C-MACHINE AND M-MACHINE

> We defined the semantics of MinHS using two different approaches
  - controll flow implicit: M-machine
  - controll flow explicit: C-machine, E-machine

> Are they equivalent?
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→ We defined the semantics of MinHS using two different approaches
  - controll flow implicit: M-machine
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→ Are they equivalent?

Proof is not obvious, as evaluation methods are very different

→ How can we make the connection?
Observation:

1. a single M-machine step usually corresponds to many C-machine steps
2. the stack in C-machine corresponds to context of subexpression which is actually evaluated in M-machine
Observation:

① a single M-machine step usually corresponds to many C-machine steps
② the stack in C-machine corresponds to context of subexpression which is actually evaluated in M-machine
③ stack and current expression provide enough information to reconstruct unevaluated portion of original expression
Reconstruction:

\[ \circ \math@{} e = e \]
\[ (\text{plus}(\Box, e_2) \triangleright s)@e_1 = s@\text{plus}(e_1, e_2) \]
\[ (\text{plus}(v_1, \Box) \triangleright s)@e_2 = s@\text{plus}(v_1, e_2) \]
\[ (\text{if}(\Box, e_1, e_2) \triangleright s)@e = s@\text{if}(e, e_1, e_2) \]
\[ \vdots \]
Show that

1. if $s \succ e \stackrel{c}{\Rightarrow} o \prec v$, then $e@s \stackrel{m}{\Rightarrow} v$

   Note: we show that the above is true for any stack $s$ (as opposed to the empty stack), since the more general statement is easier to prove as the i.h. is stronger.

   Here and in the remainder of the proof we write $s \succ e$ to mean $\succ$ or $\prec$.

2. if $e \stackrel{m}{\Rightarrow} v$, then $o \succ e \stackrel{c}{\Rightarrow} o \prec v$

Both directions can be proven using induction over the number of evaluation steps.
- **Base case:** in the C-machine, the minimum number of steps necessary to evaluate an expression is one. Show that, if $s \geq e \rightarrow_c o < e$ then $s@e \rightarrow^*_M v$

The expression has to be a value already — no evaluation step necessary in the M-machine

- **Induction step:** $n + 1$ evaluation steps necessary.

\[
s \geq e \rightarrow_c s' \geq e' \rightarrow^n_c o < v
\]

**Induction Hypothesis:**

\[
e'@s' \rightarrow^*_M v
\]

We need to prove the following lemma:

**Lemma 1:** If $s \geq e \rightarrow_c s' \geq e'$, then either $s@e = s'@e'$ or $s@e \rightarrow_M s'@e'$
② - Base case: Show that, if \( e \xrightarrow{0}_c v \) then \( o \succ e \xrightarrow{*}_c o \prec v \)

- Induction step: \( n + 1 \) evaluation steps necessary.

\[ e \xrightarrow{M} e' \xrightarrow{n}_M v \]

Induction Hypothesis:

\[ o \succ e' \xrightarrow{*}_c v \]

Again, we first need to prove a lemma:

**Lemma 2:** If \( e \xrightarrow{M} e' \) and \( s \succ e' \xrightarrow{*}_c o \prec v \) then \( s \succ e \xrightarrow{*}_c o \prec v \)
Both lemmata can be shown by rule induction over the definition of $\rightarrow_c$ and $\rightarrow_M$ respectively (see textbook)