COMP2521 26T1
Graphs (II)
Graph Traversal

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bfs and dfs
path checking
path finding

Problems on Graphs

Common problems on graphs:

- Is there a path between two vertices?
- What is the shortest path between two vertices?
- Which vertices are reachable from a particular vertex?
- Is the graph connected?
- Is there a cycle?
- How many connected components are there?
- Is there a simple path/cycle that passes through all vertices?

Graph Traversal

All of the above problems can be solved by
a systematic exploration of a graph via its edges.

This systematic exploration is called traversal or search.

Graph Traversal

Two primary methods for graph traversal/search:

Breadth-first search (BFS)

- Prioritises exploring widely over exploring deeply
  - "Go wide"
- Implemented iteratively (using a queue)

Depth-first search (DFS)

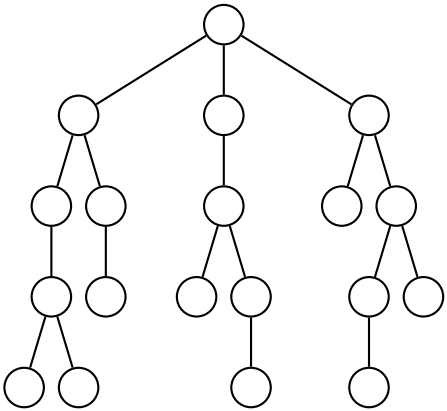
- Prioritises exploring deeply over exploring widely
  - "Go deep"
- Implemented recursively or iteratively (using a stack)

Graph Traversal

BFS vs. DFS in a tree

In what order would BFS and DFS visit the nodes of this tree?

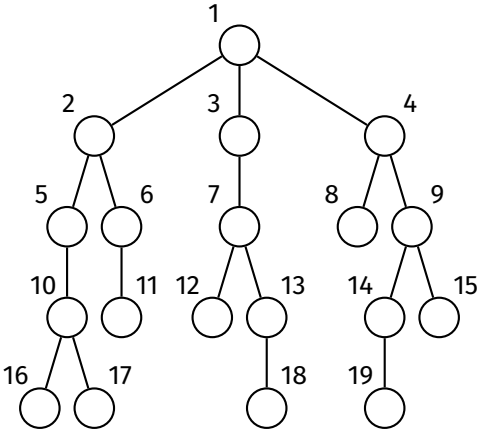
(Assume that nodes towards the left have higher priority)



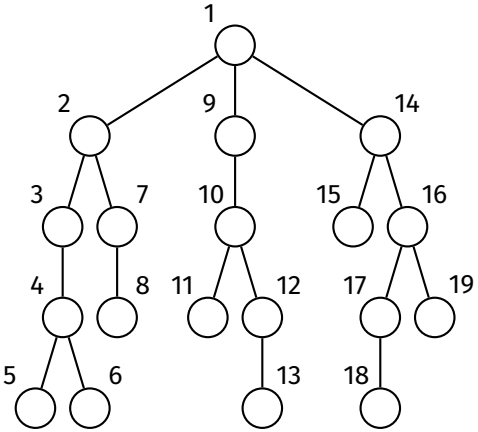
Graph Traversal

BFS vs. DFS in a tree

Breadth-first search



Depth-first search

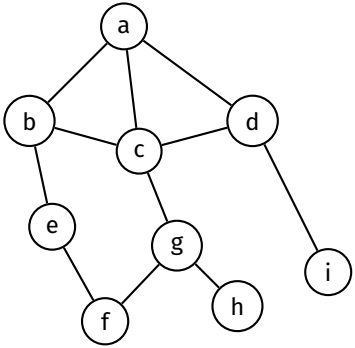


Graph Traversal

BFS vs. DFS in a graph

In what order would BFS and DFS visit the vertices of this graph?

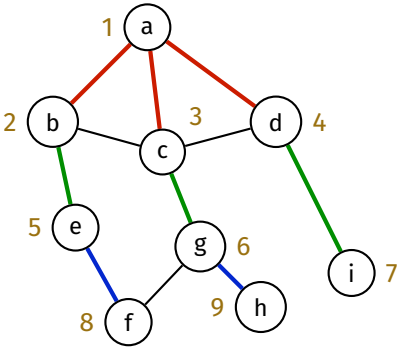
(Assume that nodes containing smaller letters have higher priority)



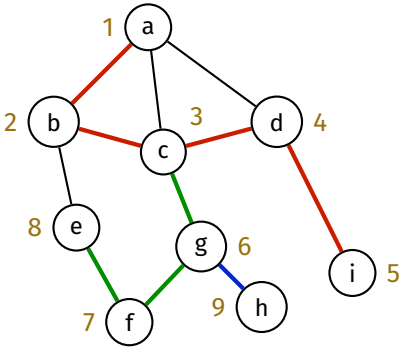
Graph Traversal

BFS vs. DFS in a graph

Breadth-first search



Depth-first search



Breadth-First Search

Breadth-first search visits vertices
in order of distance from the starting vertex.

BFS is implemented iteratively using a queue.

Breadth-First Search

Data structures

Data structures used in BFS:

- Visited array
  - To keep track of which vertices have been visited
- Predecessor array
  - To keep track of the predecessor of each vertex
  - The predecessor of v is the vertex from which we reached v
    - i.e., the vertex before v on the path to v
- Queue
  - First-in-first-out data structure
  - Stores unvisited vertices in the order that they should be visited

Breadth-First Search

Algorithm

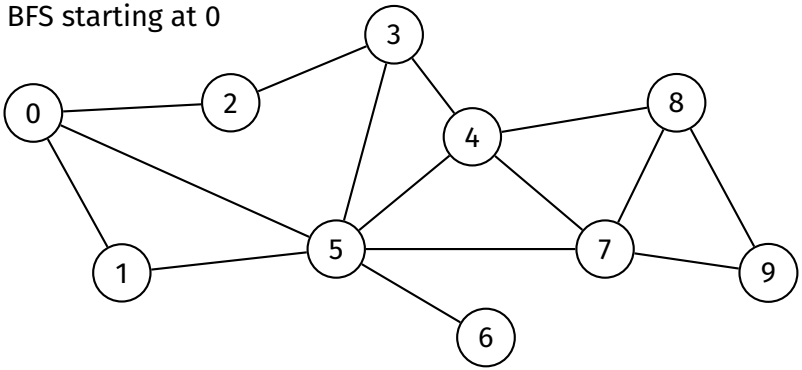
Algorithm:

1. Create/initialise data structures:
   - Create visited array, initialised to false
   - Create predecessor array, initialised to -1
   - Create empty queue
2. Mark starting vertex as visited and enqueue it
3. While the queue is not empty:
   1. Dequeue a vertex
      - Let this vertex be v
   2. **Explore** v - that is, for each of v's unvisited neighbours:
      1. Mark it as visited
      2. Set its predecessor to v
      3. Enqueue it

Breadth-First Search

Example

BFS starting at 0



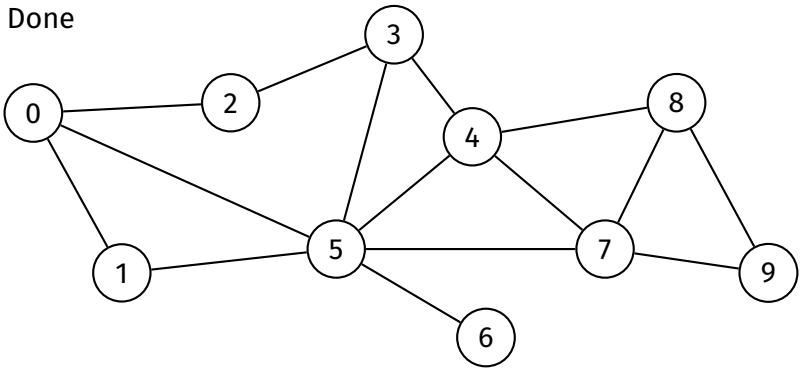
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited										
pred										

queue

Breadth-First Search

Example

Done



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

Breadth-First Search

Pseudocode


```
bfs(G, src):
    Input: graph G, starting vertex src

    create visited array, initialised to false
    create predecessor array, initialised to -1
    create queue Q

    visited[src] = true
    enqueue src into Q

    while Q is not empty:
        v = dequeue from Q
        for each neighbour w of v in G where visited[w] = false:
            visited[w] = true
            predecessor[w] = v
            enqueue w into Q
```

Breadth-First Search

Simplification

When using a predecessor array in BFS,
the predecessor array can double as a visited array

`predecessor[`v`]` $=$ `-1` means v is not visited

Breadth-First Search

Simplification


```
bfs(G, src):
    Input: graph G, starting vertex src

    create predecessor array, initialised to -1
    create queue Q

    predecessor[src] = src // <- mark src as visited
    enqueue src into Q

    while Q is not empty:
        v = dequeue from Q
        for each neighbour w of v in G where predecessor[w] = -1:
            predecessor[w] = v
            enqueue w into Q
```

Breadth-First Search

Analysis

BFS is $O(V + E)$ when using the adjacency list representation:

- Typical queue implementation has $O(1)$ enqueue and dequeue
- Each vertex is visited at most once $\Rightarrow O(V)$
- For each vertex, all of its edges are considered once $\Rightarrow O(E)$

Path-Finding with BFS

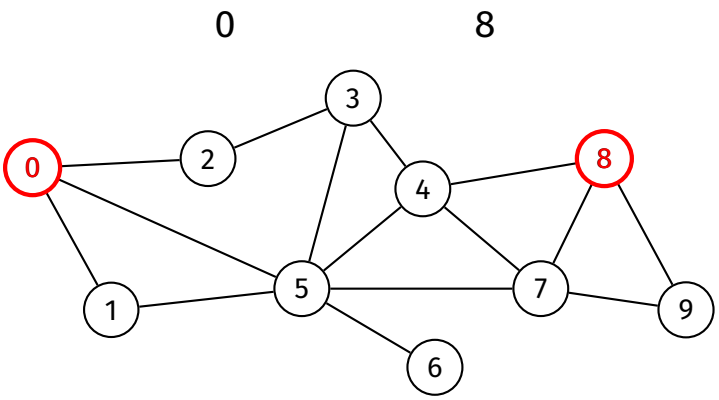
A BFS finds the shortest path between the starting vertex and all other vertices.

- Shortest path in terms of the number of edges

The shortest path between src and $dest$ can be found by tracing backwards through the predecessor array (from $dest$ to src).

Path-Finding with BFS

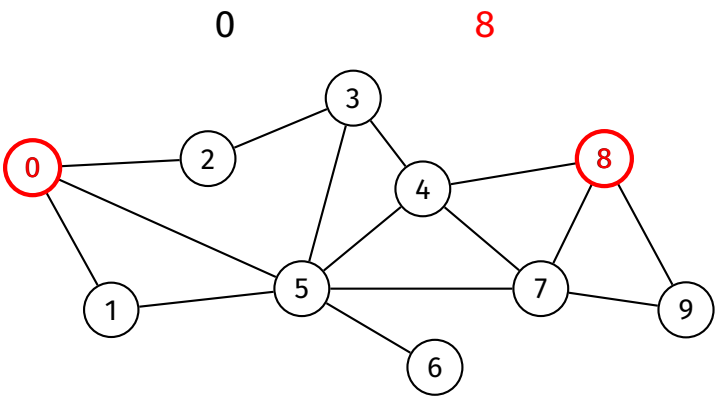
Example: Shortest path from 0 to 8



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
pred	-1	0	0	2	5	0	5	5	4	7

Path-Finding with BFS

Example: Shortest path from 0 to 8

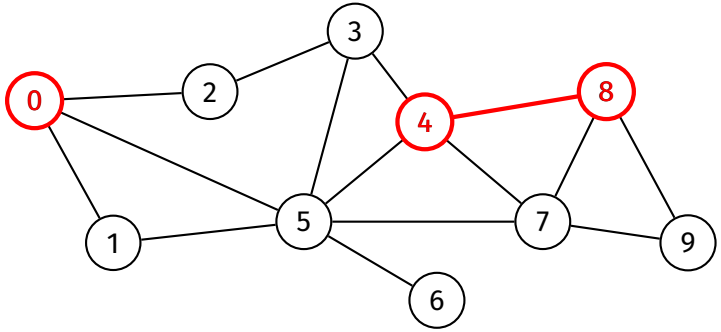


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
pred	-1	0	0	2	5	0	5	5	4	7

Path-Finding with BFS

Example: Shortest path from 0 to 8

0 $\quad\quad$ $4 \rightarrow 8$

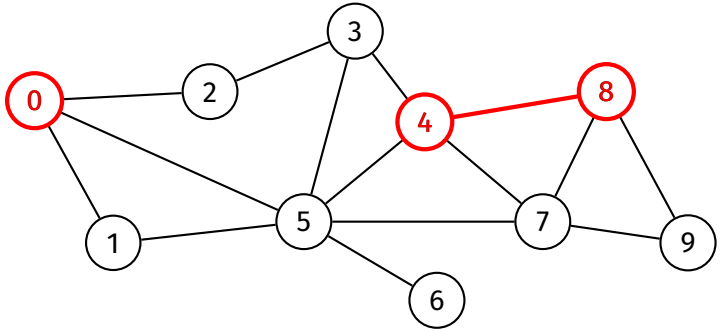


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
pred	-1	0	0	2	5	0	5	5	4	7

Path-Finding with BFS

Example: Shortest path from 0 to 8

0 $\quad\quad$ 4 $\rightarrow$ 8

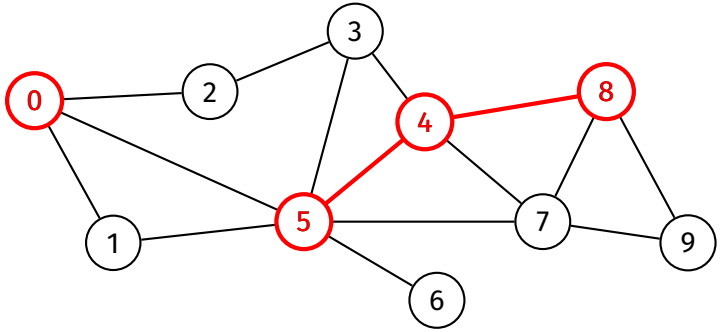


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
pred	-1	0	0	2	5	0	5	5	4	7

Path-Finding with BFS

Example: Shortest path from 0 to 8

0 $5 \rightarrow 4 \rightarrow 8$

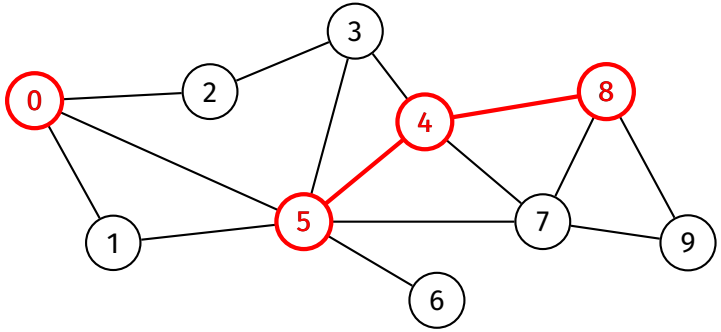


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
pred	-1	0	0	2	5	0	5	5	4	7

Path-Finding with BFS

Example: Shortest path from 0 to 8

0 $\quad$ 5 $\rightarrow$ 4 $\rightarrow$ 8

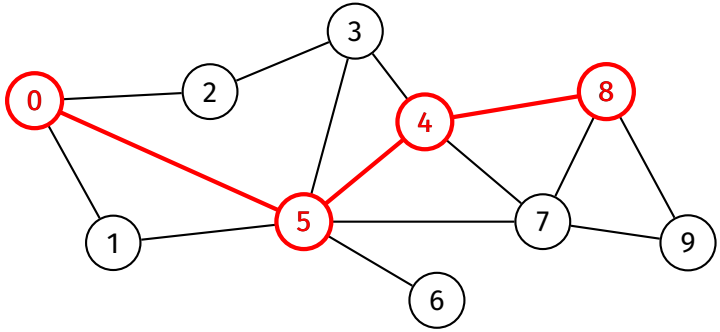


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
pred	-1	0	0	2	5	0	5	5	4	7

Path-Finding with BFS

Example: Shortest path from 0 to 8

$$0 \longrightarrow 5 \rightarrow 4 \rightarrow 8$$



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
pred	-1	0	0	2	5	0	5	5	4	7

Path-Finding with BFS


```
bfsFindPath(G, src, dest):
    Input: graph G, vertices src and dest

    ... BFS starting from src ...

    if predecessor[dest] ≠ -1:
        v = dest
        while v ≠ src:
            print v, "<-"
            v = predecessor[v]

        print src
```

Depth-First Search

Depth-first search goes as far down one path
as possible until it reaches a dead end,
then backtracks until it finds a new path to take,
then repeats

DFS can be implemented recursively or iteratively.

Recursive Depth-First Search

Depth-first search is described recursively as:

1. Mark current vertex as visited
   - The first time, this is the starting vertex
2. For each neighbour of the current vertex:
   - If it has not been visited:
     - Recursively traverse starting from that vertex

The recursion naturally induces backtracking.

Recursive Depth-First Search

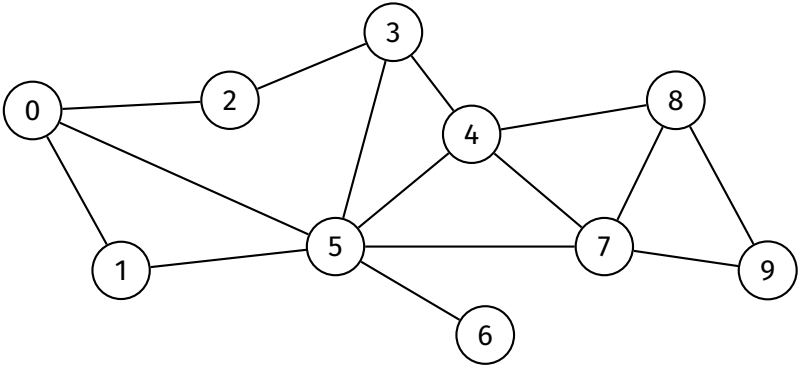
Pseudocode


```
dfs(G, src):
    Input: graph G, starting vertex src

    create visited array, initialised to false
    dfsRec(G, src, visited)

dfsRec(G, v, visited):
    Input: graph G, vertex v, visited array

    visited[v] = true // "visit" v
    for each neighbour w of v in G:
        if visited[w] = false:
            dfsRec(G, w, visited)
```

Recursive Depth-First Search

Example

DFS starting at 0



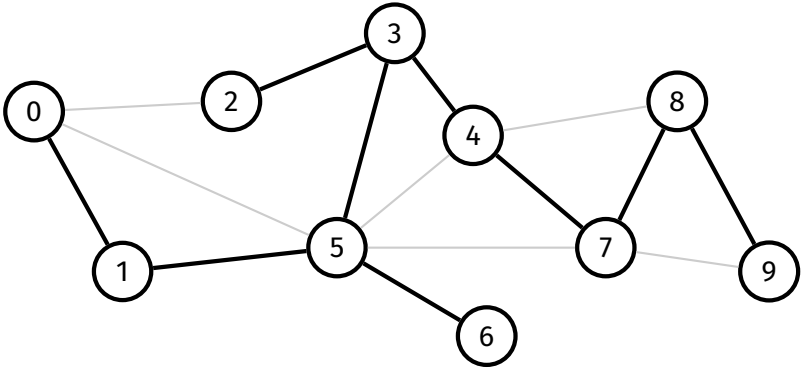
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	0	0	0	0	0	0	0	0	0	0

visit order

Recursive Depth-First Search

Example

Done



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1

visit order 0 1 5 3 2 4 7 8 9 6

Recursive Depth-First Search

Analysis

Recursive DFS is $O(V + E)$ when using the adjacency list representation:

- Each vertex is visited at most once $\Rightarrow O(V)$
  - Function is called on each vertex at most once
- For each vertex, all of its edges are considered once $\Rightarrow O(E)$

Path-Checking with Recursive DFS

Recursive DFS can be adapted to check if a path exists between two vertices.

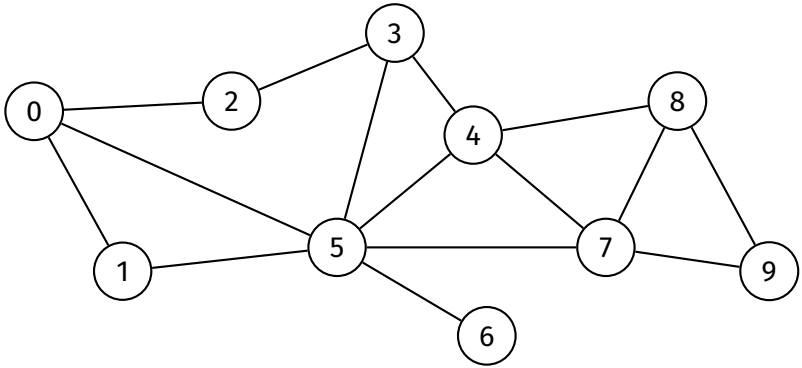
Idea:

- To check if a path exists between src and $dest$:
  - If $src = dest$, then there is a path (the empty path)
  - Otherwise, for each neighbour of src, recursively check if there is a path from that neighbour to $dest$

Path-Checking with Recursive DFS

Example

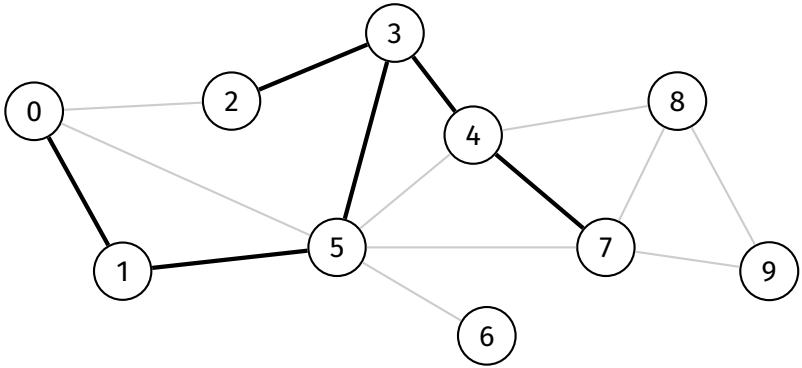
Does there exist a path between 0 and 7 in this graph?



Path-Checking with Recursive DFS

Example

Answer: Yes



Path-Checking with Recursive DFS

Pseudocode


```
dfsHasPath(G, src, dest):
    Input:  graph G, vertices src and dest
    Output: true if there is a path from src to dest
            false otherwise

    create visited array, initialised to false
    return dfsHasPathRec(G, src, dest, visited)

dfsHasPathRec(G, v, dest, visited):
    Input: graph G, vertices v and dest, visited array

    visited[v] = true
    if v = dest:
        return true

    for each neighbour w of v in G:
        if visited[w] = false:
            if dfsHasPathRec(G, w, dest, visited):
                return true

    return false
```

Path-Checking with Recursive DFS

Analysis

$O(V + E)$ when using the adjacency list representation:

- Algorithm is just a modified recursive DFS with return statements

Path-Finding with Recursive DFS

How to get the path?

