There are only 10 types of students ...

- those that understand binary
- those that don’t understand binary

**Decimal Representation**

- Can interpret decimal number $4705$ as:
  $4 \times 10^3 + 7 \times 10^2 + 0 \times 10^1 + 5 \times 10^0$

- The *base or radix* is 10 ... digits 0 – 9

- Place values:

<table>
<thead>
<tr>
<th>...</th>
<th>1000</th>
<th>100</th>
<th>10</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>$10^3$</td>
<td>$10^2$</td>
<td>$10^1$</td>
<td>$10^0$</td>
</tr>
</tbody>
</table>

- Write number as $4705_{10}$
  - Note use of subscript to denote base
Representation in Other Bases

- base 10 is an arbitrary choice
- can use any base
- e.g. could use base 7
- Place values:

<table>
<thead>
<tr>
<th></th>
<th>(7^3)</th>
<th>(7^2)</th>
<th>(7^1)</th>
<th>(7^0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>343</td>
<td>49</td>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

- Write number as \(1216_7\) and interpret as:
  \[ 1 \times 7^3 + 2 \times 7^2 + 1 \times 7^1 + 6 \times 7^0 == 454_{10} \]

Binary Representation

- Modern computing uses binary numbers
  - because digital devices can easily produce high or low level voltages which can represent 1 or 0.
- The base or radix is 2
  - Digits 0 and 1
- Place values:

<table>
<thead>
<tr>
<th></th>
<th>(2^3)</th>
<th>(2^2)</th>
<th>(2^1)</th>
<th>(2^0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

- Write number as \(1011_2\) and interpret as:
  \[ 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 == 11_{10} \]

Hexadecimal Representation

- Binary numbers hard for humans to read — too many digits!
- Conversion to decimal awkward and hides bit values
- Solution: write numbers in hexadecimal!
- The base or radix is 16 ...
  - digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
- Place values:

<table>
<thead>
<tr>
<th></th>
<th>(16^3)</th>
<th>(16^2)</th>
<th>(16^1)</th>
<th>(16^0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>4096</td>
<td>256</td>
<td>16</td>
<td>1</td>
</tr>
</tbody>
</table>

- Write number as \(3AF_{16}\) and interpret as:
  \[ 3 \times 16^3 + 10 \times 16^2 + 15 \times 16^1 + 1 \times 16^0 == 15089_{10} \]
- in C, \texttt{0x} prefix denotes hexadecimal, e.g. \texttt{0x3AF1}
Octal & Binary C constants

- Octal (based 8) representation used to be popular for binary numbers
- Similar advantages to hexadecimal
- In C, a leading 0 denotes octal, e.g., \texttt{07563}
- Standard C doesn't have a way to write binary constants
- Some C compilers let you write \texttt{0b}
  - OK to use \texttt{0b} in experimental code but don't use in important code

```c
printf("%d", 0x2A); // prints 42
printf("%d", 052); // prints 42
printf("%d", 0b101010); // might compile and print 42
```

### Binary Constants

In hexadecimal, each digit represents 4 bits

```
0100 1000 1111 1010 1011 1100 1001 0111
0x 4 8 F A B C 9 7
```

In octal, each digit represents 3 bits

```
01 001 000 111 110 101 011 110 010 010 111
0 1 1 0 7 6 5 3 6 2 2 7
```

In binary, each digit represents 1 bit

```
0b01001000111110101011110010010111
```

### Binary to Hexadecimal

- Example: Convert \texttt{1011111000101001}_2 to Hex:

- Example: Convert \texttt{10111101011100}_2 to Hex:
Hexadecimal to Binary

- Reverse the previous process ...
- Convert each hex digit into equivalent 4-bit binary representation
- Example: Convert \( AD_{16} \) to Binary:

Representing Negative Integers

- Modern computers almost always use two's complement to represent integers
- Positive integers and zero represented in obvious way
- Negative integers represented in clever way to make arithmetic in silicon fast/simpler
- For an \( n \)-bit binary number the representation of \(-b\) is \( 2^n - b \)
- E.g. in 8-bit two's complement \(-5\) is represented as \( 2^8 - 5 = \text{11111011}_2 \)

Code example: printing all 8 bit two's complement bit patterns

- Some simple code to examine all 8 bit two's complement bit patterns.

```c
for (int i = -128; i < 128; i++) {
    printf("%4d ", i);
    print_bits(i, 8);
    printf("\n");
}
```

