• COMP1511 focuses on writing programs.
• Efficiency is also important. Often need to consider:
  ▶ execution time
  ▶ memory use.
• A **correct** but slow program can be useless.
• Efficiency often depends on the size of the data being processed.
• Understanding this dependency lets us predict program performance on larger data
• Informal exploration in COMP1511 - much more in COMP2521 and COMP3121
Analysis of Algorithms

How can we find out whether a program is efficient or not?

• empirical approach - run the program, several times with different input sizes and measure the time taken

• theoretical approach - try to count the number of ‘operations’ performed by the algorithm on input of size $n$
```cpp
int linear_search(int array[], int length, int x) {
    for (int i = 0; i < length; i = i + 1) {
        if (array[i] == x) {
            return 1;
        }
    }
    return 0;
}
```
Operations:

- start at first element
- inspect each element in turn
- stop when find $X$ or reach end

If there are $N$ elements to search:

- Best case check 1 element
- Worst case check $N$ elements
- If in list on average check $N/2$ elements
- If not in list check $N$ elements
int linear_ordered(int array[], int length, int x) {
    for (int i = 0; i < length; i = i + 1) {
        if (array[i] == x) {
            return 1;
        } else if (array[i] > x) {
            return 0;
        }
    }
    return 0;
}
Linear Search Ordered Array - Informal Analysis

Operations:

- start at first element
- inspect each element in turn
- stop when find \( X \) or find value \( \nabla X \) or reach end

If there are \( N \) elements to search:

- Best case check 1 element
- Worst case check \( N \) elements
- If in list on average check \( N/2 \) elements
- If not in list on average check \( N/2 \) elements
int binary_search(int array[], int length, int x) {
    int lower = 0;
    int upper = length - 1;
    while (lower <= upper) {
        int mid = (lower + upper) / 2;
        if (array[mid] == x) {
            return 1;
        } else if (array[mid] > x) {
            upper = mid - 1;
        } else {
            lower = mid + 1;
        }
    }
    return 0;
}
Binary Search Ordered Array - Informal Analysis

Operations:

- start with entire array
- at each step halve the range the element may be in
- stop when find $X$ or range is empty

If there are $N$ elements to search

- Best case check 1 element
- Worst case check $\log_2(N)+1$ elements
- If in list on average check $\log_2(N)$ elements
log₂(N) grows very slowly:

- log₂(10) = 3.3
- log₂(1000) = 10
- log₂(1000000) = 20
- log₂(1000000000) = 30
- log₂(1000000000000) = 40

Physicists estimate $10^{80}$ atoms in universe: $log₂(10^{80}) = 240$

Binary search all atoms in universe in $< 1$ microsecond
• Aim: rearrange a sequence so it is in non-decreasing order
• Advantages
  ▶ sorted sequence can be searched efficiently
  ▶ items with equal keys are located together
• The problem of sorting
  ▶ simple obvious algorithms too slow to sort large sequences
  ▶ better algorithms can sort very large sequences
• sorting extensively studied and many algorithms proposed.
• We’ll look at one slow obvious algorithm: **bubblesort**
• And at one fast algorithm: **quicksort**
• We’ll assume sorting array of ints.
• Straight-forward to extend code to handle other types of items (e.g. strings) and other data structures.
void bubblesort(int array[], int length) {
    int swapped = 1;
    while (swapped) {
        swapped = 0;
        for (int i = 1; i < length; i = i + 1) {
            if (array[i] < array[i - 1]) {
                int tmp = array[i];
                array[i] = array[i - 1];
                array[i - 1] = tmp;
                swapped = 1;
            }
        }
    }
}
void bubblesort(int array[], int length) {
    int swapped = 1;
    while (swapped) {
        swapped = 0;
        for (int i = 1; i < length; i = i + 1) {
            if (array[i] < array[i - 1]) {
                int tmp = array[i];
                array[i] = array[i - 1];
                array[i - 1] = tmp;
                swapped = 1;
            }
        }
    }
}
void quicksort(int array[], int length) {
    quicksort1(array, 0, length - 1);
}

void quicksort1(int array[], int lo, int hi) {
    if (lo >= hi) {
        return;
    }
    int p = partition(array, lo, hi);
    // sort lower part of array
    quicksort1(array, lo, p);
    // sort upper part of array
    quicksort1(array, p + 1, hi);
}
```c
int partition(int array[], int lo, int hi) {
    int i = lo, j = hi;
    int pivotValue = array[(lo + hi) / 2];
    while (1) {
        while (array[i] < pivotValue) {
            i = i + 1;
        }
        while (array[j] > pivotValue) {
            j = j - 1;
        }
        if (i >= j) {
            return j;
        }
        int temp = array[i];
        array[i] = array[j];
        array[j] = temp;
        i = i + 1;
        j = j - 1;
    }
    return j;
}
```
Quicksort and Bubblesort Compared

If we instrument quicksort and bubble sort code, we see:

<table>
<thead>
<tr>
<th>Array size (n)</th>
<th>bubblesort operations</th>
<th>quicksort operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>81</td>
<td>24</td>
</tr>
<tr>
<td>100</td>
<td>8415</td>
<td>457</td>
</tr>
<tr>
<td>1000</td>
<td>981018</td>
<td>9351</td>
</tr>
<tr>
<td>10000</td>
<td>98790120</td>
<td>102807</td>
</tr>
</tbody>
</table>

- bubblesort is proportional to $n^2$
- quicksort is proportional to $n \log_2(n)$
- if $n$ is small, little difference
- if $n$ is large, huge difference
- for large $n$, you need a good sorting algorithm like quicksort