Idea:

- Record the predecessor of each vertex during the DFS
- Trace backwards through the path after the DFS

Path-Finding with Recursive DFS

Pseudocode


```
dfsFindPath(G, src, dest):
    Input: graph G, vertices src and dest

    create predecessor array, initialised to -1
    predecessor[src] = src

    if dfsFindPathRec(G, src, dest, predecessor):
        v = dest
        while v ≠ src:
            print v, "<-"
            v = predecessor[v]

        print src
```

Path-Finding with Recursive DFS

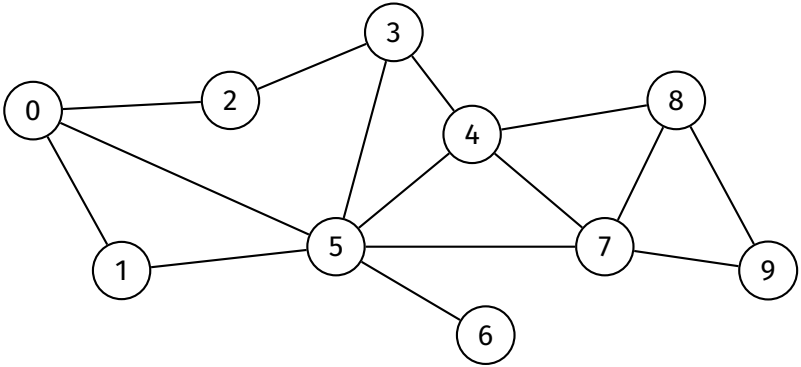
Pseudocode


```
dfsFindPathRec(G, v, dest, predecessor):
    if v = dest:
        return true

    for each neighbour w of v in G:
        if predecessor[w] = -1:
            predecessor[w] = v
            if dfsFindPathRec(G, w, dest, predecessor):
                return true

    return false
```

Path-Finding with Recursive DFS

Example

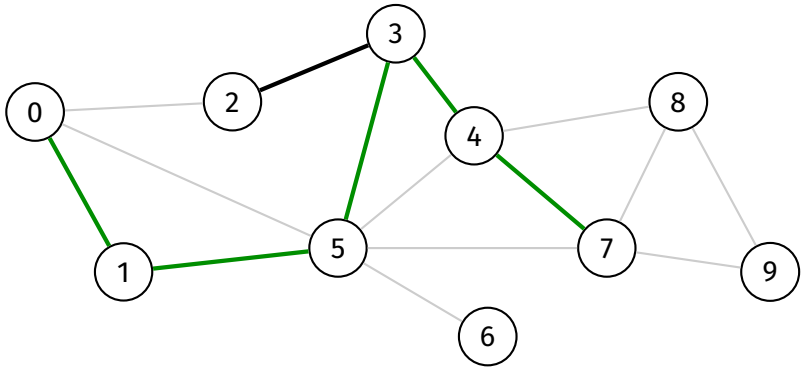
Find a path from 0 to 7



Path-Finding with Recursive DFS

Example

Path found:



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
pred	0	0	3	5	3	1	-1	4	-1	-1

Clearly, DFS is not guaranteed to find the shortest path.

Iterative Depth-First Search

DFS can be implemented iteratively.

Iterative DFS is similar to BFS, but there are a few crucial differences:

- DFS uses a stack instead of a queue
- DFS marks a vertex as visited after removing it from the stack, not when adding it (which is what BFS does, but with a queue)

Iterative Depth-First Search

Pseudocode


```
dfs(G, src):
    Input: graph G, vertex src

    create visited array, initialised to false
    create predecessor array, initialised to -1
    create stack S

    push src onto S

    while S is not empty:
        v = pop from S
        if visited[v] = true:
            continue // i.e., return to start of loop

        visited[v] = true

        for each neighbour w of v in G where visited[w] = false:
            predecessor[w] = v
            push w onto S
```

Iterative Depth-First Search

Analysis

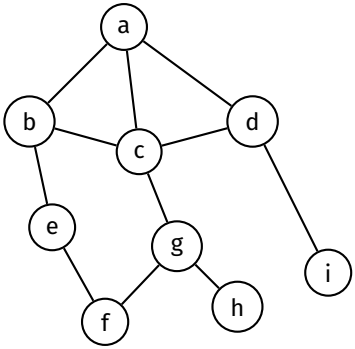
Iterative DFS is $O(V + E)$ when using the adjacency list representation.

- Typical stack implementation has $O(1)$ push and pop
- Each vertex visited at most once $\Rightarrow O(V)$
- For each vertex, all of its edges are considered $\Rightarrow O(E)$

Spanning Trees

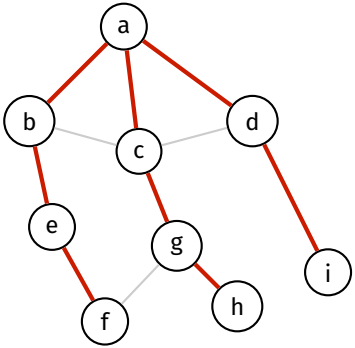
The edges traversed in a graph traversal form a spanning tree.

Consider the following graph:

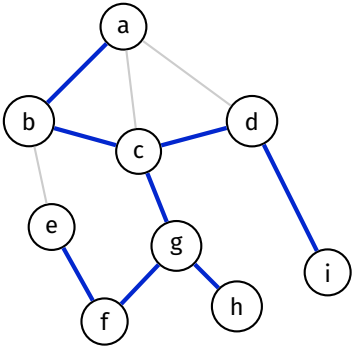


Spanning Trees

A traversal starting at vertex ‘a’ forms the following spanning trees:



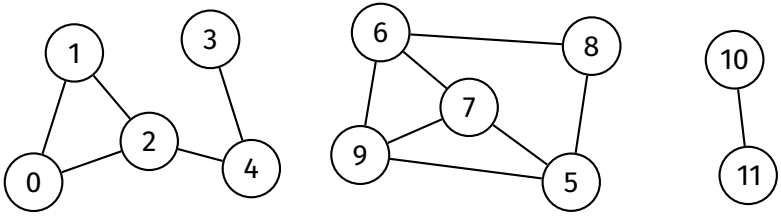
Breadth-first search



Depth-first search

Disconnected Graphs

If a graph is not connected,
a graph traversal starting from a given vertex
will not traverse the entire graph

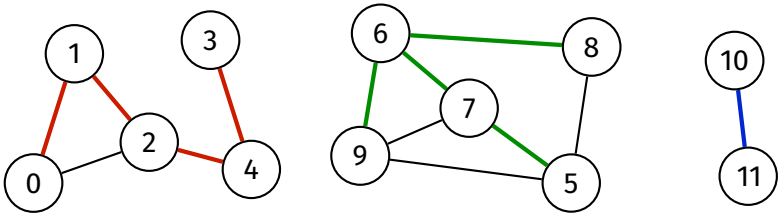


Disconnected Graphs

Solution

After initial traversal is complete,
perform traversal again on an unvisited vertex,
repeat until all vertices are visited

This produces a spanning forest



Disconnected Graphs

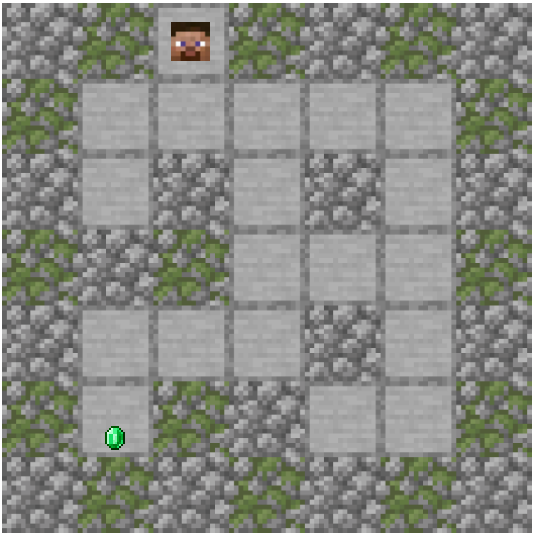

```
dfs(G):
    Input: graph G

    create predecessor array, initialised to -1

    for each vertex v in G:
        if predecessor[v] = -1:
            dfsRec(G, v, predecessor)

    ...
```

Feedback

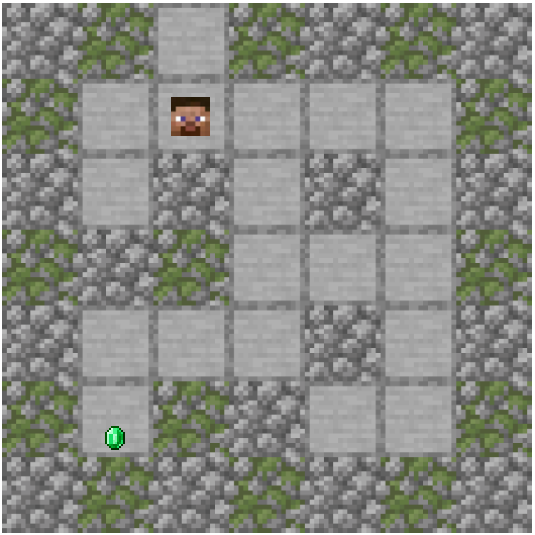
https://forms.office.com/r/ucsJ4r99Ag

Appendix

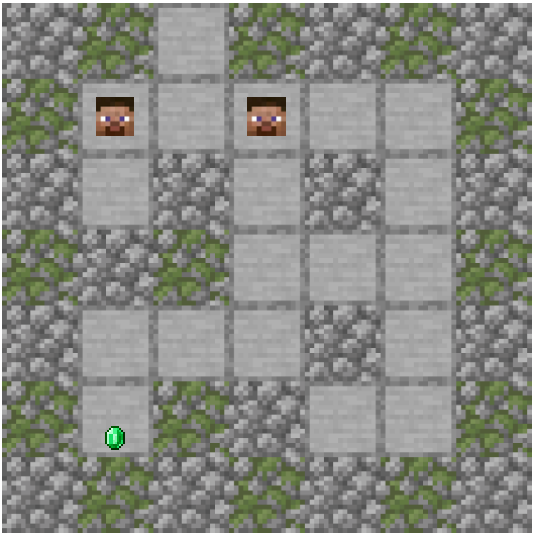
BFS



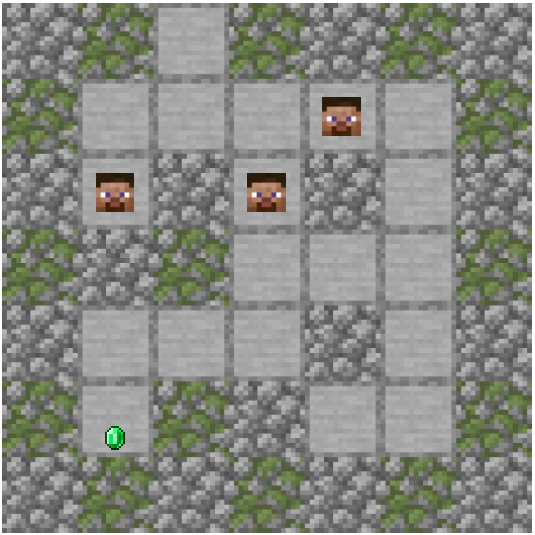
BFS



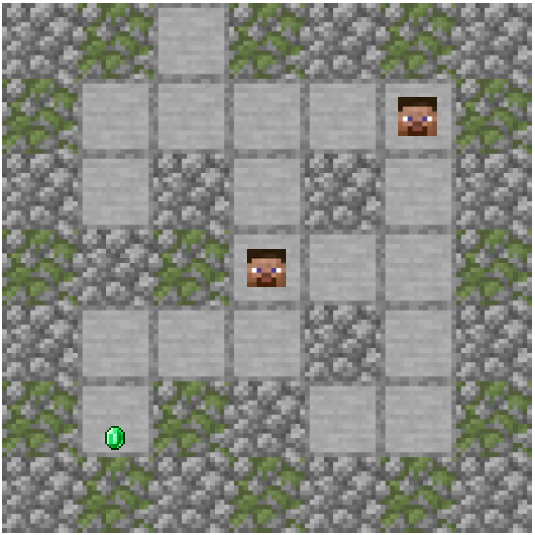
BFS



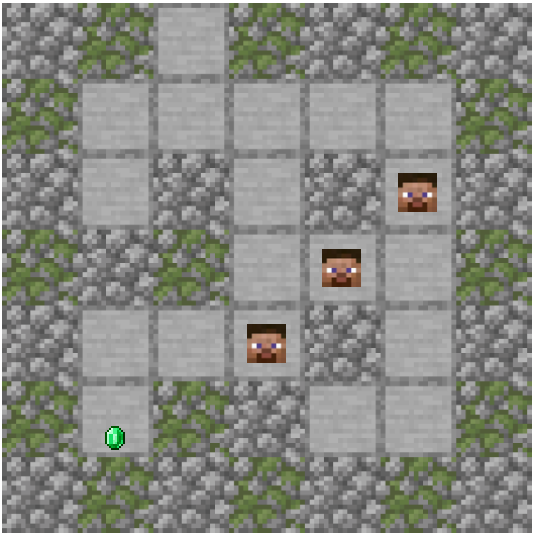
BFS



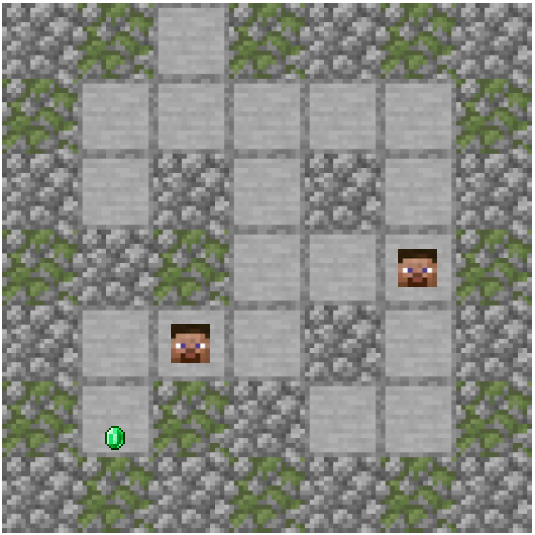
BFS



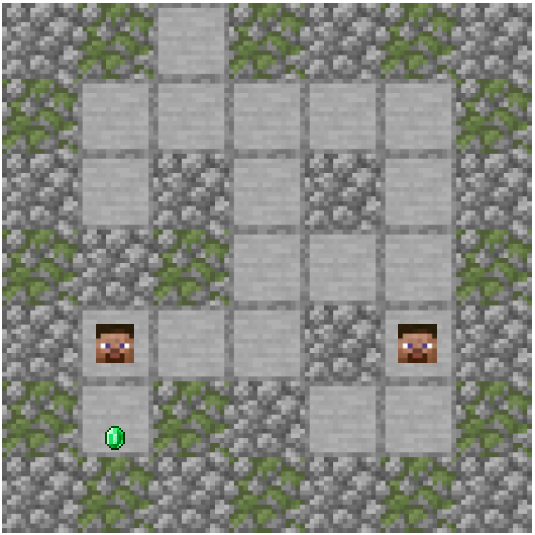
BFS



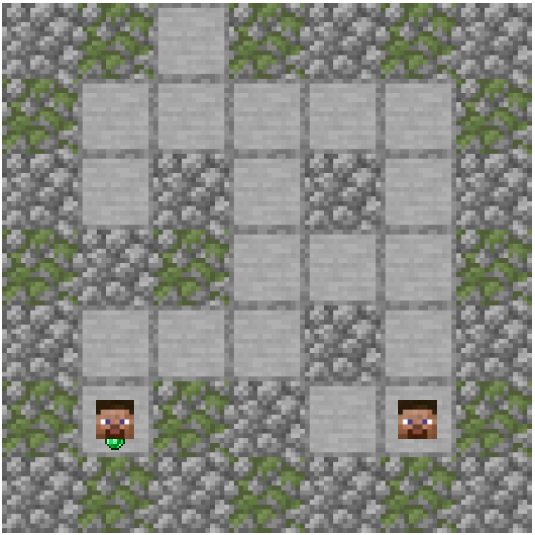
BFS



BFS



BFS



BFS

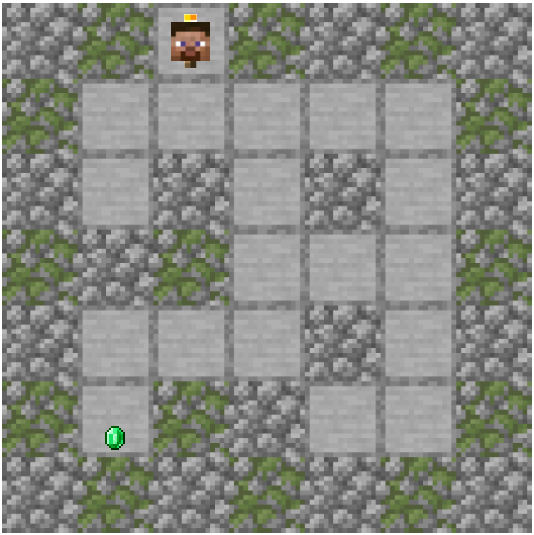
How do we avoid revisiting the same tiles?

BFS

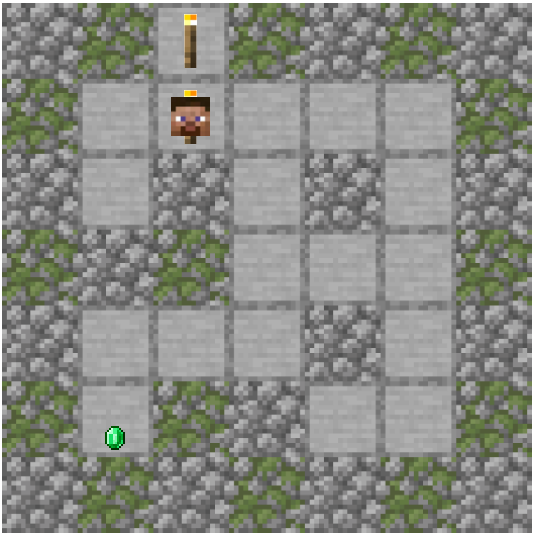
How do we avoid revisiting the same tiles?

Mark tiles as they are visited!

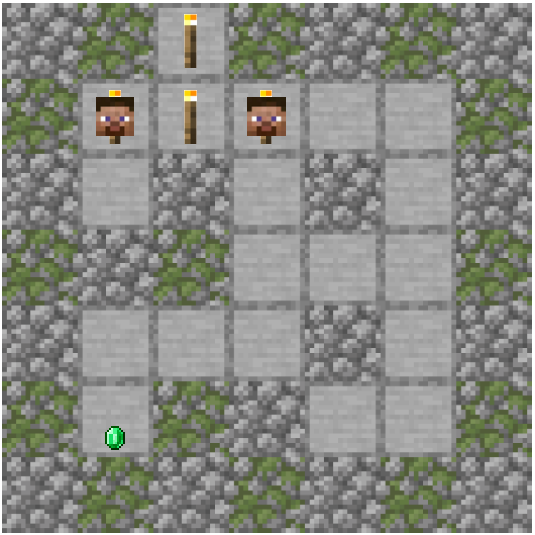
BFS



BFS



BFS



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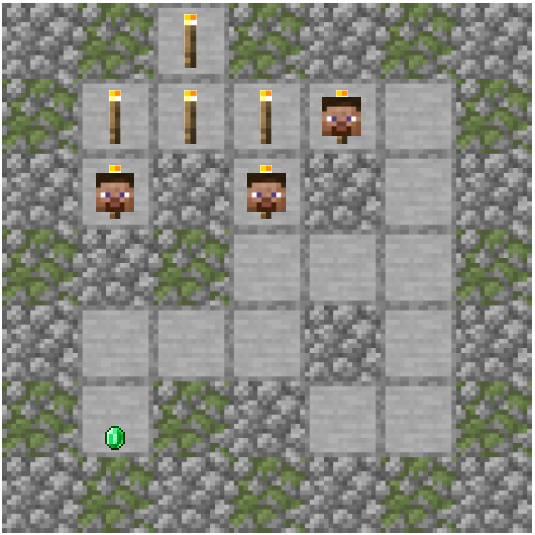
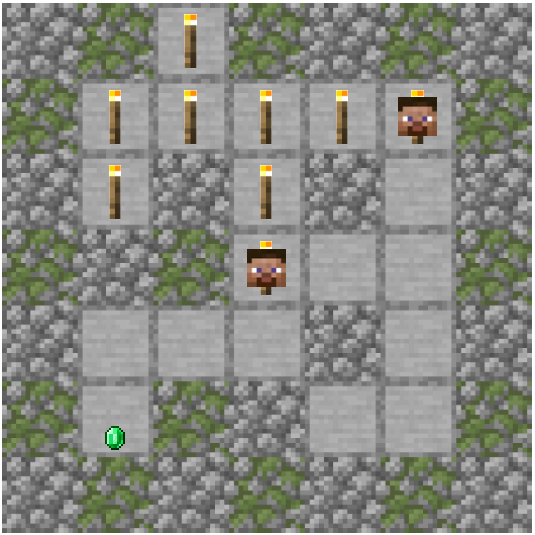
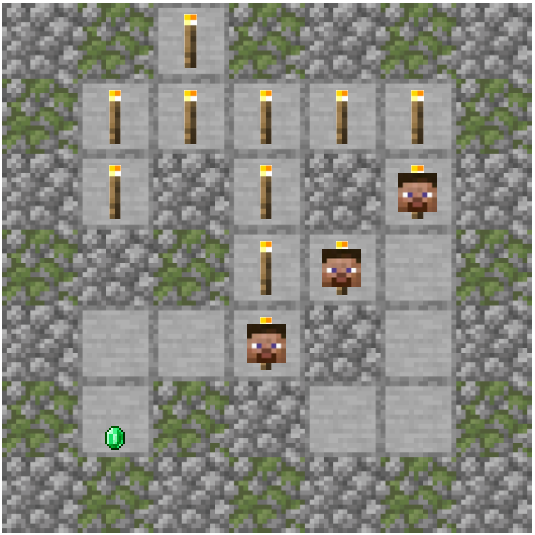


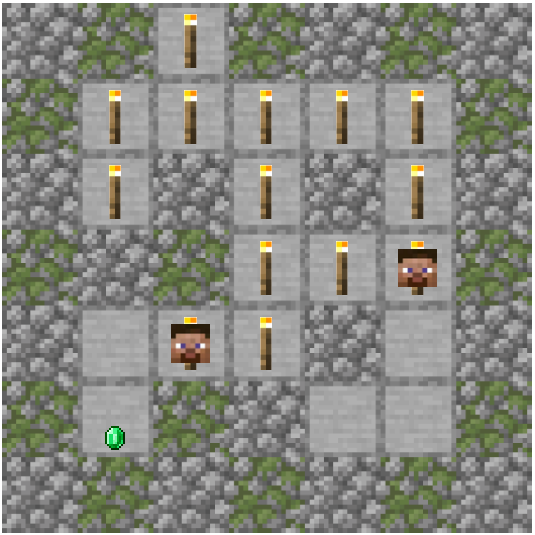
BFS



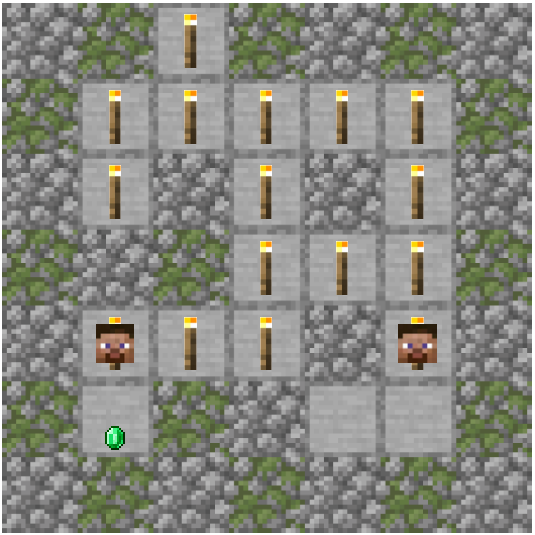
BFS



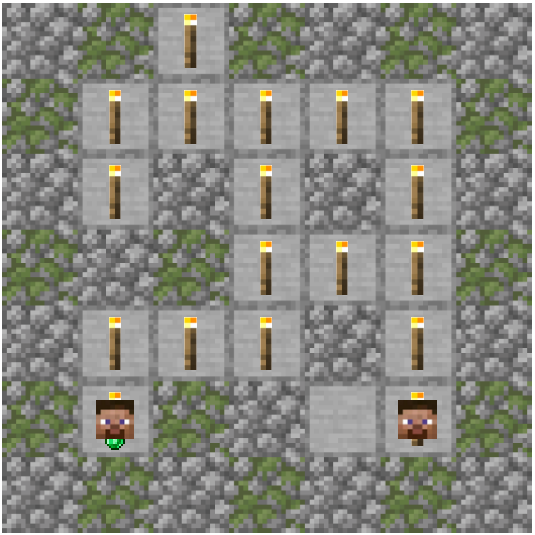
BFS



BFS



BFS



BFS

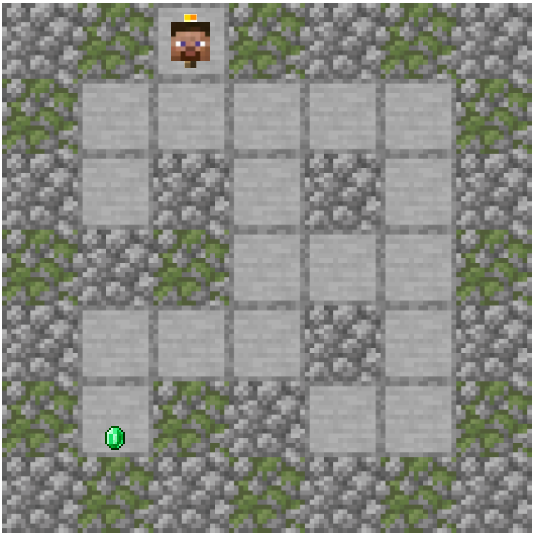
How do we find our way back to the entrance?

