bytes to store ***type***
- On CSE servers, C types have these sizes
  - `char` = 1 byte = 8 bits, 42 is 00101010
  - `short` = 2 bytes = 16 bits, 42 is 0000000000101010
  - `int` = 4 bytes = 32 bits, 42 is 00000000000000000000000000101010
  - `double` = 8 bytes = 64 bits, 42 = ?
- above are common sizes but not universal on a small embedded CPU
  `sizeof (int)` might be 2 (bytes)

Code example: `integer_types.c` - exploring integer types

We can use **`sizeof`** and **`limits.h`** to explore the range of values which can be represented by standard C integer types on our machine...

```
$ dcc integer_types.c -o integer_types
$ ./integer_types
              Type Bytes  Bits
              char     1     8
       signed char     1     8
     unsigned char     1     8
             short     2    16
    unsigned short     2    16
               int     4    32
      unsigned int     4    32
              long     8    64
     unsigned long     8    64
         long long     8    64
unsigned long long     8    64
```


Code example: `integer_types.c` - exploring integer types

```
              Type                  Min                  Max
              char                 -128                  127
       signed char                 -128                  127
     unsigned char                    0                  255
             short               -32768                32767
    unsigned short                    0                65535
               int          -2147483648           2147483647
      unsigned int                    0           4294967295
              long -9223372036854775808  9223372036854775807
     unsigned long                    0 18446744073709551615
         long long -9223372036854775808  9223372036854775807
unsigned long long                    0 18446744073709551615
```

source code for integer_types.c

stdint.h - integer types with guaranteed sizes

```
#include <stdint.h>
```

- to get below integer types (and more) with guaranteed sizes
- we will use these heavily in COMP1521

```
             // range of values for type
             //             minimum               maximum
int8_t   i1; //                -128                   127
uint8_t  i2; //                   0                   255
int16_t  i3; //              -32768                 32767
uint16_t i4; //                   0                 65535
int32_t  i5; //         -2147483648            2147483647
uint32_t i6; //                   0            4294967295
int64_t  i7; // -9223372036854775808  9223372036854775807
uint64_t i8; //                   0 18446744073709551615
```

source code for stdint.c

Code example: char_bug.c

Common C bug:

```
char c;  // c should be declared int   (int16_t would work, int is better)
while ((c = getchar()) != EOF) {
    putchar(c);
}
```

Typically stdio.h contains:

```
#define EOF -1
```

- most platforms: char is signed (-128..127)
  - loop will incorrectly exit for a byte containing 0xFF
- rare platforms: char is unsigned (0..255)
  - loop will never exit

source code for char_bug.c

Endian-ness

- The bytes of a multi-byte (2 byte, 4 byte, ...) quantity can be stored in various orders.
- *Endian-ness* is the order.
- Two common orders: big-endian & little-endian
- *big-endian* - most significant byte at the smallest memory address.
- *little-endian* - least significant byte at the smallest memory address.
- Most modern general-purpose computers little-endian
- Endian-ness configurable on some architectures e.g ARM

Testing Endian-ness

C

```
uint8_t b;
uint32_t u;
u = 0x03040506;
// load first byte of u
b = *(uint8_t *)&u;
// prints 6 if little-endian
// and 3 if big-endian
printf("%d\n", b);
```

source code for endian.c

MIPS

```
    li   $t0, 0x03040506
    la   $t1, u
    sw   $t0, 0($t1) # u = 0x03040506;
    lb   $a0, 0($t1) # b = *(uint8_t *)&u;
    li   $v0, 1      # printf("%d", a0);
    syscall
    li   $a0, '\n'   # printf("%c", '\n');
    li   $v0, 11
    syscall
    li   $v0, 0     # return 0
    jr   $ra
    .data
u:
    .space 4
```

source code for endian.s