Exercise 1. We are given

1. \( H \land \neg R \)
2. \((H \land N) \Rightarrow R\)

From 1 we can conclude \( H \) and \( \neg R \). If \( N \) were true, then from \( H \) and \( N \) we could conclude \( R \) by 2, which contradicts \( \neg R \). Hence, \( N \) cannot be true, which proves \( \neg N \).

Hint: You can also use a truth table to show that \( \neg N \) is true in every row in which formulae 1 and 2 are true.

Exercise 2.

• First question: Yes. In fact, the conclusion follows directly from just the first requirement.

• Second question: No. The third requirement states that the alarm should sound whenever there is a fire. On the other hand, the first requirement does not require the alarm to sound at all (it only states a requirement about when the alarm should not sound); and the second requirement mentions nothing about fire at all.

Exercise 3. If \( n \) is odd, then \( n - 1 \) and \( n + 1 \) are both even and one of them must be divisible by 4. It follows that \( n^2 - 1 = (n + 1)(n - 1) = 2k \cdot 4\ell = 8k\ell \), for some \( k, \ell \in \mathbb{N} \). Therefore, \( 8 \mid (n^2 - 1) \).

Hint: Other proofs are possible.

Exercise 4. Model the “character” of each of the three persons (Joan, Shane and Peter) with a proposition \( J, S, P \). These are true if and only if that person is a truar. Then, we write their statements as follows:

\[
\begin{align*}
J & \iff \neg S \land \neg P \\
S & \iff \neg \neg S \\
P & \iff \neg S
\end{align*}
\]

Using a truth table we can see that the only assignments consistent with the above are: \( J=F, S=T, P=F \) or \( J=F, S=F, P=T \). In both cases there are two liars and one truar.