BFS

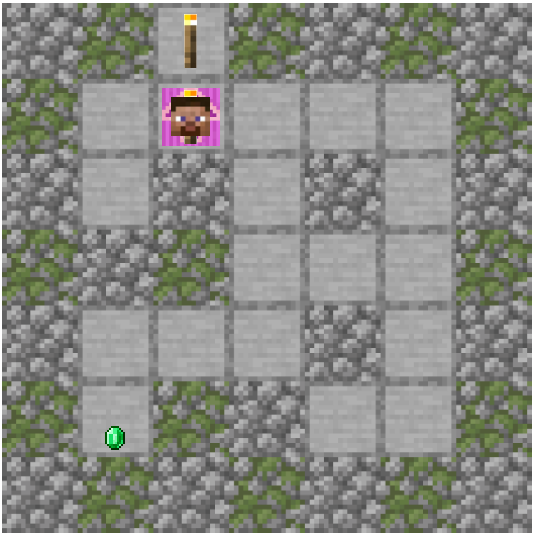
How do we find our way back to the entrance?

For each tile that we visit,
keep note of the tile we were on directly before it!
This is called the *predecessor*.

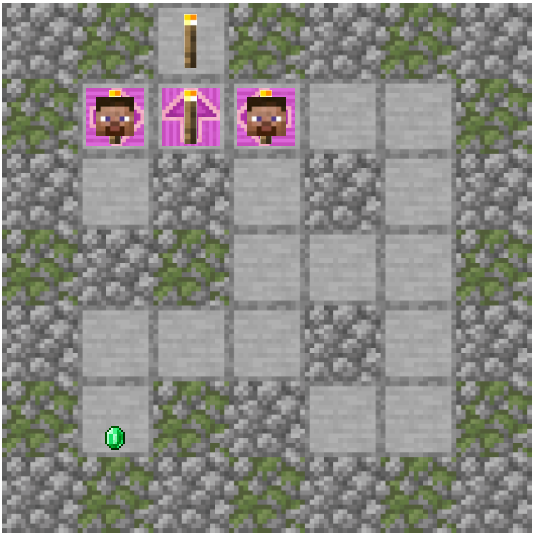
BFS



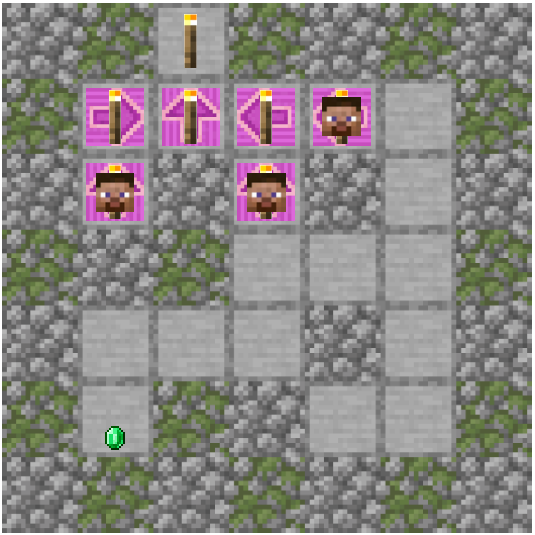
BFS



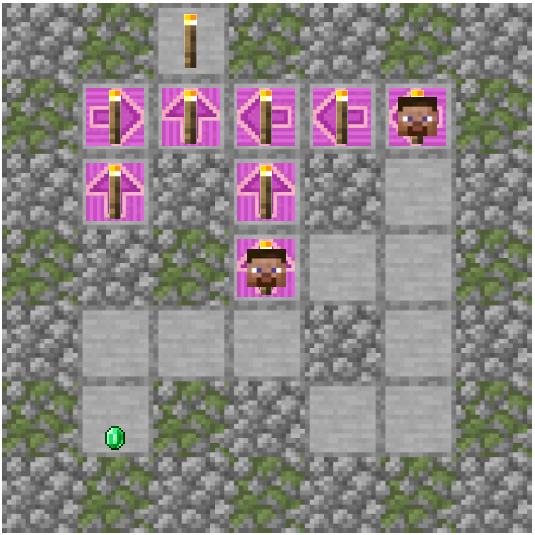
BFS



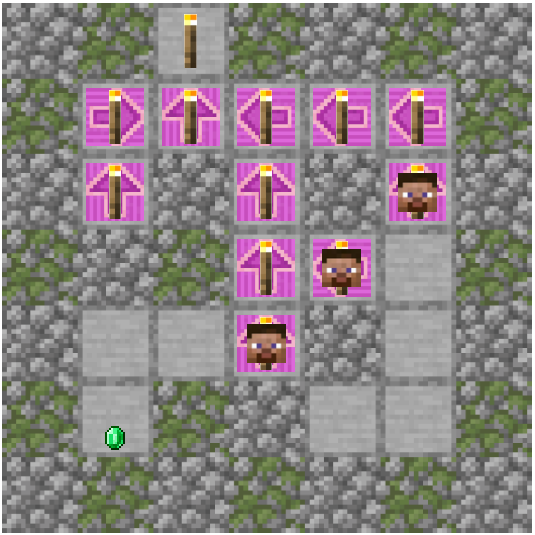
BFS



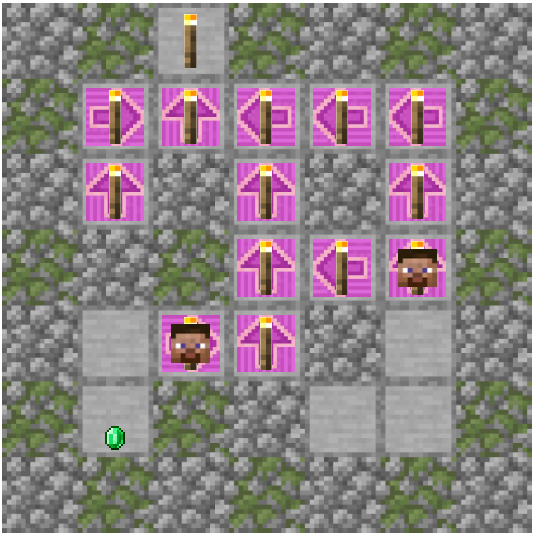
BFS



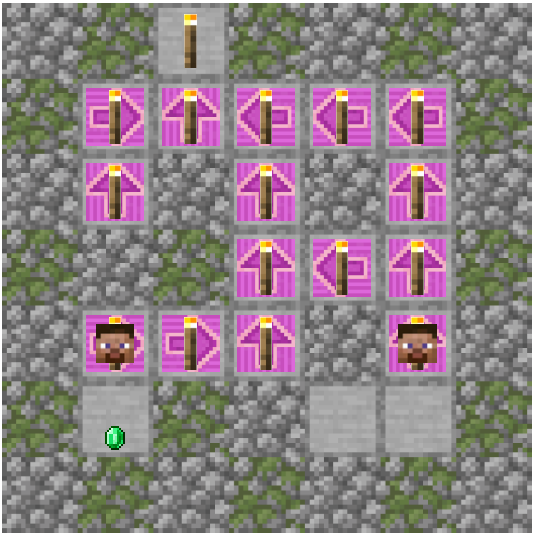
BFS



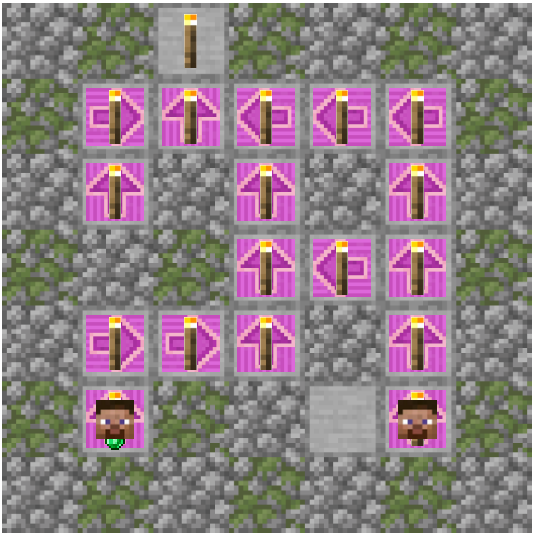
BFS



BFS

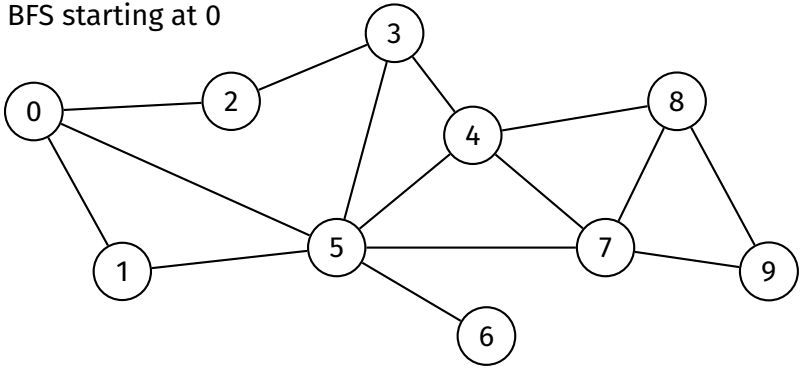


BFS



BFS Example

BFS starting at 0



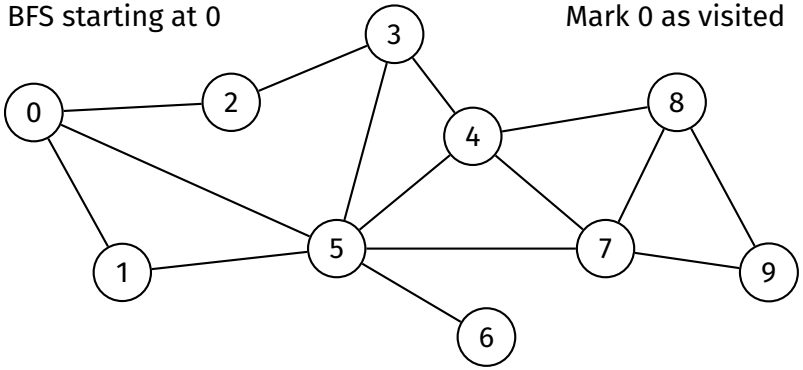
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	0	0	0	0	0	0	0	0	0	0
pred	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

queue

BFS Example

BFS starting at 0

Mark 0 as visited

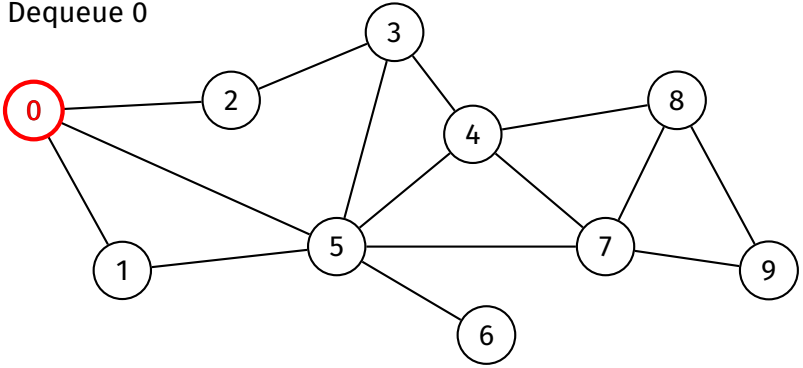


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0
pred	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

queue 0

BFS Example

Dequeue 0

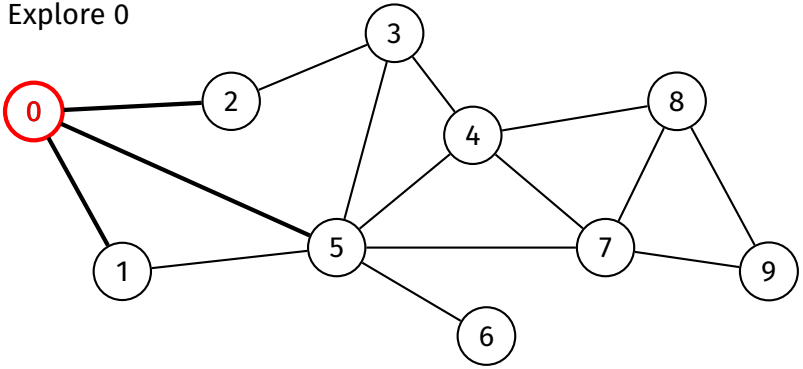


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0
pred	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

queue 0

BFS Example

Explore 0

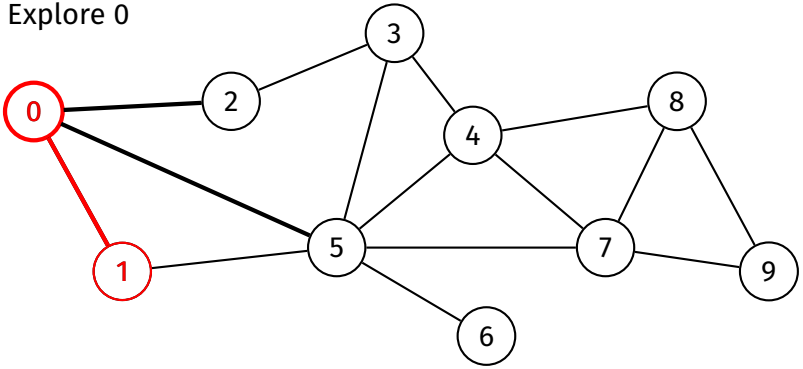


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0
pred	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

queue 0

BFS Example

Explore 0



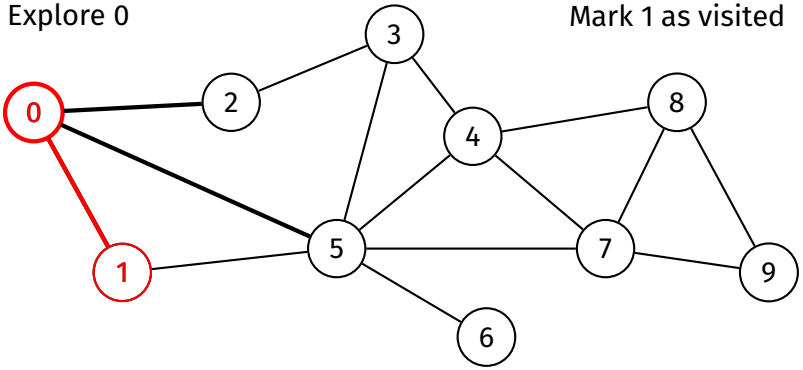
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0
pred	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

queue 0

BFS Example

Explore 0

Mark 1 as visited

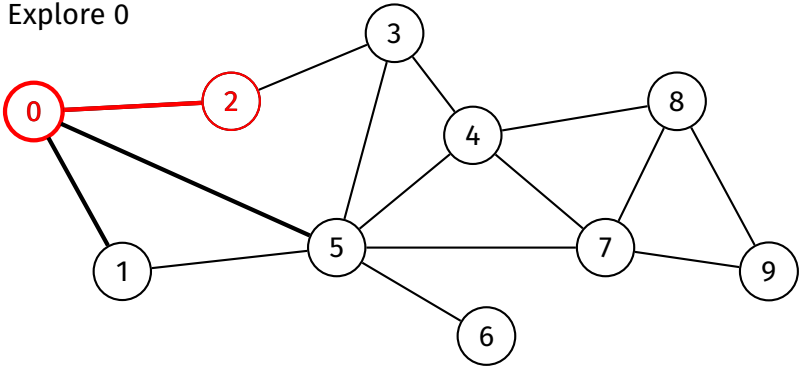


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0
pred	-1	0	-1	-1	-1	-1	-1	-1	-1	-1

queue 0 1

BFS Example

Explore 0



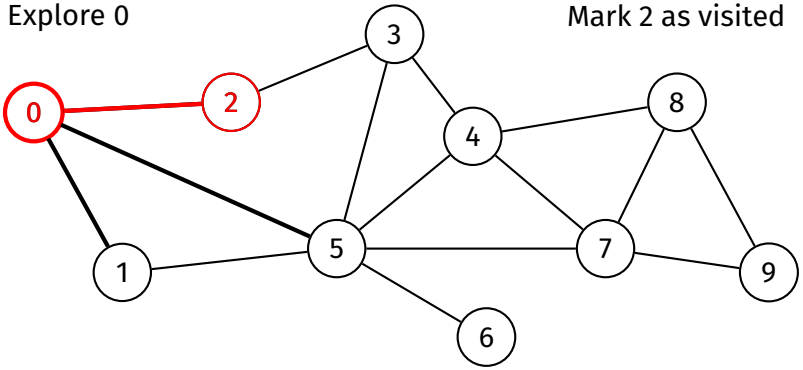
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0
pred	-1	0	-1	-1	-1	-1	-1	-1	-1	-1

queue 0 1

BFS Example

Explore 0

Mark 2 as visited

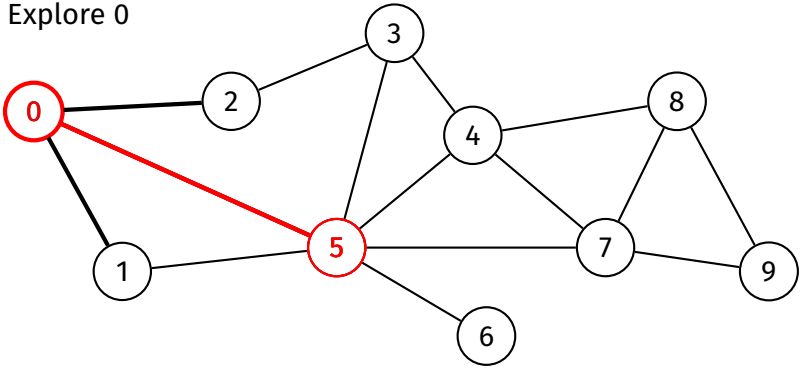


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	0	0	0	0	0
pred	-1	0	0	-1	-1	-1	-1	-1	-1	-1

queue 0 1 2

BFS Example

Explore 0



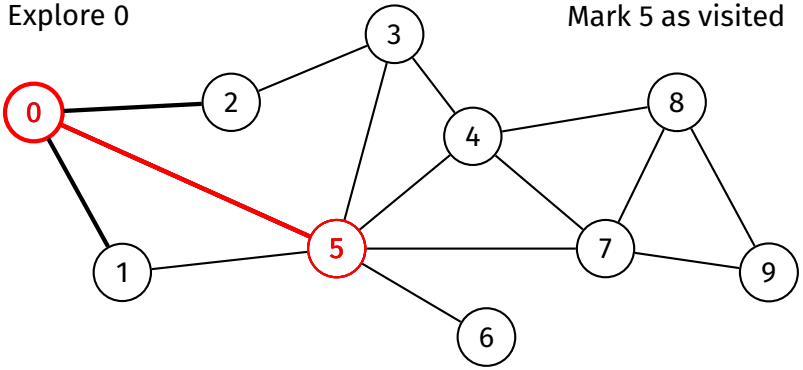
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	0	0	0	0	0
pred	-1	0	0	-1	-1	-1	-1	-1	-1	-1

queue 0 1 2

BFS Example

Explore 0

Mark 5 as visited



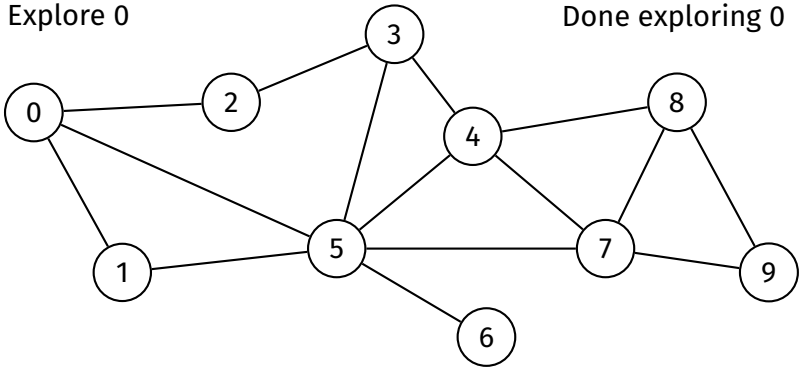
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