$ dcc 8_bit_twos_complement.c print_bits.c -o 8_bit_twos_complement

source code for print_bits.c source code for print_bits.h
Code example: printing all 8 bit twos complement bit patterns

```shell
$ ./8_bit_twos_complement
-128 10000000
-127 10000001
-126 10000010
...
-3 11111101
-2 11111110
-1 11111111
0 00000000
1 00000001
2 00000010
3 00000011
...
125 01111101
126 01111110
127 01111111
```

Code example: printing bits of int

```c
int a = 0;
printf("Enter an int: ");
scanf("%d", &a);
// sizeof returns number of bytes, a byte has 8 bits
int n_bits = 8 * sizeof a;
print_bits(a, n_bits);
printf("\n");
```

```shell
$ dcc print_bits_of_int.c print_bits.c -o print_bits_of_int
$ ./print_bits_of_int
Enter an int: 42
00000000000000000000000000101010
$ ./print_bits_of_int
Enter an int: -42
11111111111111111111111111010110
```

Code example: printing bits of int

```shell
$ ./print_bits_of_int
Enter an int: 0
00000000000000000000000000000000
$ ./print_bits_of_int
Enter an int: 1
00000000000000000000000000000001
$ ./print_bits_of_int
Enter an int: -1
11111111111111111111111111111111
$ ./print_bits_of_int
Enter an int: 2147483647
01111111111111111111111111111111
$ ./print_bits_of_int
Enter an int: -2147483648
10000000000000000000000000000000
$ 
```
Many hardware operations work with bytes: 1 byte == 8 bits

C’s `sizeof` gives you number of bytes used for variable or type

`sizeof variable` - returns number of bytes to store `variable`

`sizeof (type)` - returns number of bytes to store `type`

On CSE servers, C types have these sizes:
- `char` = 1 byte = 8 bits, 42 is 00101010
- `short` = 2 bytes = 16 bits, 42 is 0000000000101010
- `int` = 4 bytes = 32 bits, 42 is 00000000000000000000000000101010
- `double` = 8 bytes = 64 bits, 42 = ?

above are common sizes but not universal on a small embedded CPU
`sizeof (int)` might be 2 (bytes)

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Code example: `integer_types.c` - exploring integer types

We can use `sizeof` and `limits.h` to explore the range of values
which can be represented by standard C integer types on our machine...

```
$ dcc integer_types.c -o integer_types
$ ./integer_types
```

<table>
<thead>
<tr>
<th>Type</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>char</td>
<td>-128</td>
<td>127</td>
</tr>
<tr>
<td>signed char</td>
<td>-128</td>
<td>127</td>
</tr>
<tr>
<td>unsigned char</td>
<td>0</td>
<td>255</td>
</tr>
<tr>
<td>short</td>
<td>-32768</td>
<td>32767</td>
</tr>
<tr>
<td>unsigned short</td>
<td>0</td>
<td>65535</td>
</tr>
<tr>
<td>int</td>
<td>-2147483648</td>
<td>2147483647</td>
</tr>
<tr>
<td>unsigned int</td>
<td>0</td>
<td>4294967295</td>
</tr>
<tr>
<td>long</td>
<td>-9223372036854775808</td>
<td>9223372036854775807</td>
</tr>
<tr>
<td>unsigned long</td>
<td>0</td>
<td>18446744073709551615</td>
</tr>
<tr>
<td>long long</td>
<td>-9223372036854775808</td>
<td>9223372036854775807</td>
</tr>
</tbody>
</table>

source code for `integer_types.c`
#include <stdint.h>

- to get below integer types (and more) with guaranteed sizes
- we will use these heavily in COMP1521

```
// range of values for type
// minimum maximum
int8_t i1; // -128 127
uint8_t i2; // 0 255
int16_t i3; // -32768 32767
uint16_t i4; // 0 65535
int32_t i5; // -2147483648 2147483647
uint32_t i6; // 0 4294967295
int64_t i7; // -9223372036854775808 9223372036854775807
uint64_t i8; // 0 18446744073709551615
```

Common C bug:
```
char c; // c should be declared int
while ((c = getchar()) != EOF) {
    putchar(c);
}
```

Typically stdio.h contains:
```
#define EOF -1
```

- most platforms: char is signed (-128..127)
  - loop will incorrectly exit for a byte containing 0xFF
- rare platforms: char is unsigned (0..255)
  - loop will never exit