Floating Point Numbers

Floating point numbers model a (tiny finite) subset of reals:
- almost all real values don’t have exact representation (e.g. 1/3)
- numbers close to zero have higher precision (more accurate)

C has two floating point types
- float ... typically 32-bit quantity (lower precision, narrower range)
- double ... typically 64-bit quantity (higher precision, wider range)

Literal floating point values: 3.14159, 1.0/3, 1.0e-9

printf("%10.4lf", (double)2.718281828459);
// displays 2.7183
printf("%20.20lf", (double)4.0/7);
// displays 0.57142857142857139685

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IEEE 754 standard ...
- scientific notation with fraction \( F \) and exponent \( E \)
- numbers have form \( F \times 2^E \), where both \( F \) and \( E \) can be -ve
- INFINITY = representation for &infin; and -&infin; (e.g. 1.0/0)
- NAN = representation for invalid value (e.g. sqrt(-1.0))

Fraction part is normalised (i.e. 1.2345 × 10² rather than 123.45)

In binary, exponent is represented relative to a bias value \( B \)
- if the unsigned exponent value is \( e \), the actual value is \( e - B \)

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Example of normalising the fraction part in binary:
- 1010.1011 is normalized as 1.0101011 × 2⁰¹¹
- 1010.1011 = 10 + 11/16 = 10.6875
- 1.0101011 × 2⁰¹¹ = (1 + 43/128) * 2³ = 1.3359375 * 8 = 10.6875

The normalised fraction part always has 1 before the decimal point.

Example of determining the exponent in binary:
- assume an 8-bit exponent, then bias \( B = 2^8 - 1 = 127 \)
- valid bit patterns for exponent 00000000 .. 11111111
- exponent values -126 .. 127

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Internal structure of floating point values

More complex representation than int because 1.difference
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Example (single-precision):

\[
150.75 = 10010110.11 \\
// normalise fraction, compute exponent \\
= 1.001011011 * 2 ** 7 \\
// determine sign bit, \\
// map fraction to 24 bits, \\
// map exponent relative to baseline \\
= 0 100000110 001011011 0000000000000000
\]

where red is sign bit, green is exponent, blue is fraction

Note: \( B = 127 \), \( e = 2^7 \), so exponent \( = 134 = 1000110 \)

Exercise: Floating point → Decimal

Convert the following floating point numbers to decimal. Assume that they are in IEEE 754 single-precision format.

\[
0 \ 10000000 \ 11000000000000000000000 \\
1 \ 01111110 \ 100000000000000000000000
\]