BFS Example

Explore 0

Done exploring 0

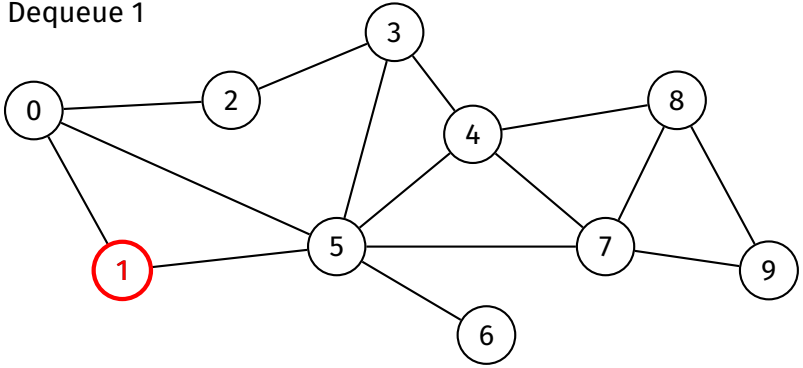


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

BFS Example

Dequeue 1

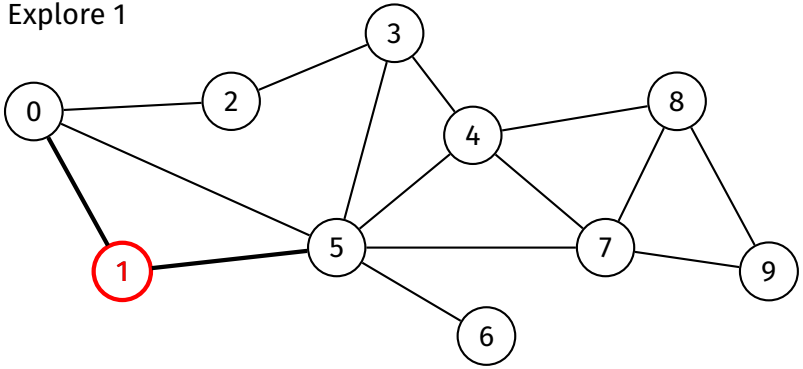


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

BFS Example

Explore 1



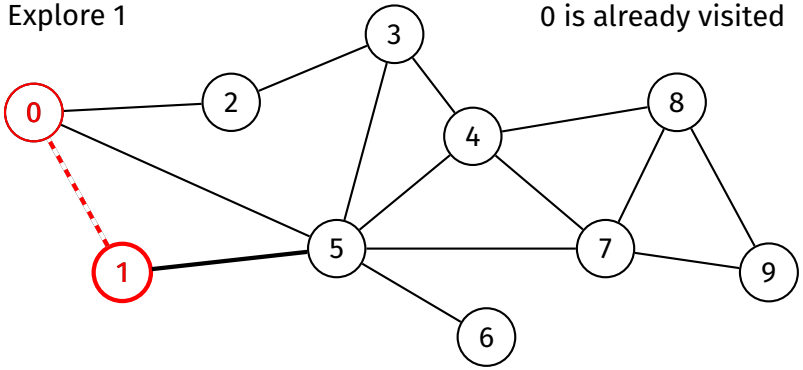
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

BFS Example

Explore 1

0 is already visited



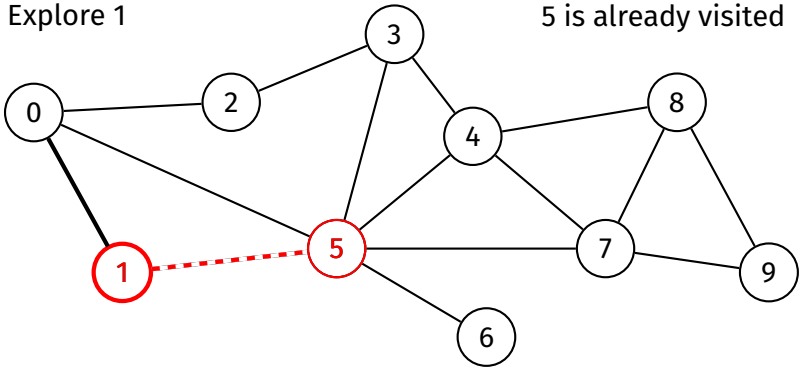
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

BFS Example

Explore 1

5 is already visited



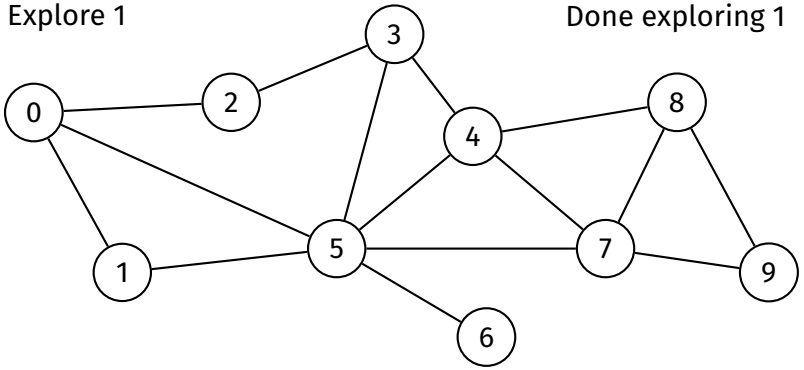
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

BFS Example

Explore 1

Done exploring 1

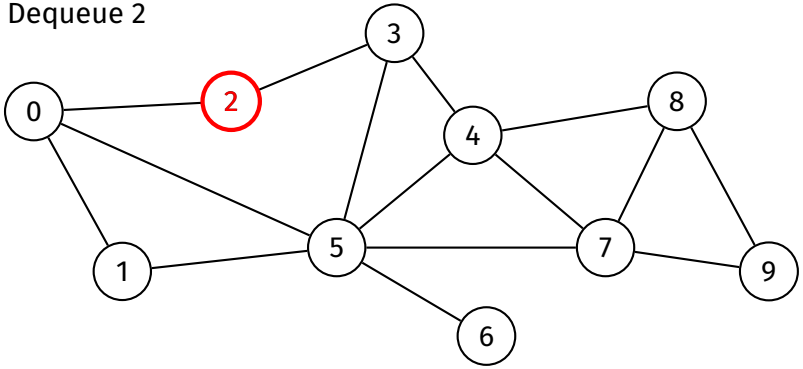


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

BFS Example

Dequeue 2

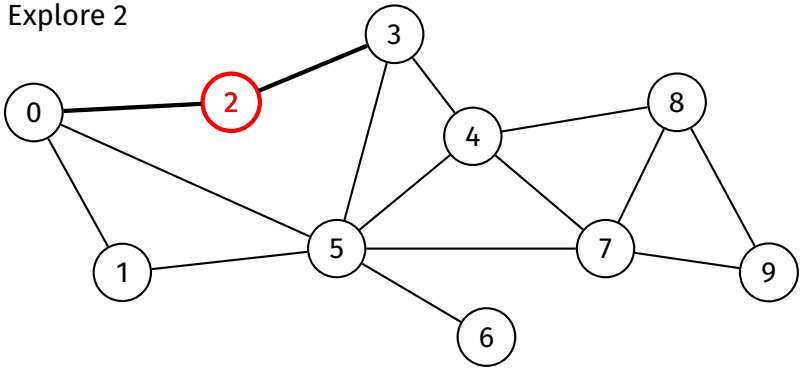


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

BFS Example

Explore 2



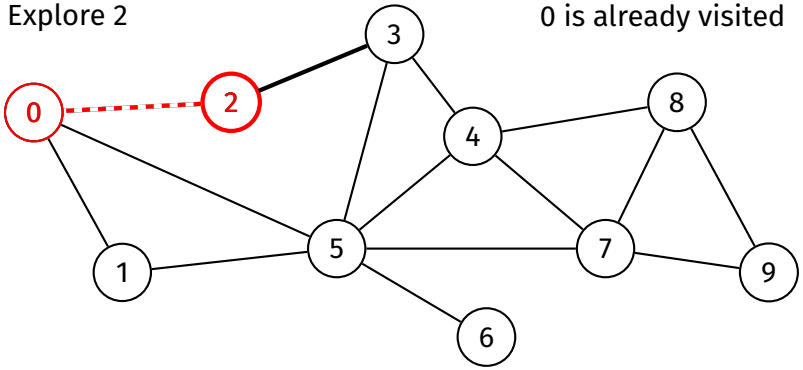
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

BFS Example

Explore 2

0 is already visited

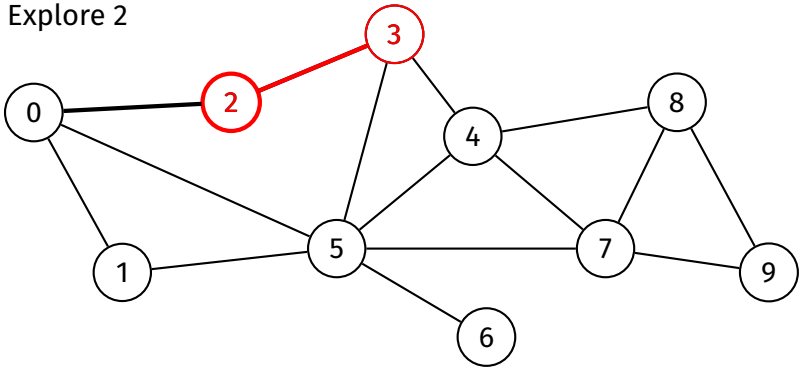


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

BFS Example

Explore 2



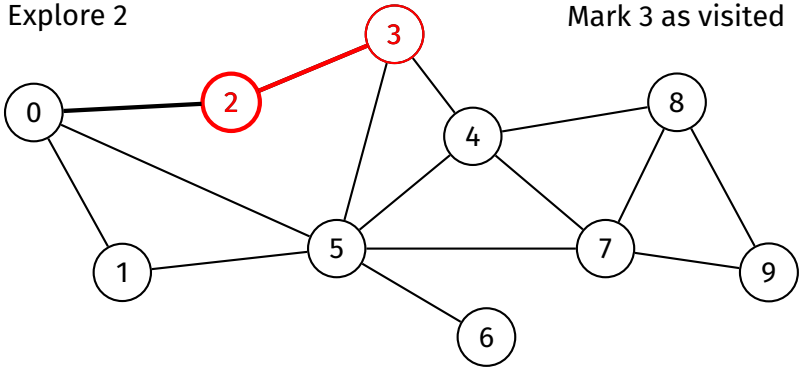
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	0	0	1	0	0	0	0
pred	-1	0	0	-1	-1	0	-1	-1	-1	-1

queue 0 1 2 5

BFS Example

Explore 2

Mark 3 as visited



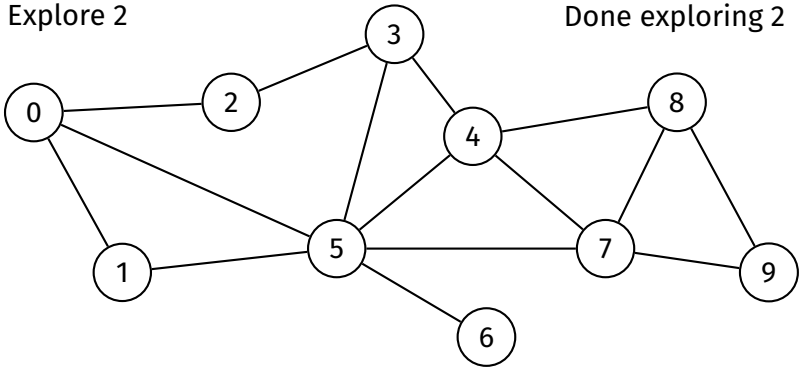
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

BFS Example

Explore 2

Done exploring 2

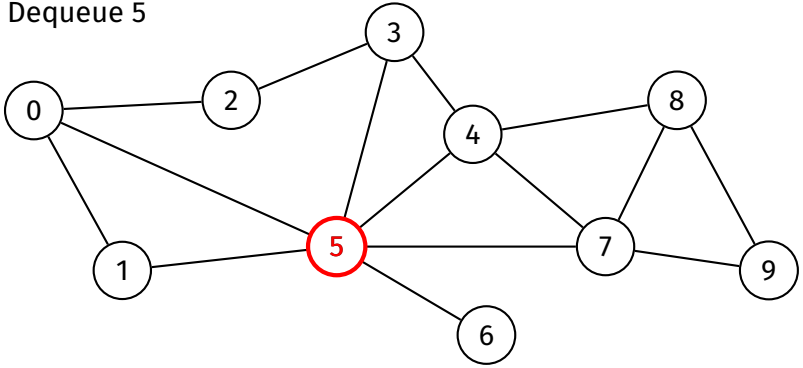


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

BFS Example

Dequeue 5

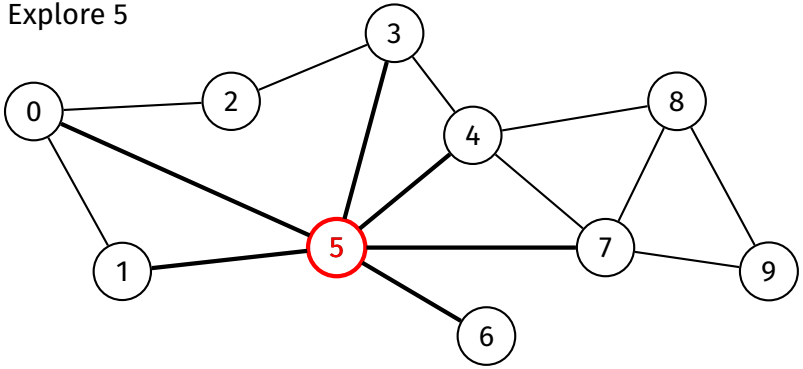


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

BFS Example

Explore 5



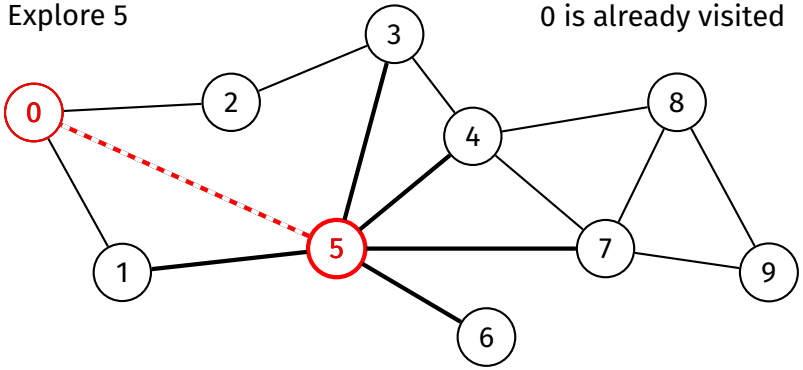
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

BFS Example

Explore 5

0 is already visited



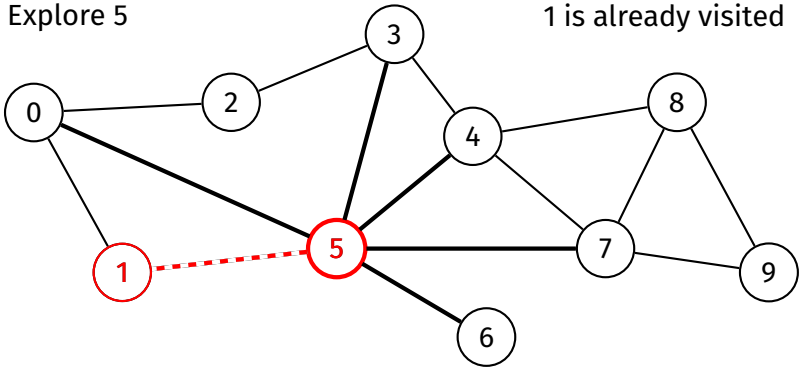
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

BFS Example

Explore 5

1 is already visited



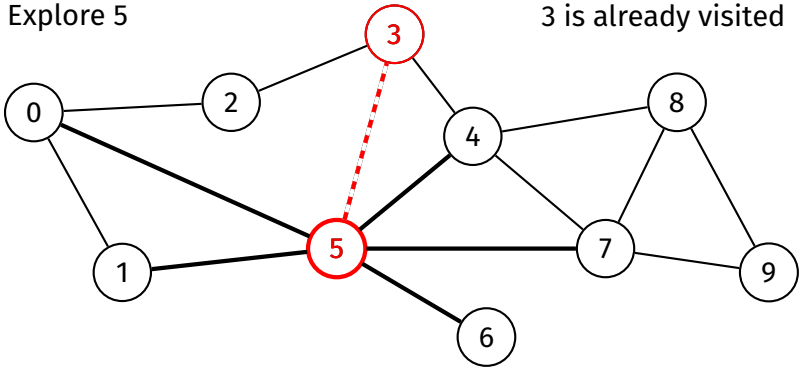
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

BFS Example

Explore 5

3 is already visited

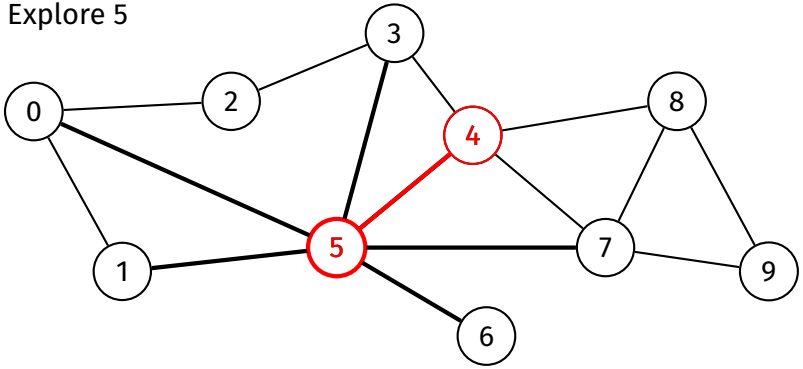


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

BFS Example

Explore 5



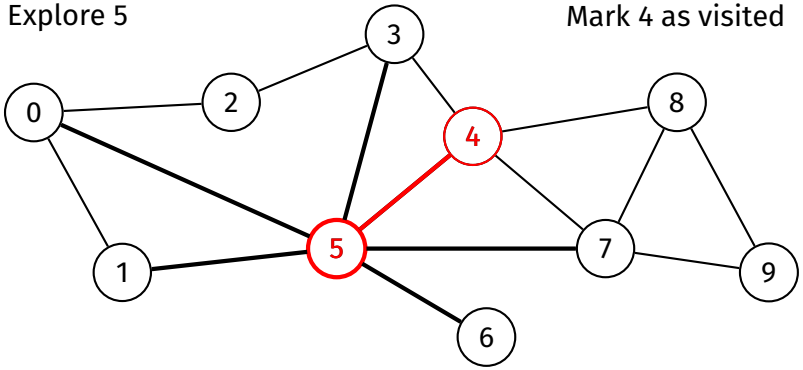
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0
pred	-1	0	0	2	-1	0	-1	-1	-1	-1

queue 0 1 2 5 3

BFS Example

Explore 5

Mark 4 as visited

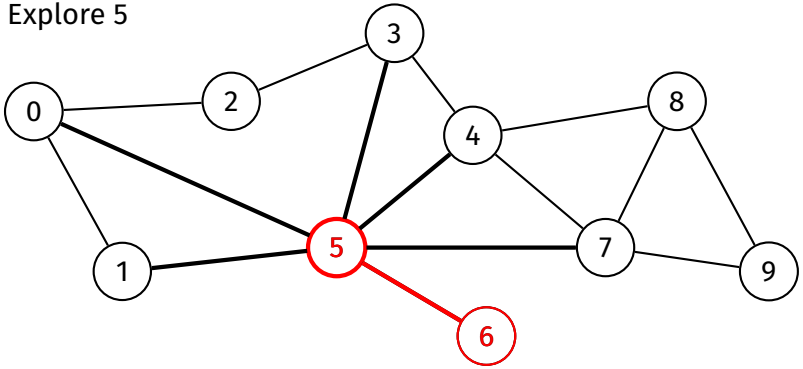


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0
pred	-1	0	0	2	5	0	-1	-1	-1	-1

queue 0 1 2 5 3 4

BFS Example

Explore 5



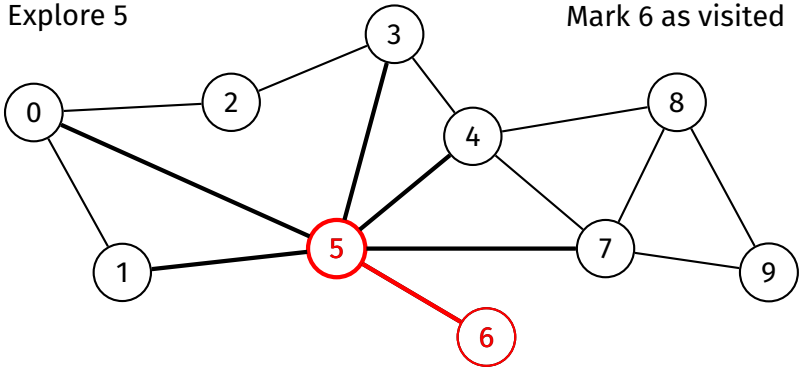
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0
pred	-1	0	0	2	5	0	-1	-1	-1	-1

queue 0 1 2 5 3 4

BFS Example

Explore 5

Mark 6 as visited

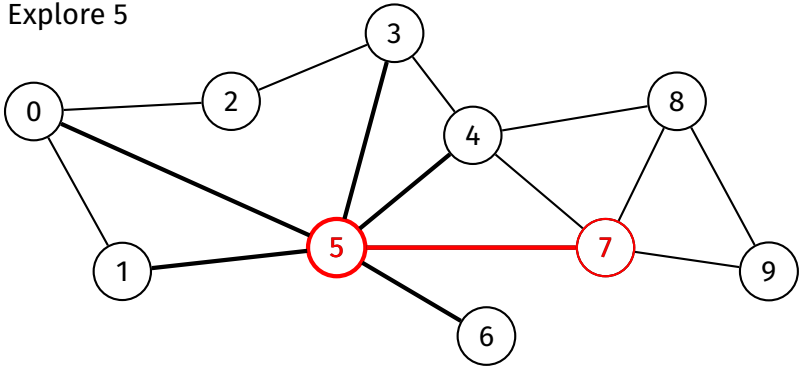


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	0	0	0
pred	-1	0	0	2	5	0	5	-1	-1	-1

queue 0 1 2 5 3 4 6

BFS Example

Explore 5



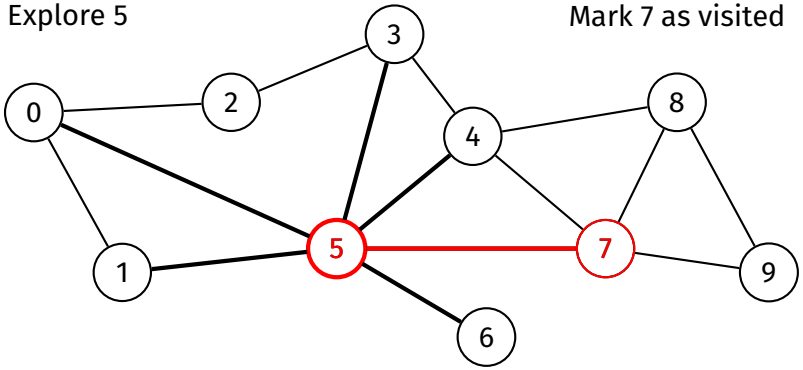
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	0	0	0
pred	-1	0	0	2	5	0	5	-1	-1	-1

queue 0 1 2 5 3 4 6

BFS Example

Explore 5

Mark 7 as visited



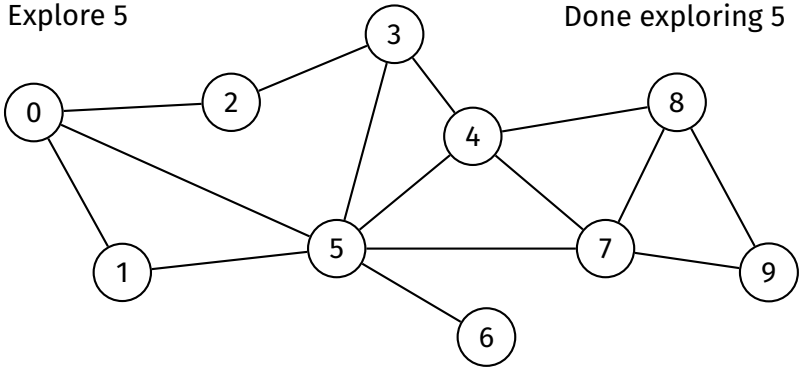
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

BFS Example

Explore 5

Done exploring 5

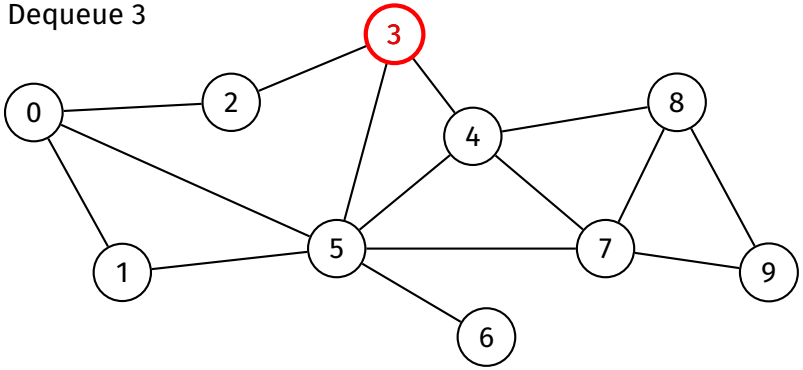


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

BFS Example

Dequeue 3

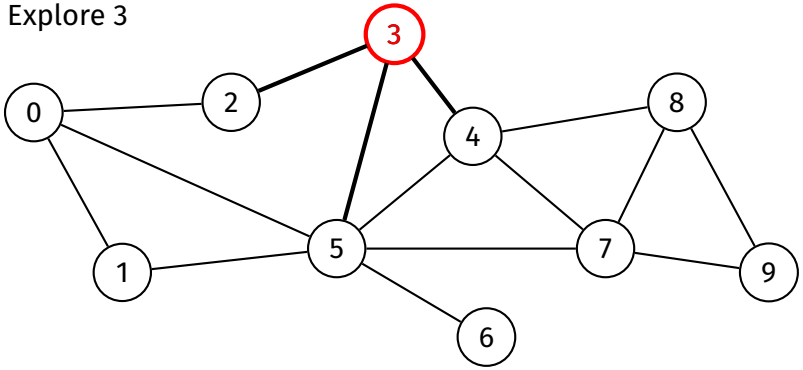


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

BFS Example

Explore 3



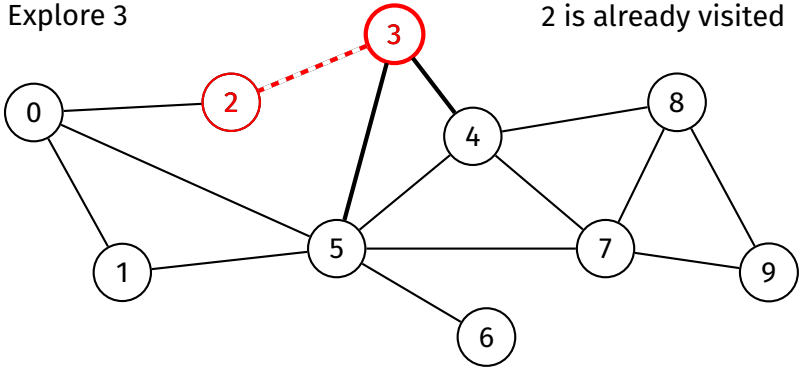
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

BFS Example

Explore 3

2 is already visited



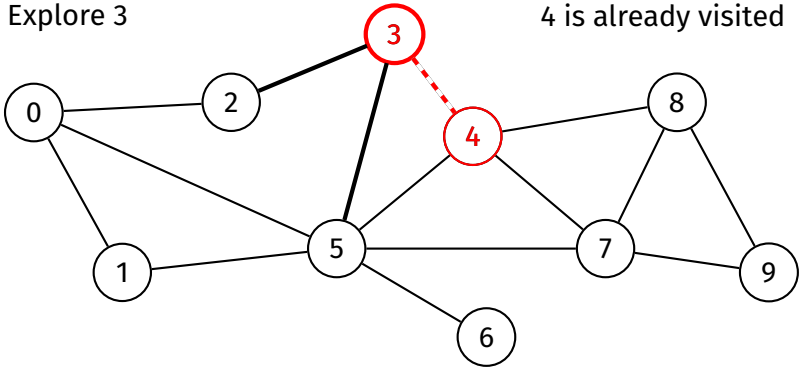
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

BFS Example

Explore 3

4 is already visited



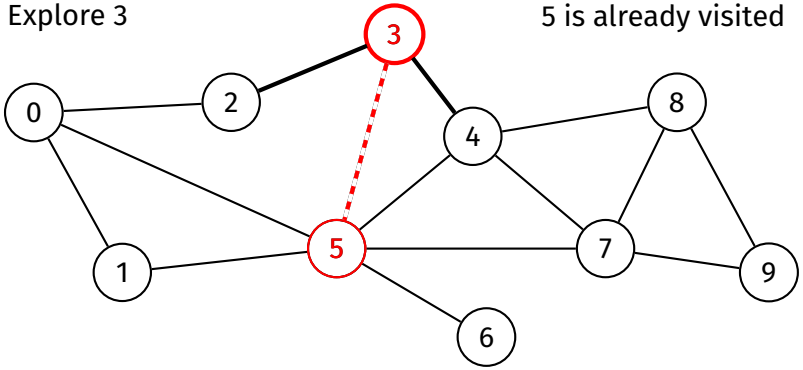
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

BFS Example

Explore 3

5 is already visited



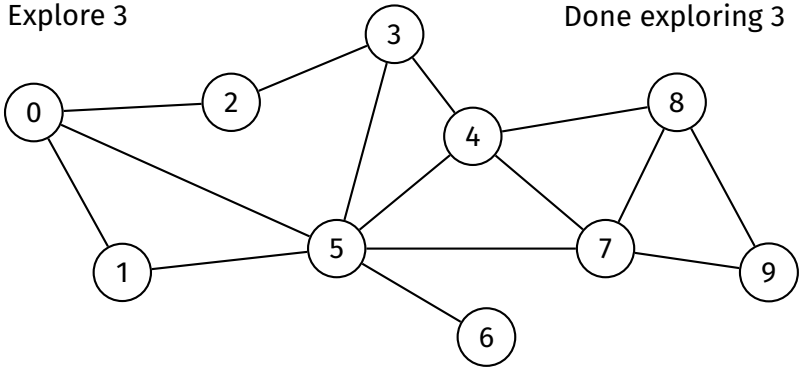
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

BFS Example

Explore 3

Done exploring 3

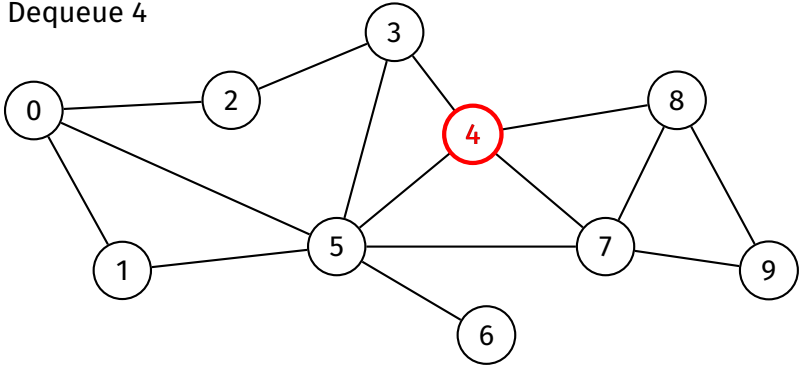


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

BFS Example

Dequeue 4

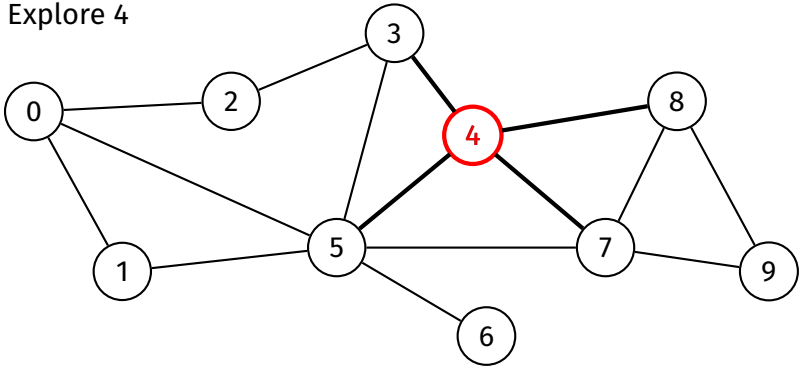


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

BFS Example

Explore 4



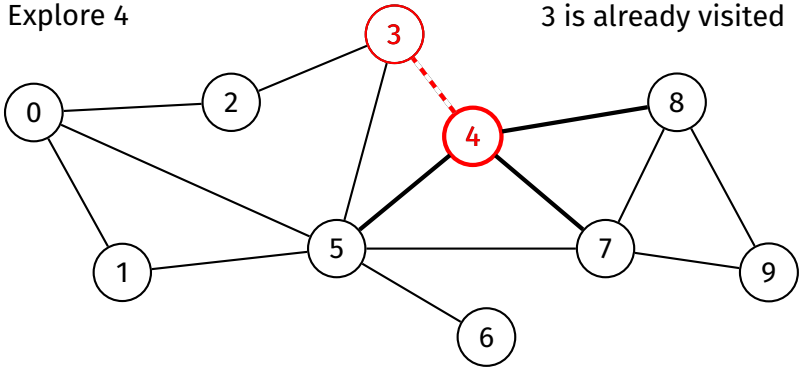
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

BFS Example

Explore 4

3 is already visited



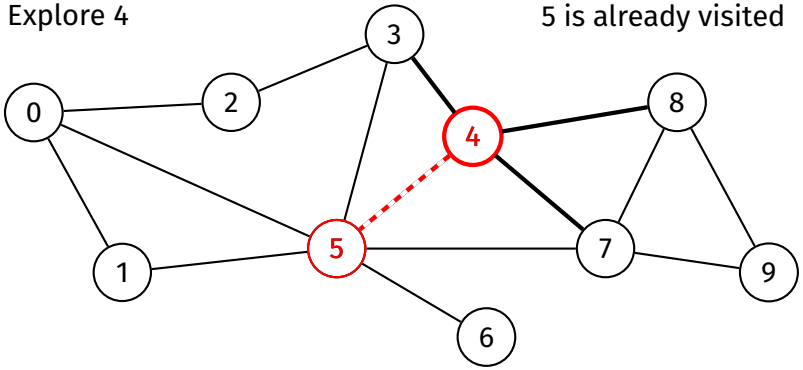
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

BFS Example

Explore 4

5 is already visited



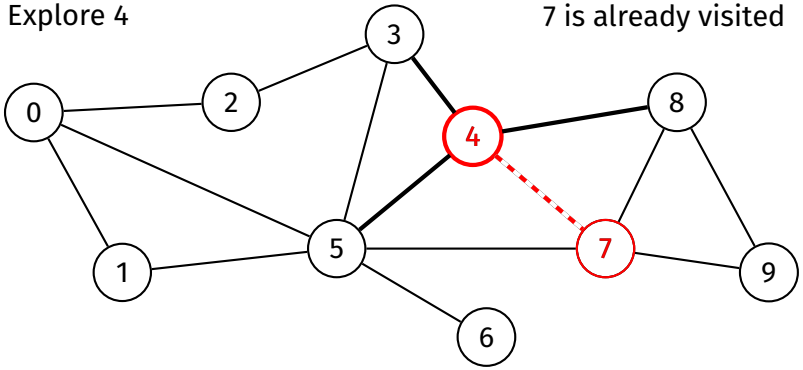
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

BFS Example

Explore 4

7 is already visited

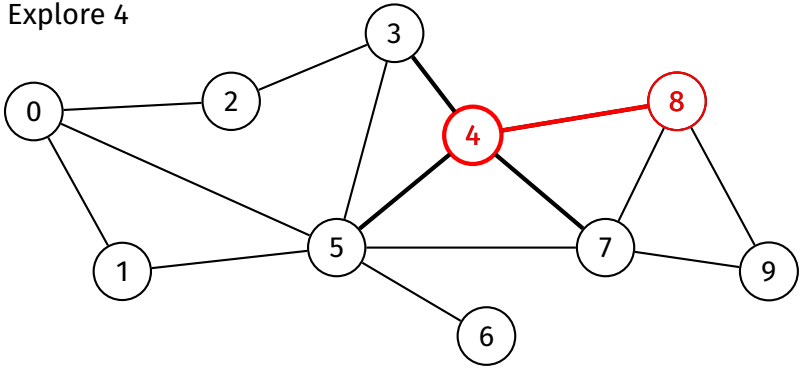


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

BFS Example

Explore 4



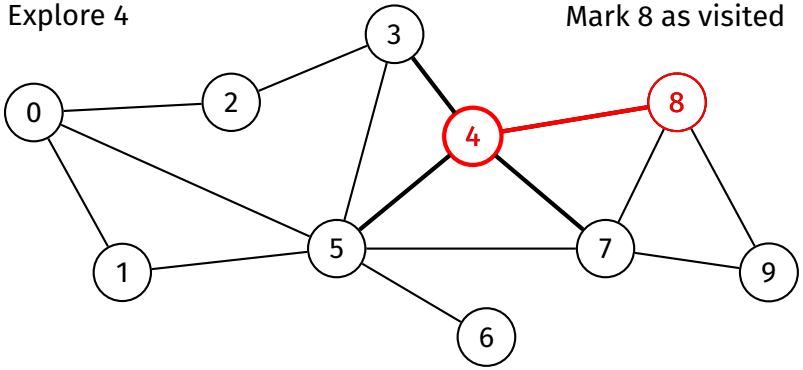
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	0	0
pred	-1	0	0	2	5	0	5	5	-1	-1

queue 0 1 2 5 3 4 6 7

BFS Example

Explore 4

Mark 8 as visited



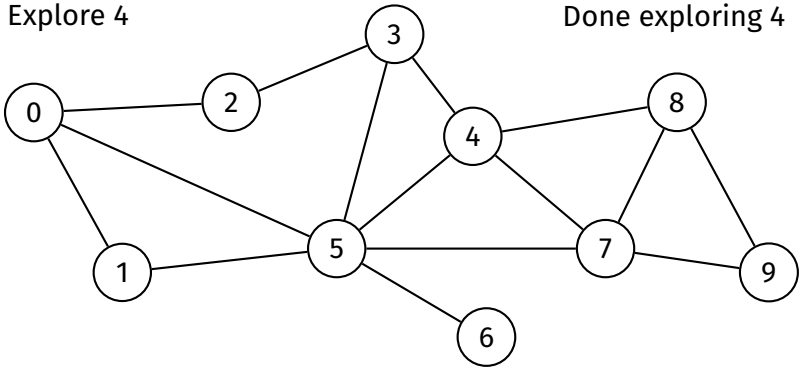
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

BFS Example

Explore 4

Done exploring 4

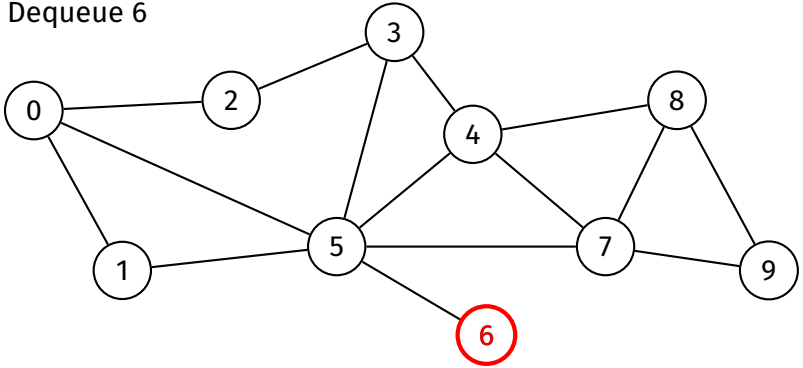


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

BFS Example

Dequeue 6

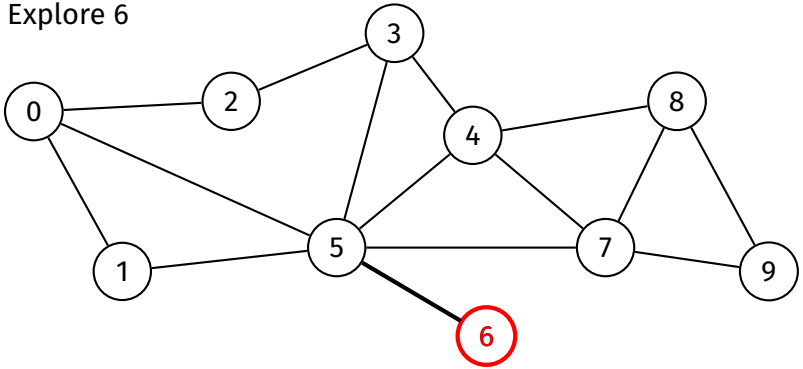


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

BFS Example

Explore 6



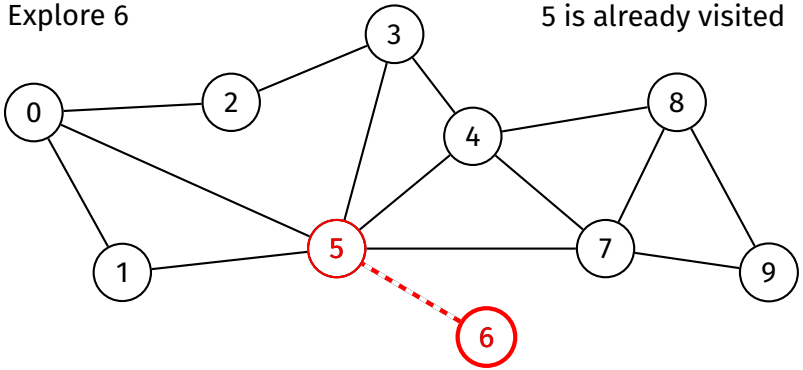
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

BFS Example

Explore 6

5 is already visited



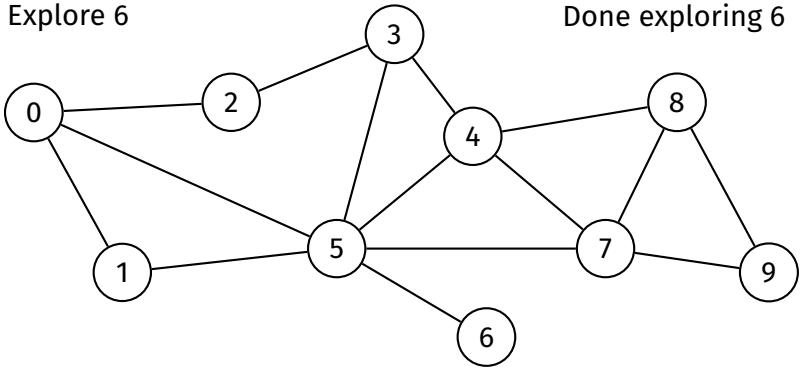
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

BFS Example

Explore 6

Done exploring 6

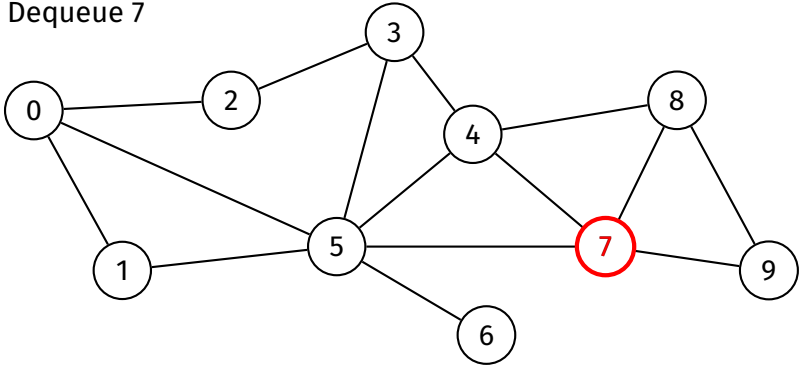


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

BFS Example

Dequeue 7

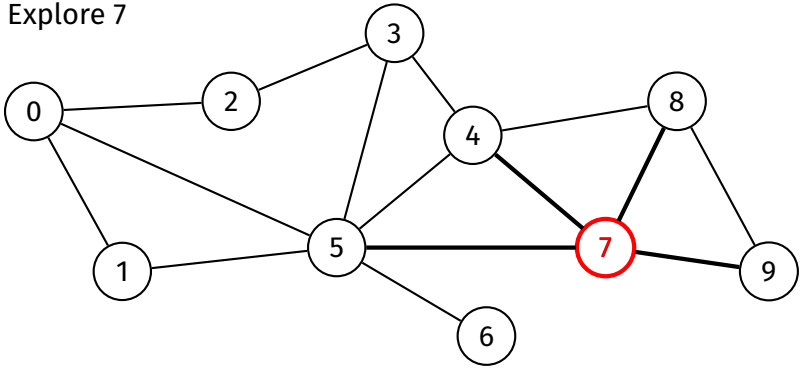


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

BFS Example

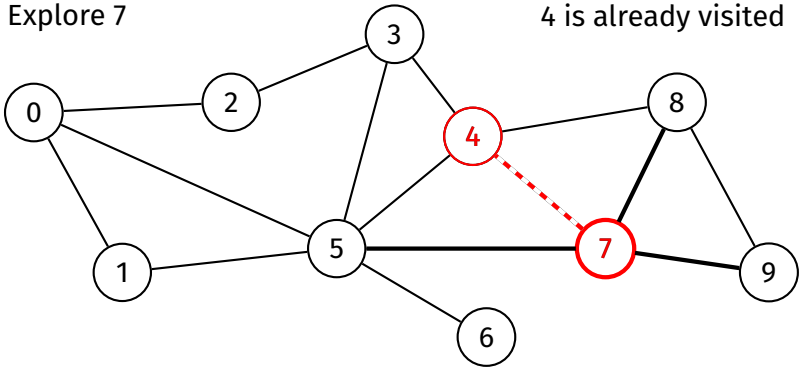
Explore 7



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

BFS Example



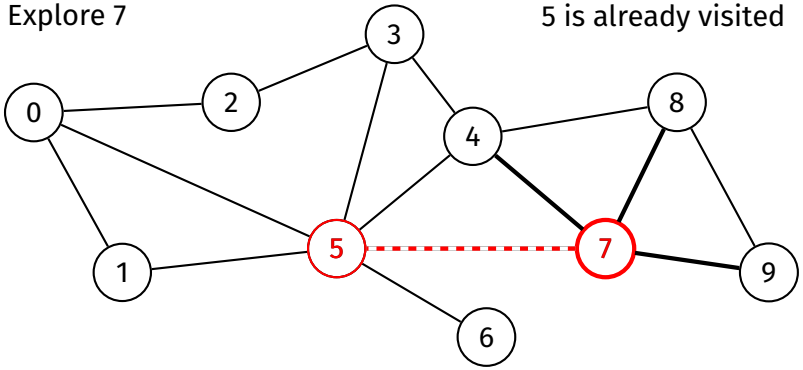
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

BFS Example

Explore 7

5 is already visited



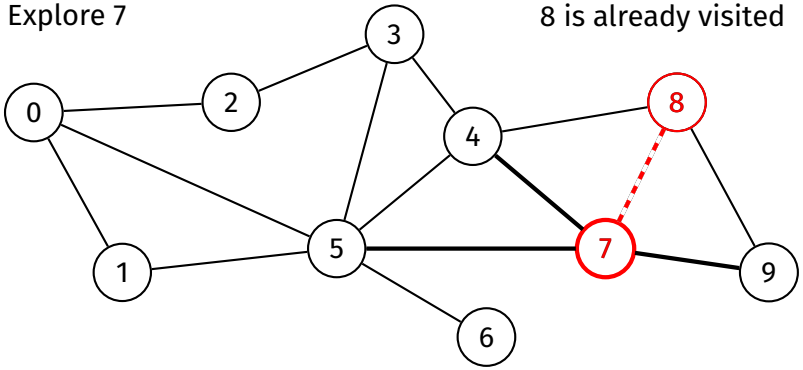
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

BFS Example

Explore 7

8 is already visited

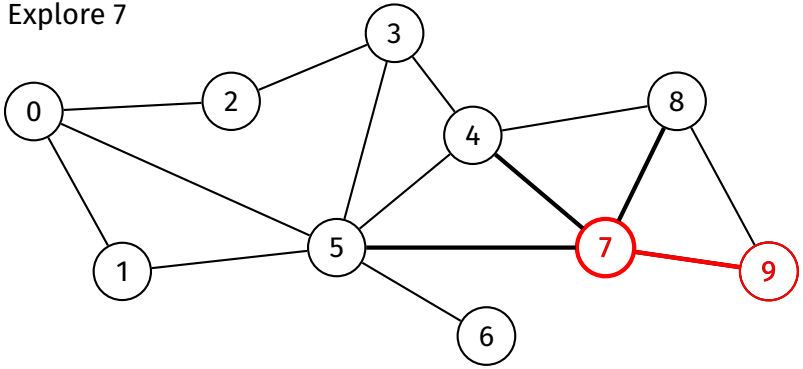


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

BFS Example

Explore 7



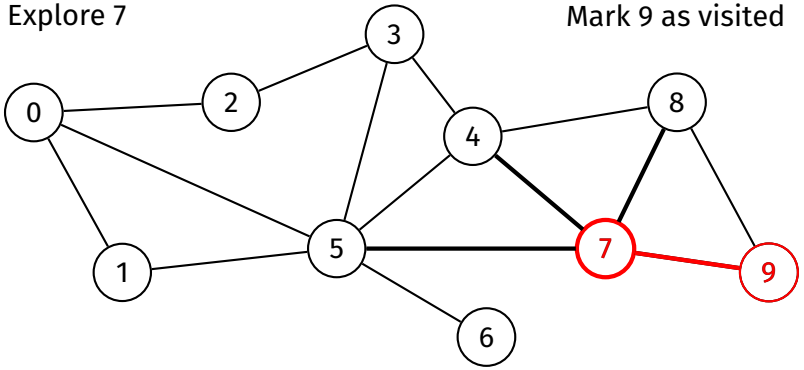
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	0
pred	-1	0	0	2	5	0	5	5	4	-1

queue 0 1 2 5 3 4 6 7 8

BFS Example

Explore 7

Mark 9 as visited



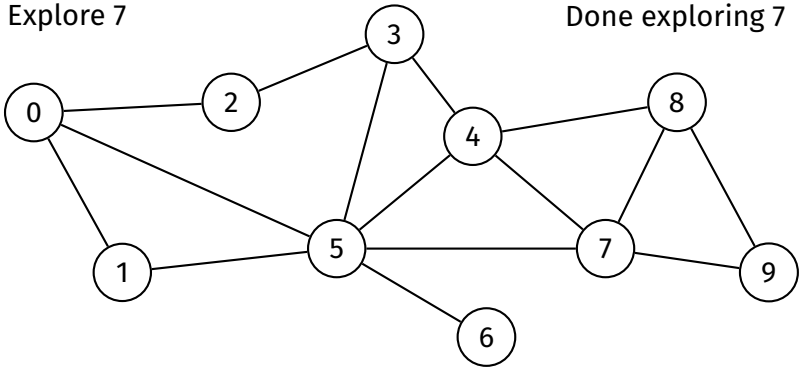
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

BFS Example

Explore 7

Done exploring 7

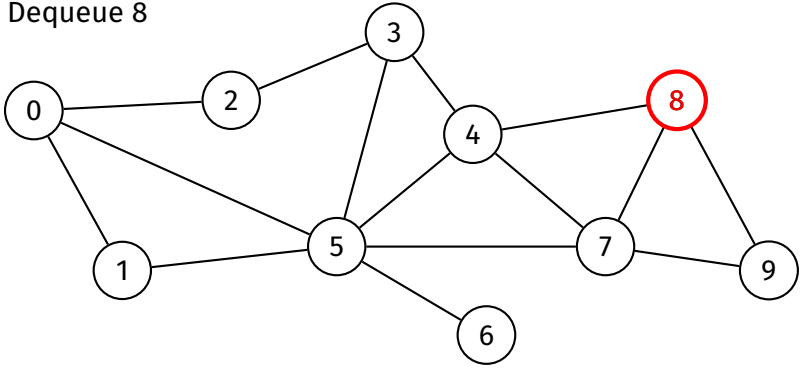


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

BFS Example

Dequeue 8

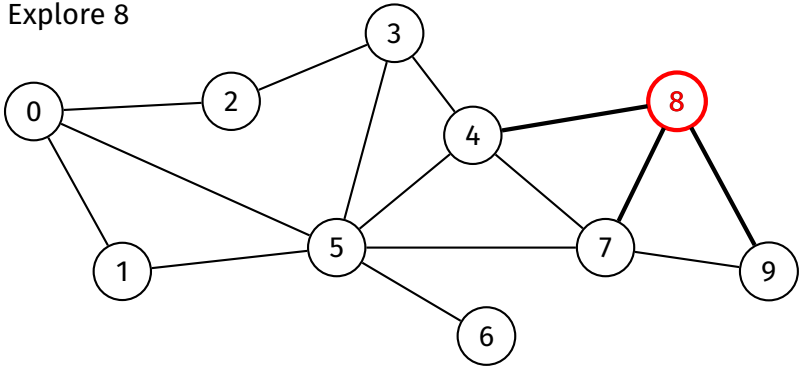


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

BFS Example

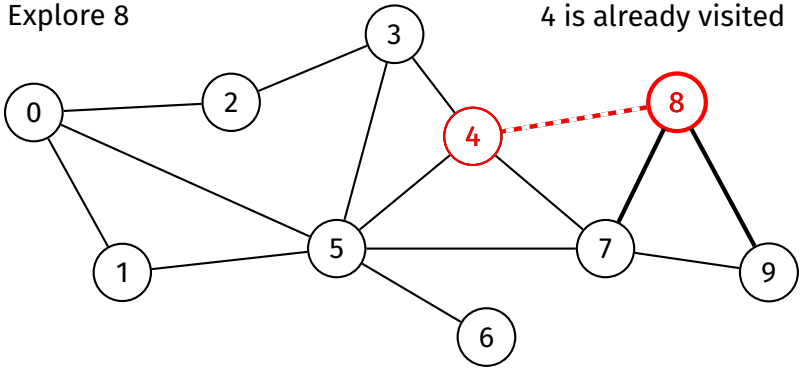
Explore 8



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

BFS Example



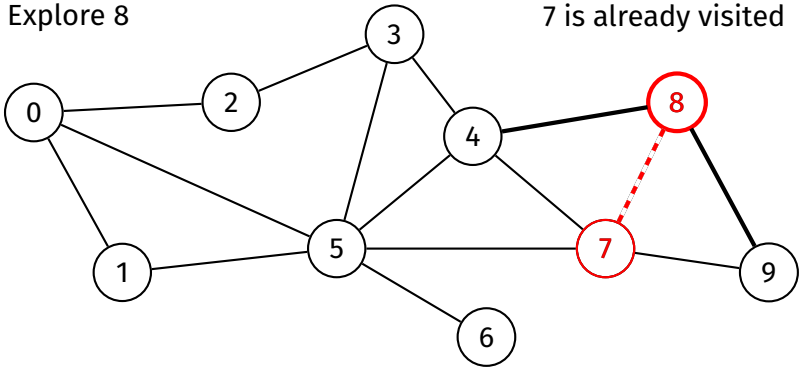
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

BFS Example

Explore 8

7 is already visited



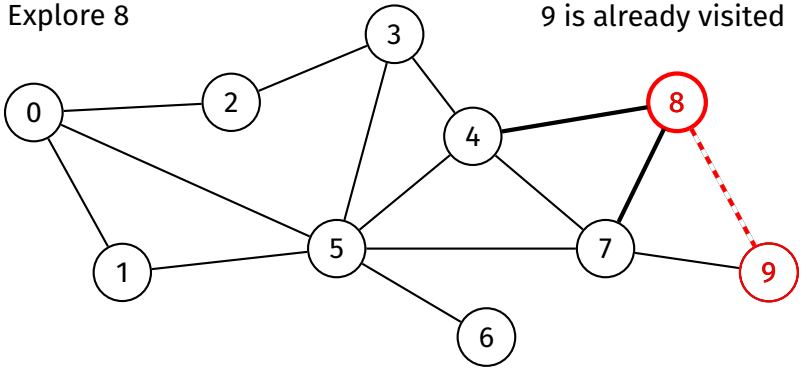
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

BFS Example

Explore 8

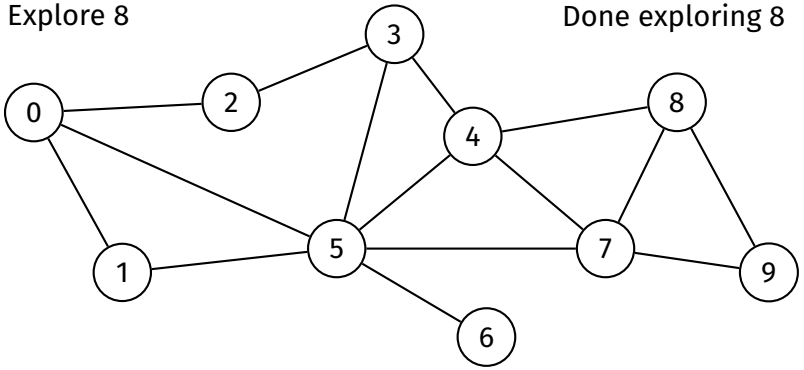
9 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

BFS Example

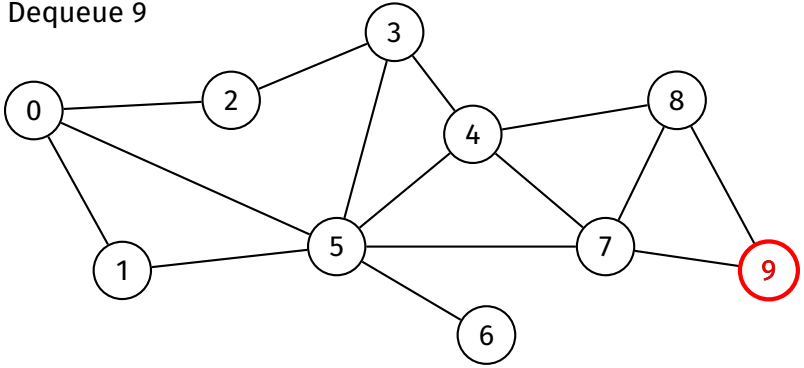


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

BFS Example

Dequeue 9

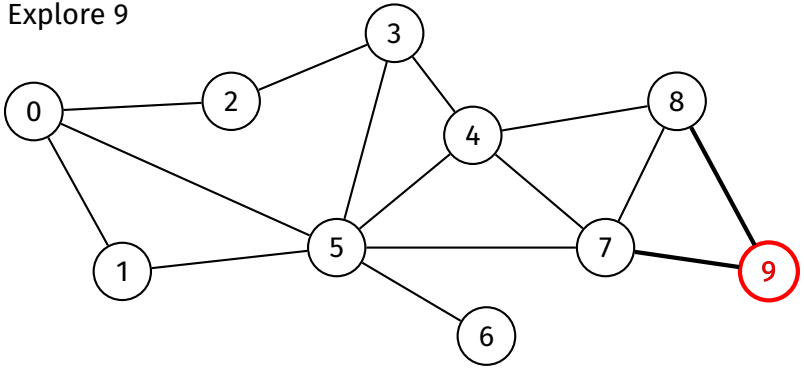


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

BFS Example

Explore 9



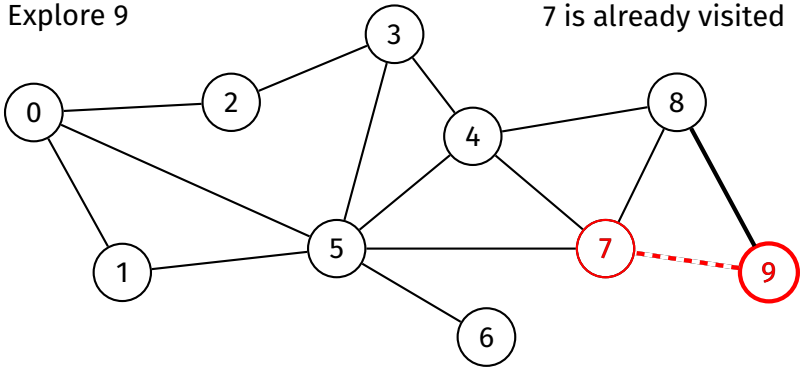
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

BFS Example

Explore 9

7 is already visited



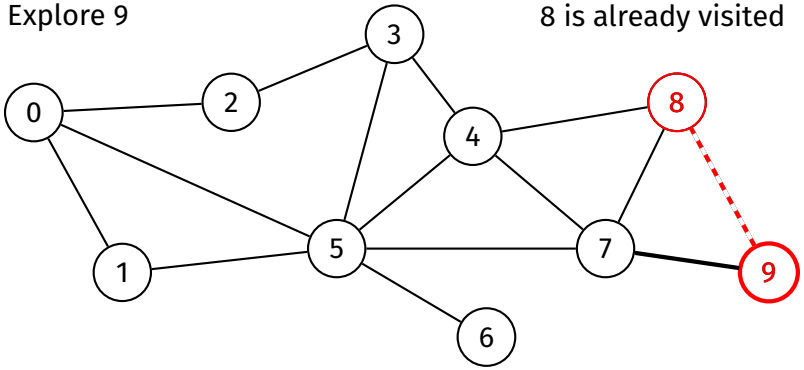
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

BFS Example

Explore 9

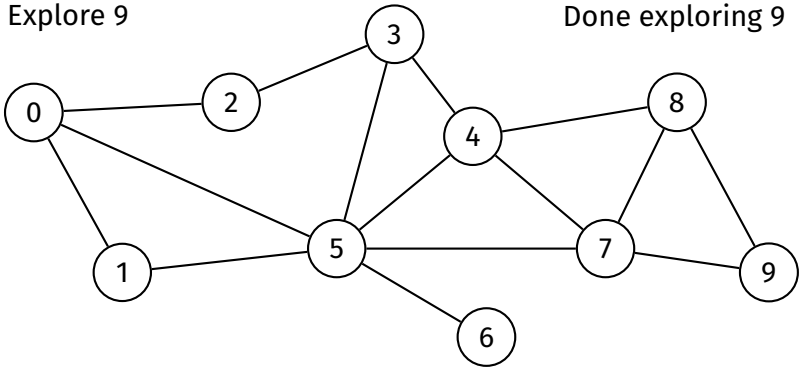
8 is already visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

BFS Example

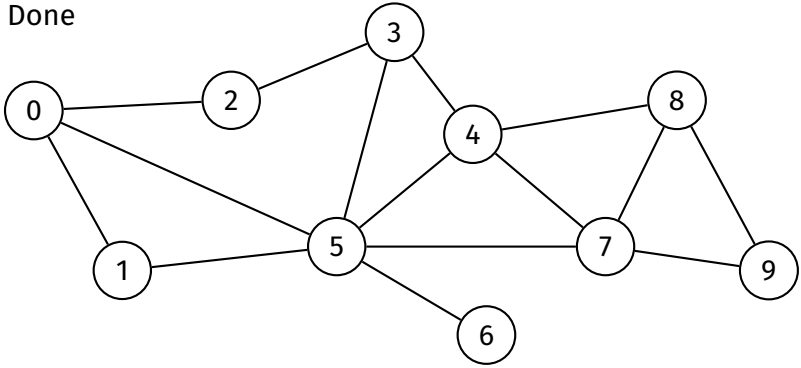


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

BFS Example

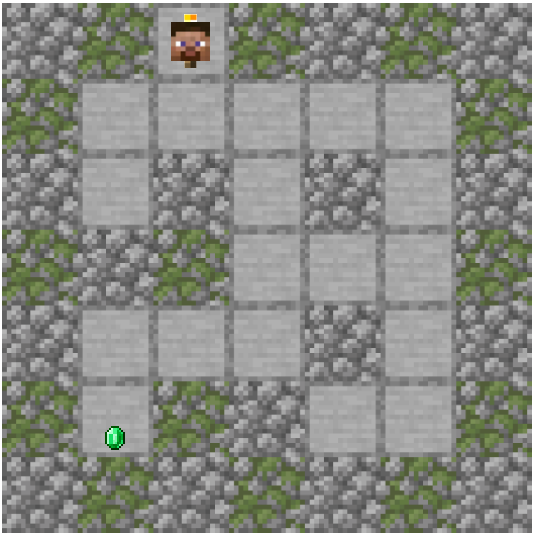
Done



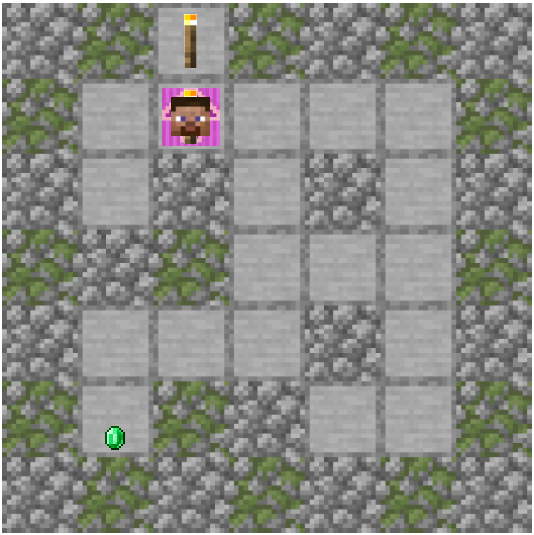
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1
pred	-1	0	0	2	5	0	5	5	4	7

queue 0 1 2 5 3 4 6 7 8 9

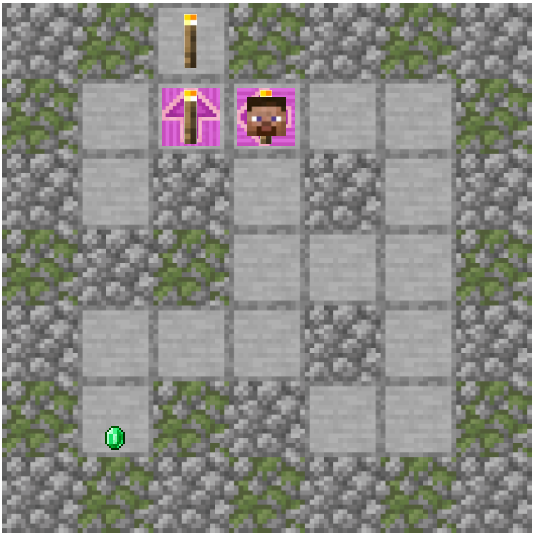
DFS



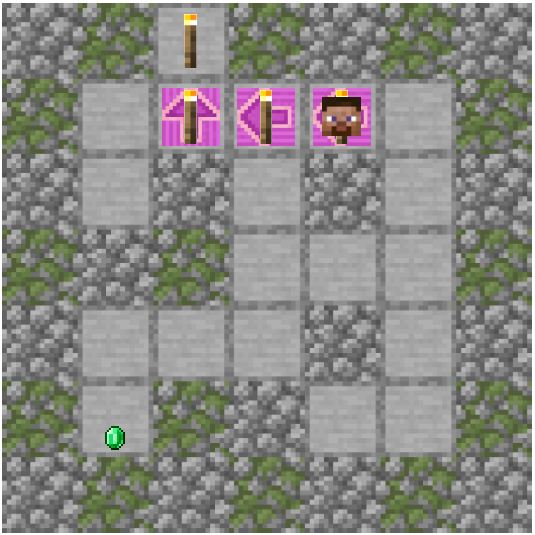
DFS



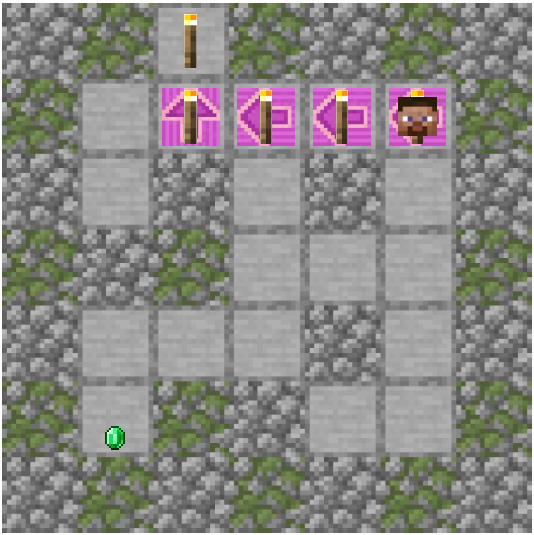
DFS



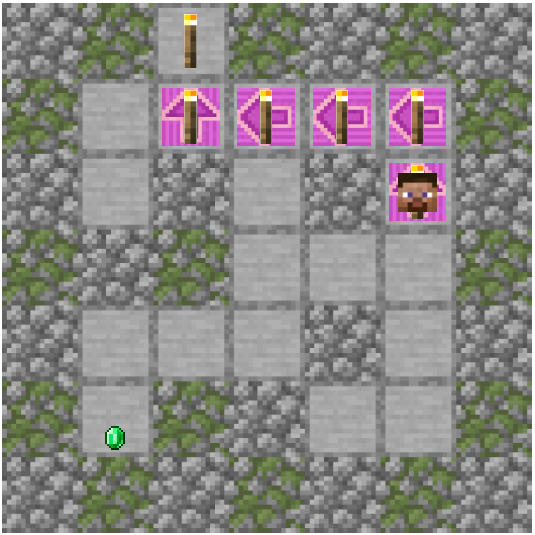
DFS



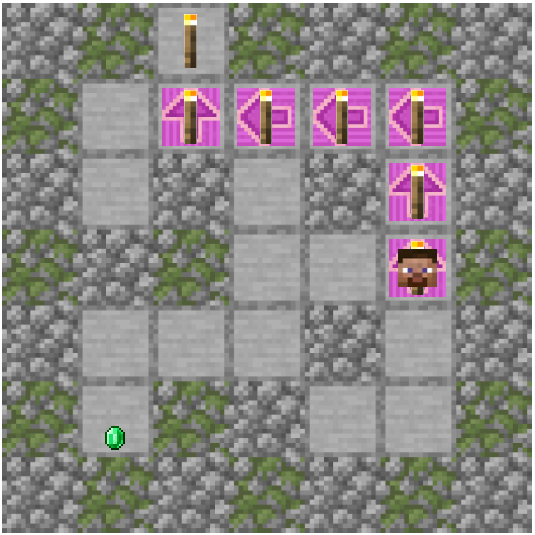
DFS



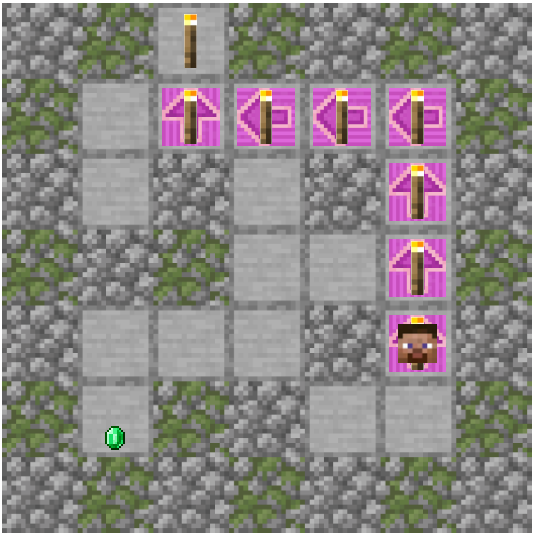
DFS



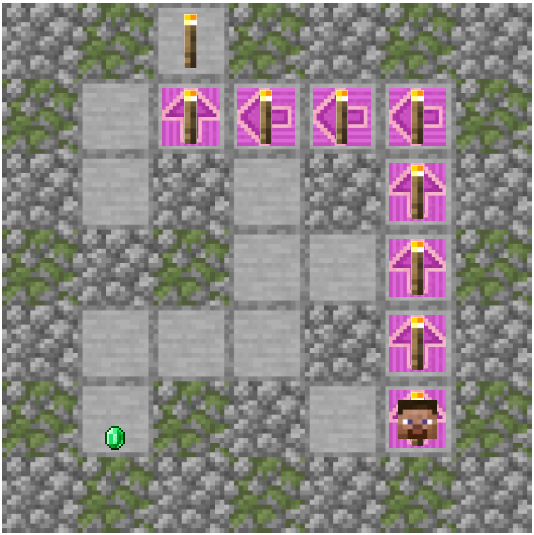
DFS



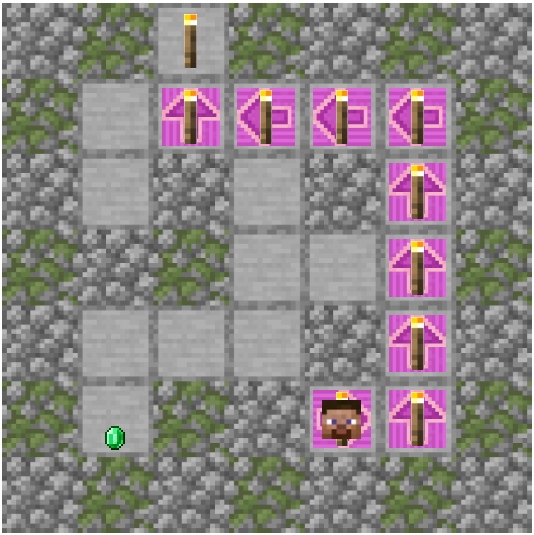
DFS



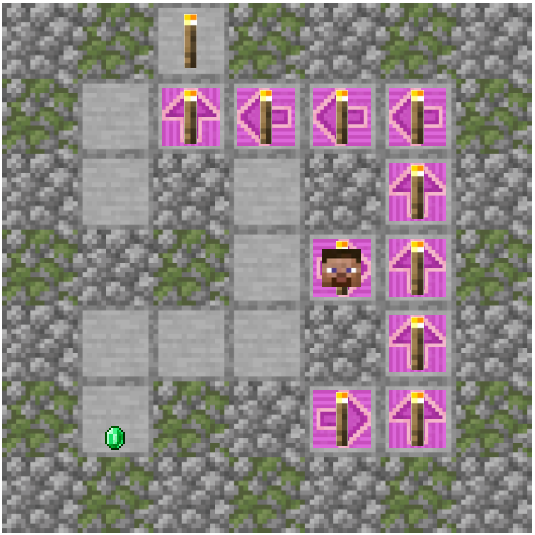
DFS



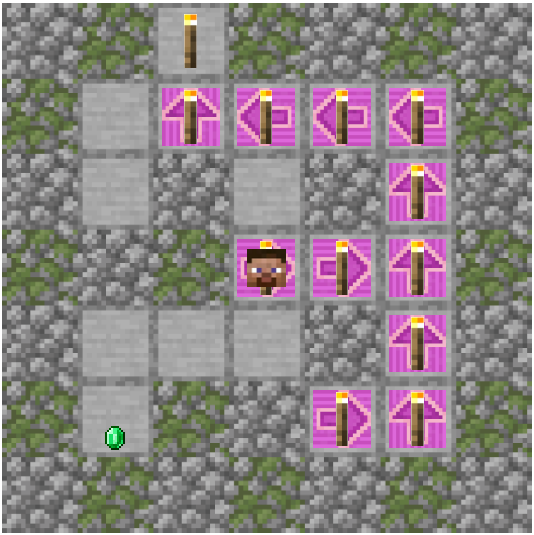
DFS



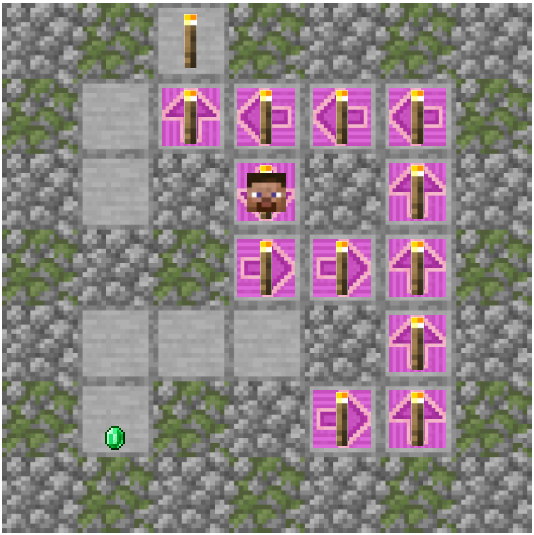
DFS



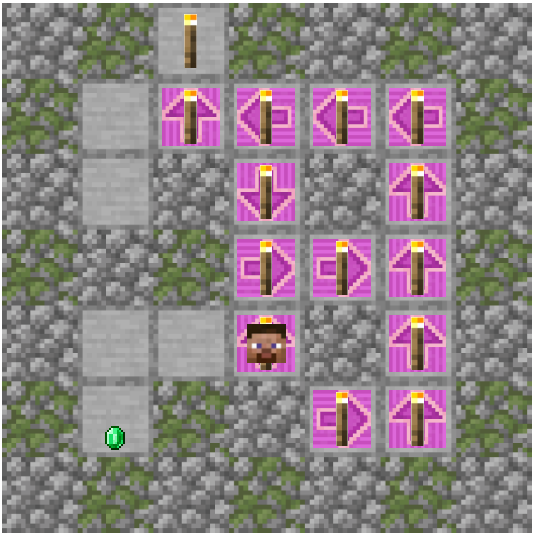
DFS



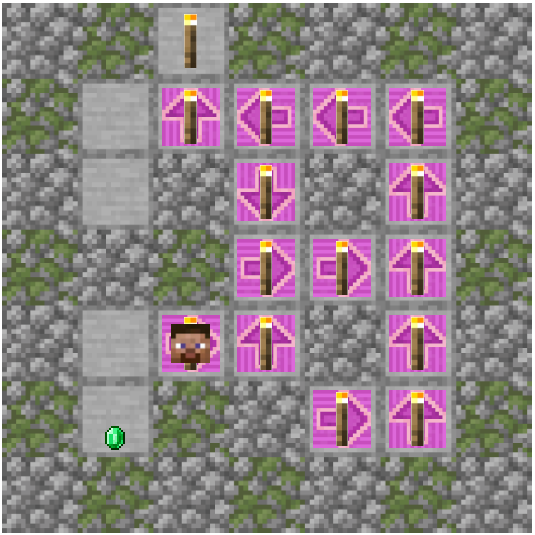
DFS



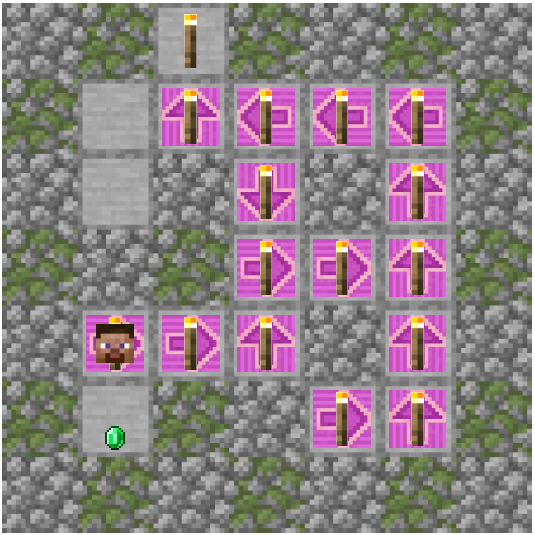
DFS



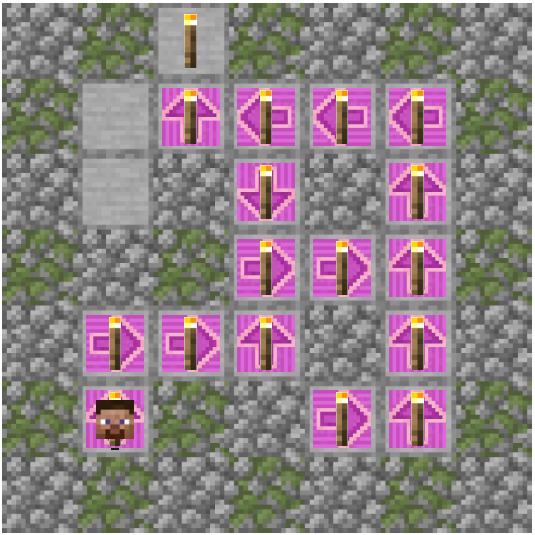
DFS



DFS

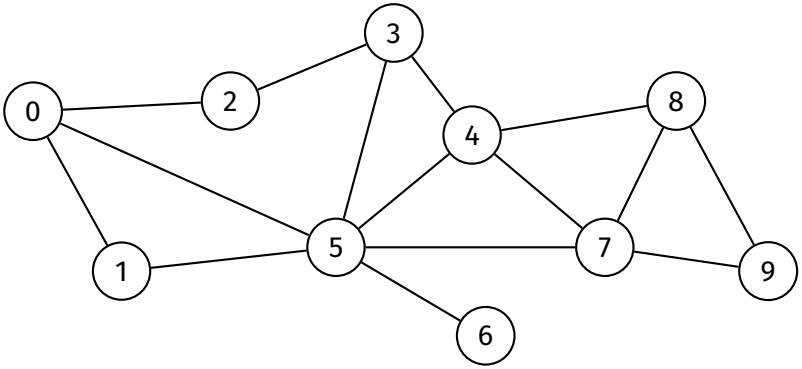


DFS



DFS Example

DFS starting at 0

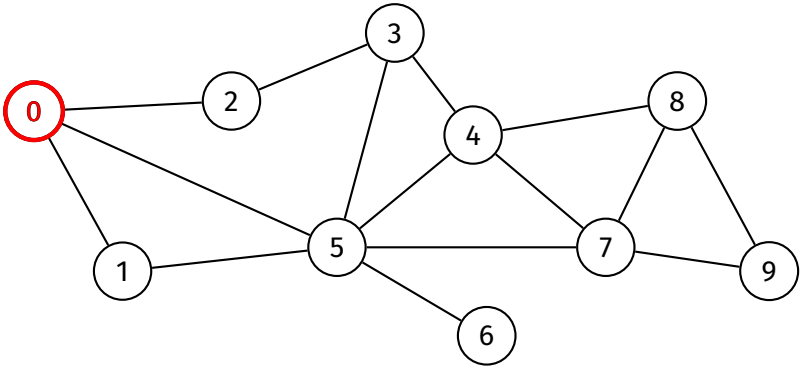


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	0	0	0	0	0	0	0	0	0	0

call stack

visit order

DFS Example



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	0	0	0	0	0	0	0	0	0	0

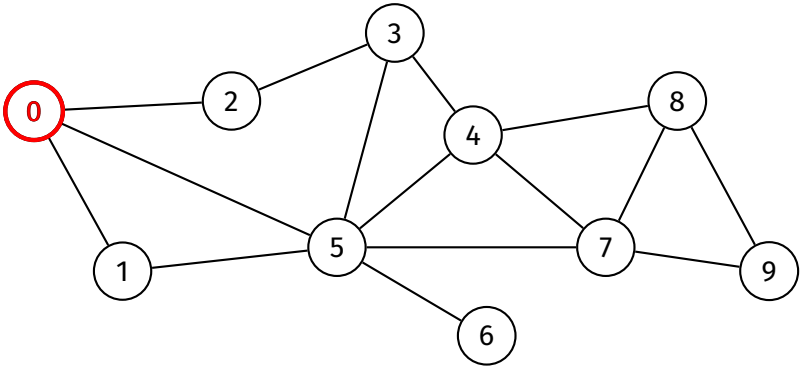
visit order

dfs(0)

call stack

DFS Example

Mark 0 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0

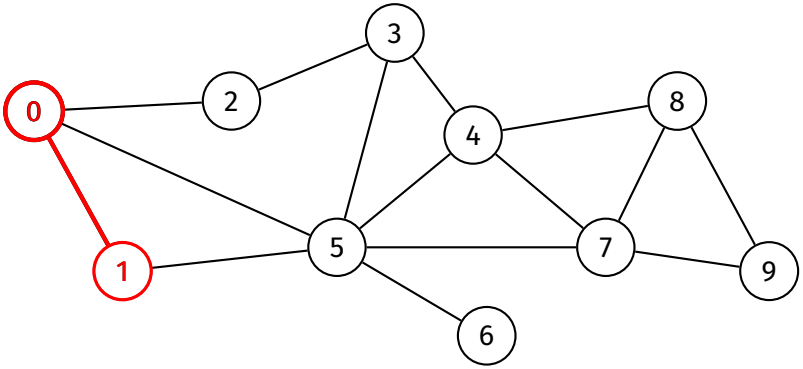
visit order 0

dfs(0)

call stack

DFS Example

1 has not been visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0

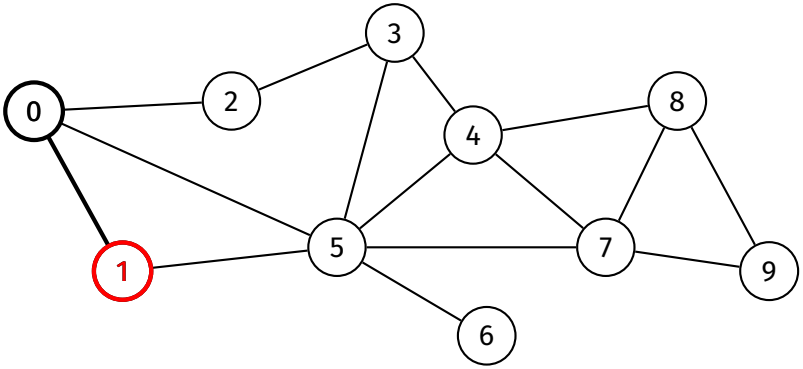
visit order 0

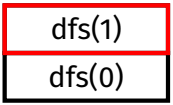
dfs(0)

call stack

DFS Example

Recurse into 1



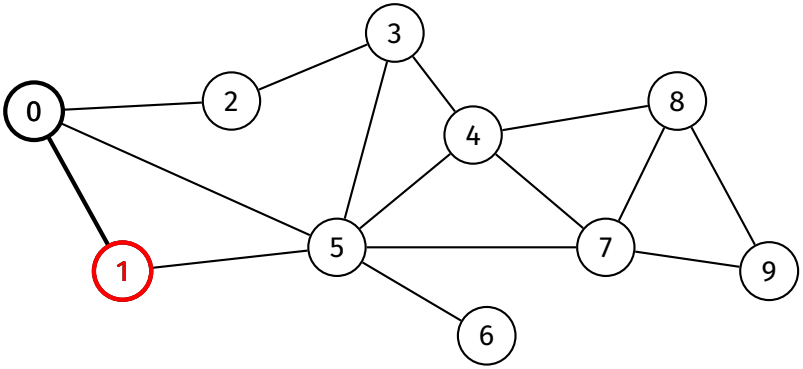


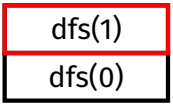
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0

visit order 0

DFS Example

Mark 1 as visited



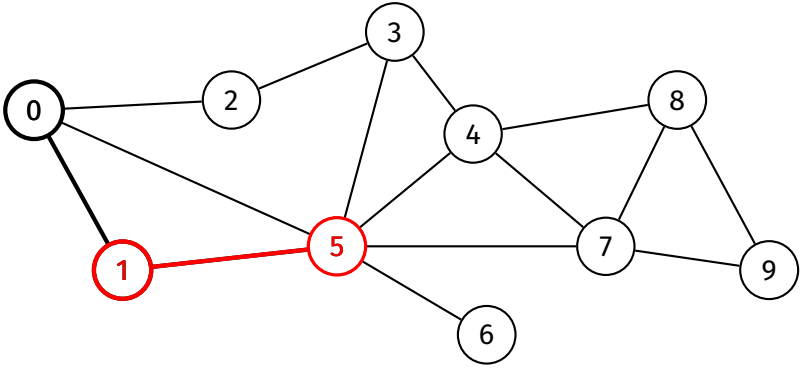


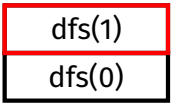
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0

visit order 0 1

DFS Example

5 has not been visited



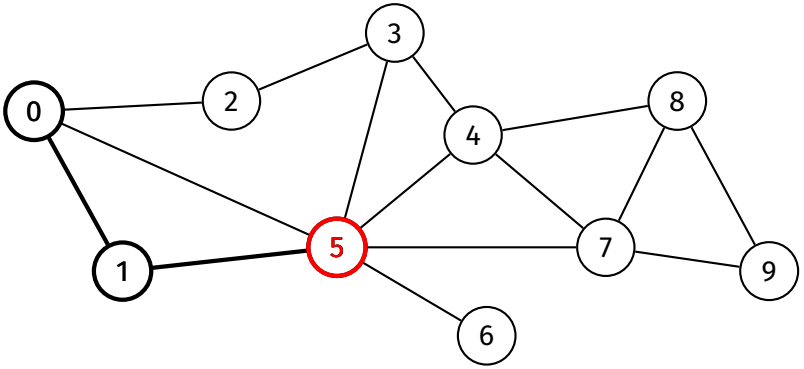


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0

visit order 0 1

DFS Example

Recurse into 5

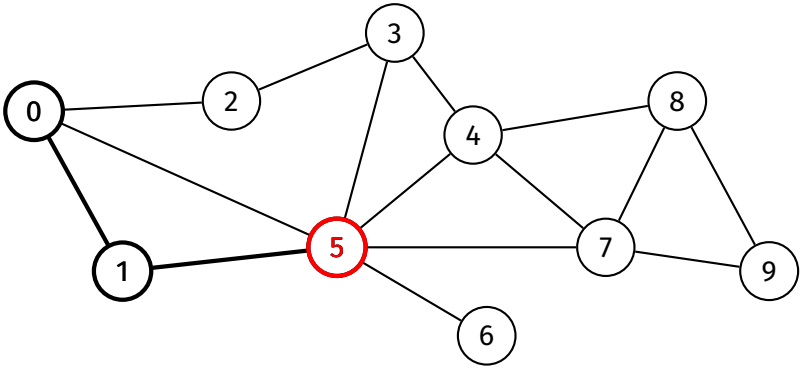


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0

visit order 0 1

DFS Example

Mark 5 as visited



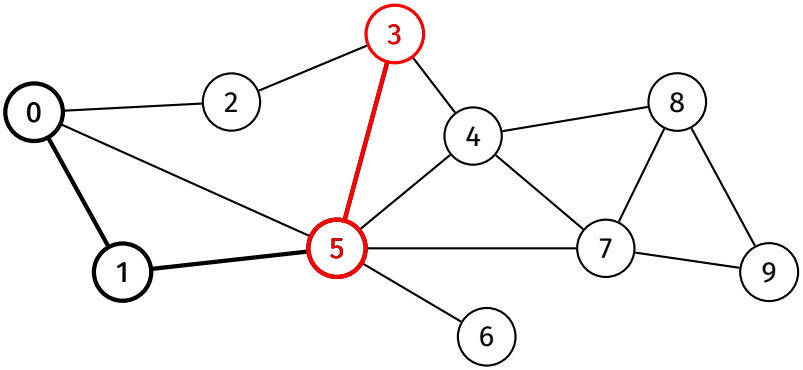
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	1	0	0	0	0

visit order 0 1 5

DFS Example

3 has not been visited



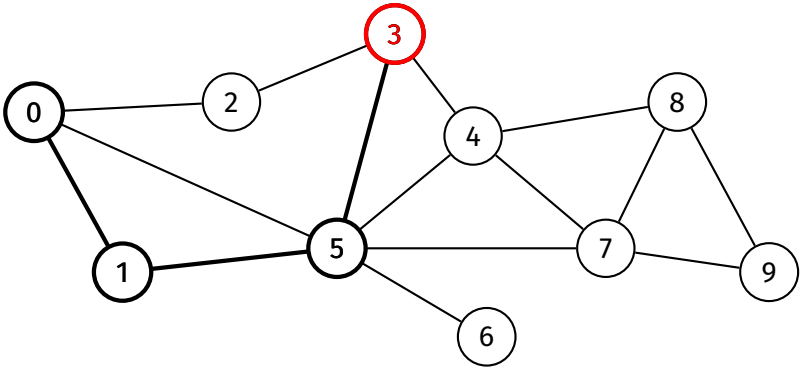
call stack

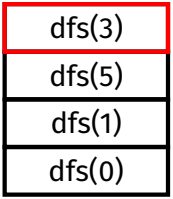
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	1	0	0	0	0

visit order 0 1 5

DFS Example

Recurse into 3





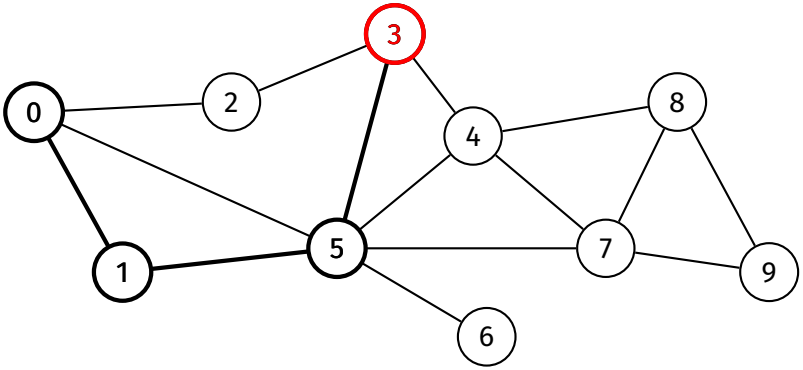
call stack

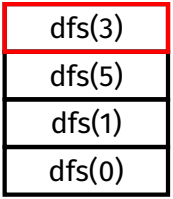
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	1	0	0	0	0

visit order 0 1 5

DFS Example

Mark 3 as visited





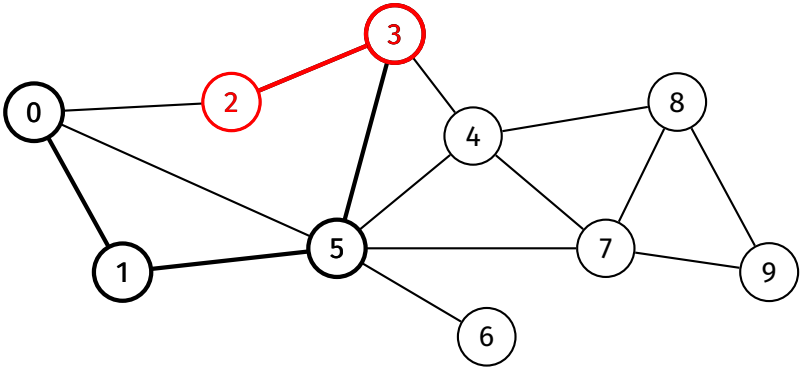
call stack

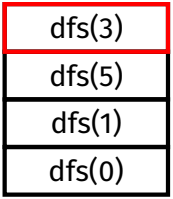
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	1	0	1	0	0	0	0

visit order 0 1 5 3

DFS Example

2 has not been visited





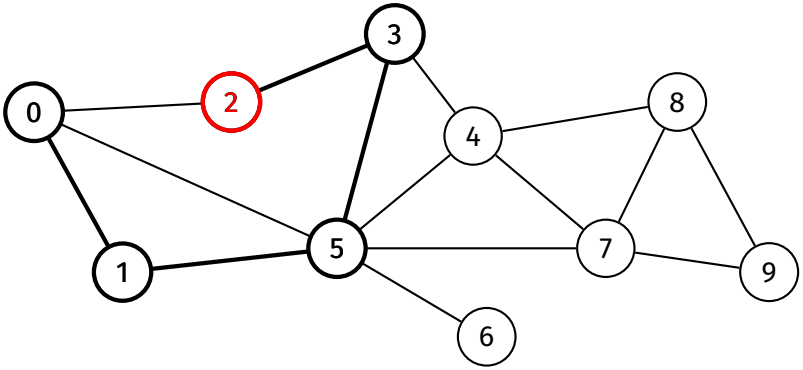
call stack

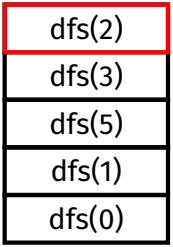
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	1	0	1	0	0	0	0

visit order 0 1 5 3

DFS Example

Recurse into 2





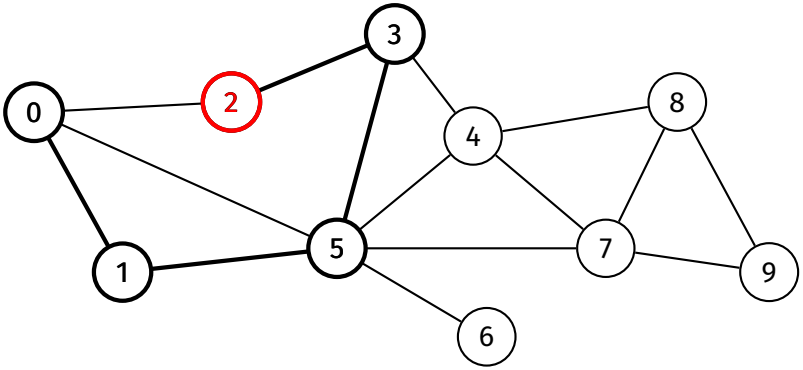
call stack

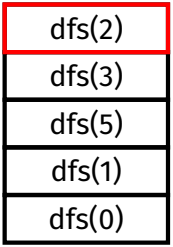
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	1	0	1	0	0	0	0

visit order 0 1 5 3

DFS Example

Mark 2 as visited





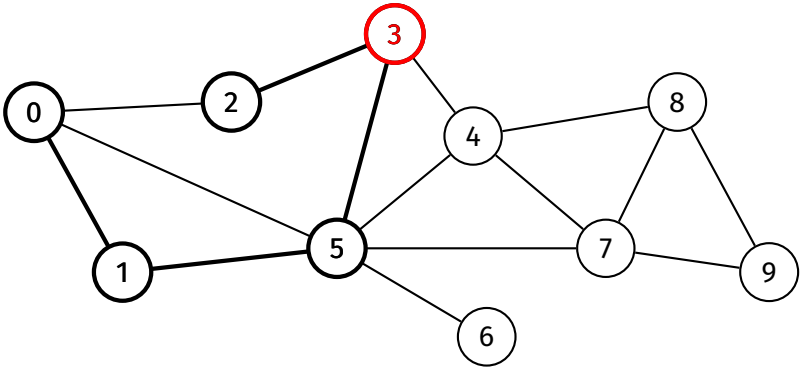
call stack

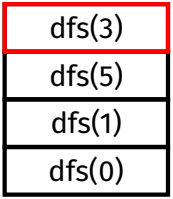
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

visit order 0 1 5 3 2

DFS Example

Return





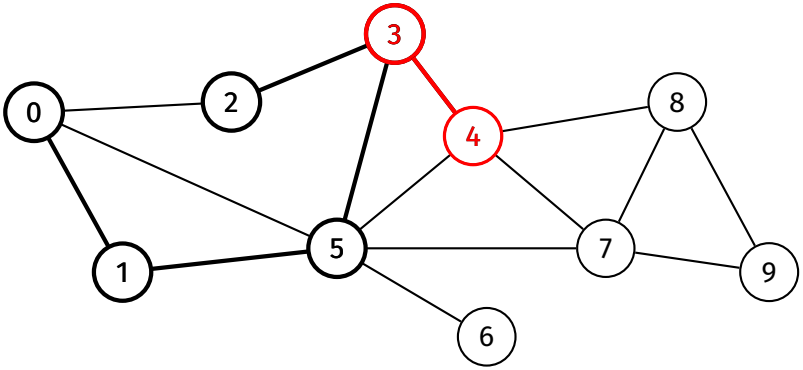
call stack

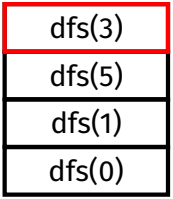
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

visit order 0 1 5 3 2

DFS Example

4 has not been visited





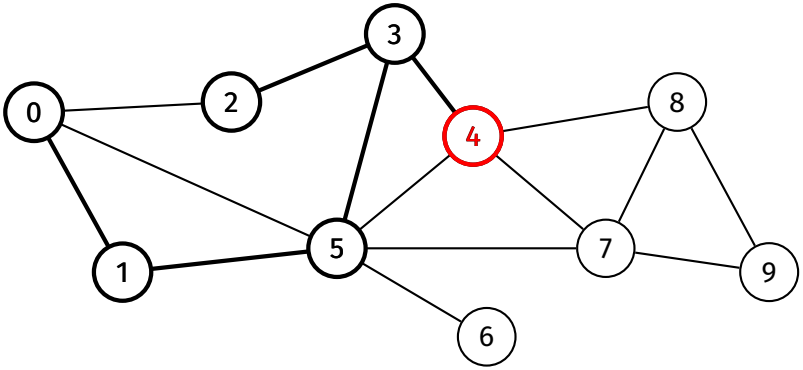
call stack

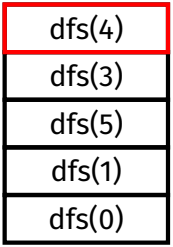
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

visit order 0 1 5 3 2

DFS Example

Recurse into 4





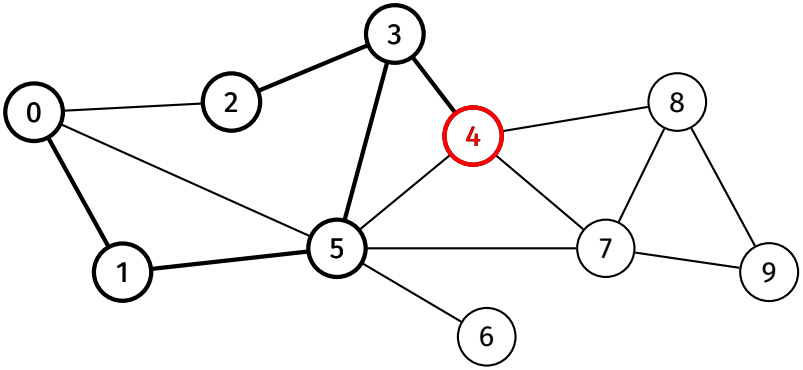
call stack

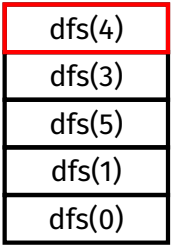
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

visit order 0 1 5 3 2

DFS Example

Mark 4 as visited





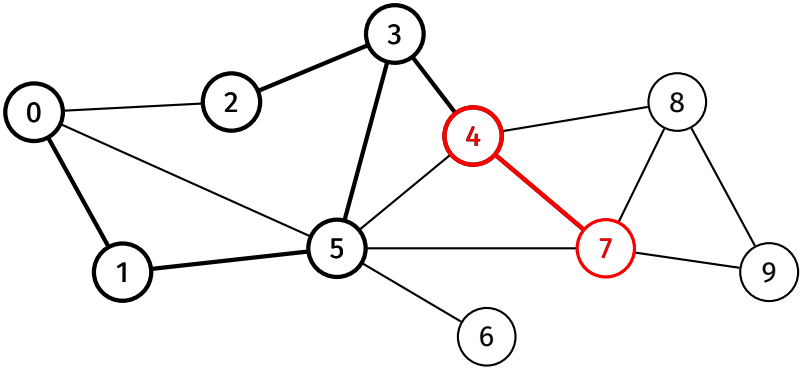
call stack

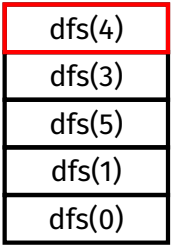
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0

visit order 0 1 5 3 2 4

DFS Example

7 has not been visited





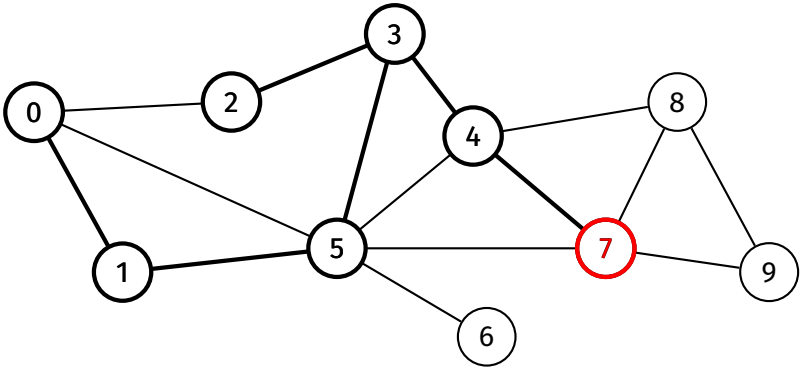
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0

visit order 0 1 5 3 2 4

DFS Example

Recurse into 7



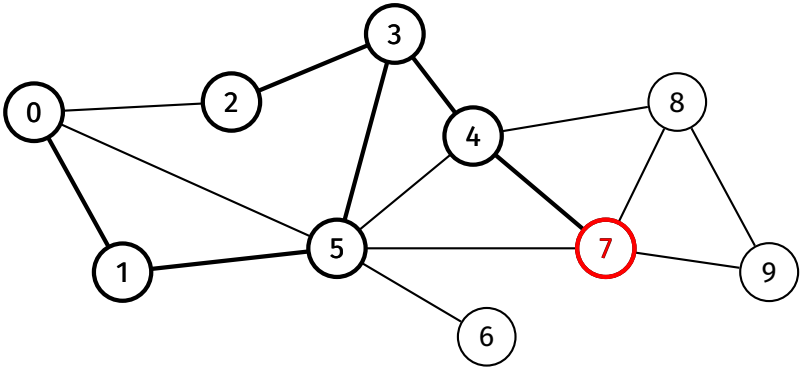
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0

visit order 0 1 5 3 2 4

DFS Example

Mark 7 as visited



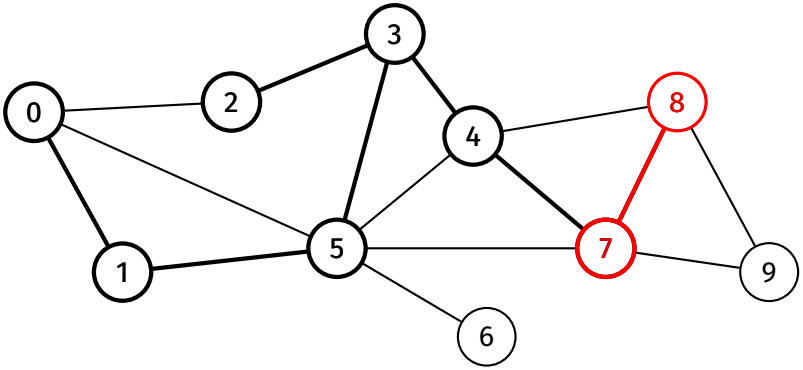
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

visit order 0 1 5 3 2 4 7

DFS Example

8 has not been visited



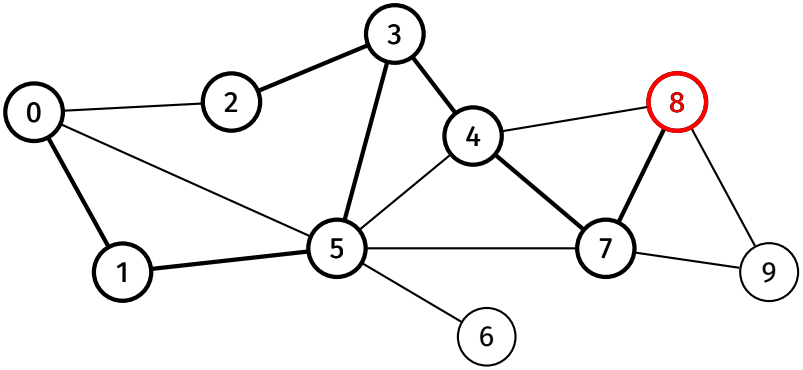
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

visit order 0 1 5 3 2 4 7

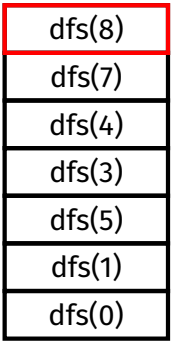
DFS Example

Recurse into 8



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

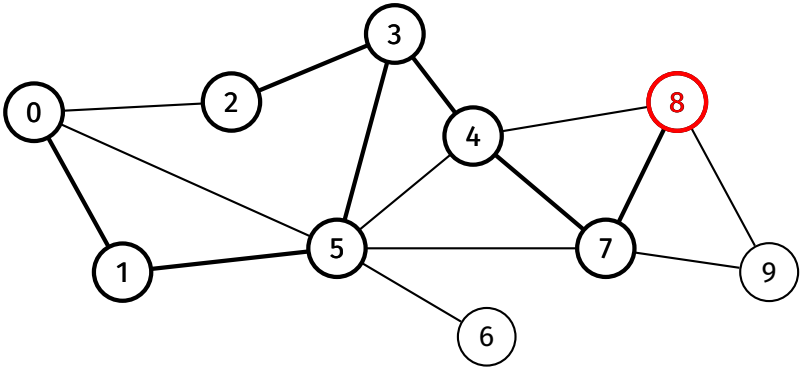
visit order 0 1 5 3 2 4 7

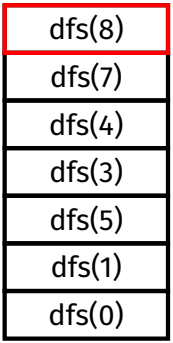


call stack

DFS Example

Mark 8 as visited





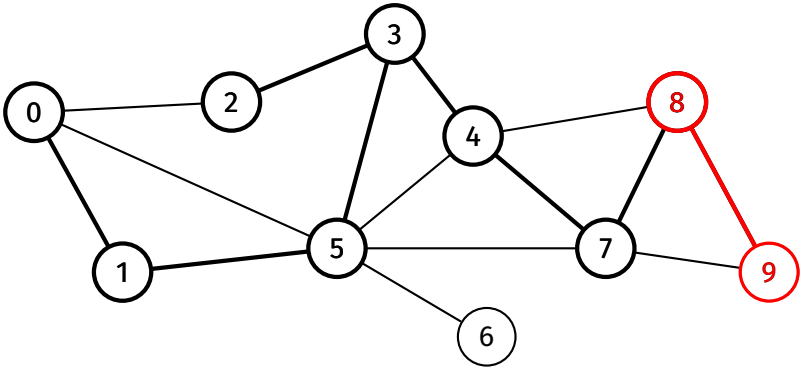
call stack

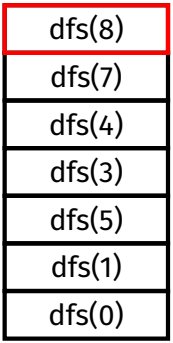
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	0

visit order 0 1 5 3 2 4 7 8

DFS Example

9 has not been visited





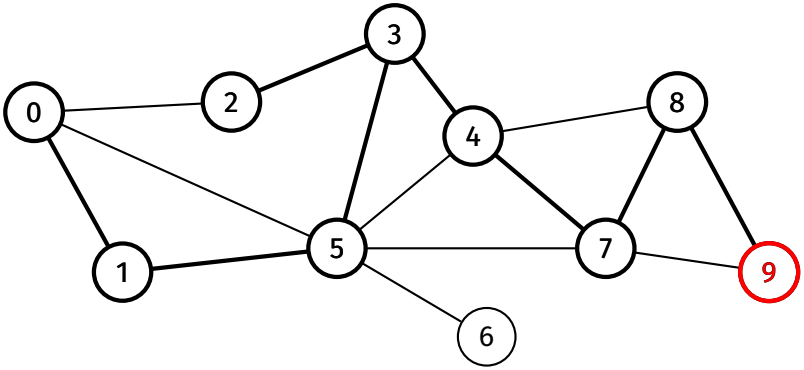
call stack

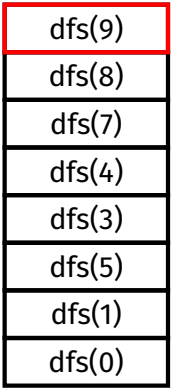
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	0

visit order 0 1 5 3 2 4 7 8

DFS Example

Recurse into 9





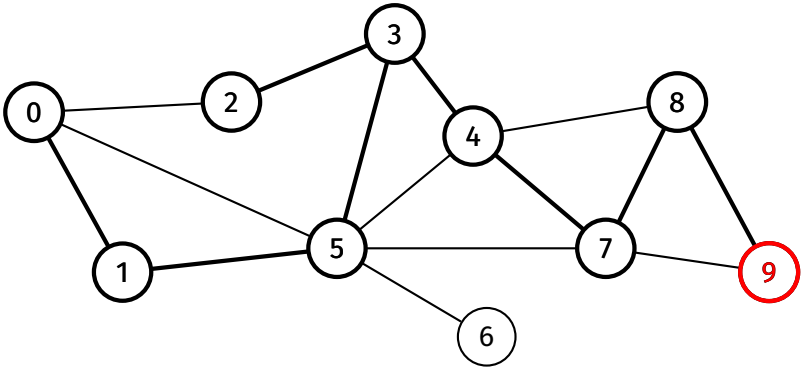
call stack

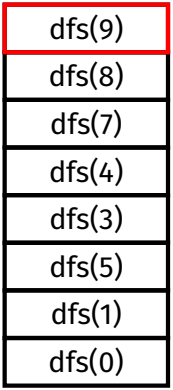
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	0

visit order 0 1 5 3 2 4 7 8

DFS Example

Mark 9 as visited





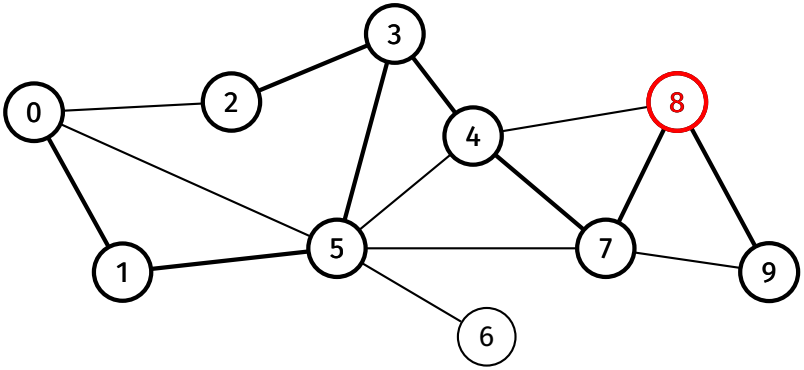
call stack

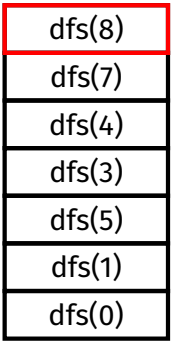
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

DFS Example

Return





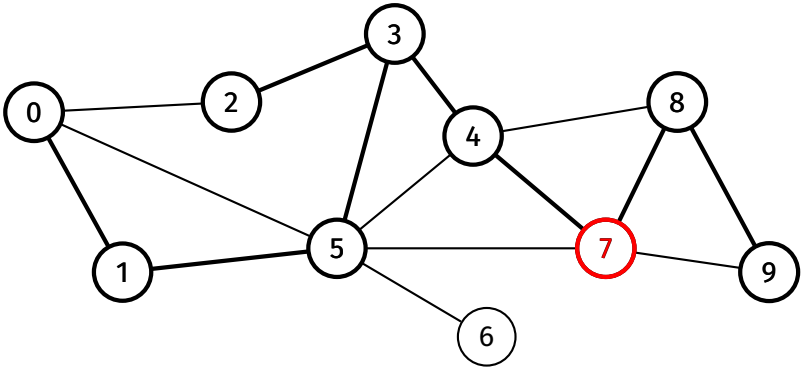
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

DFS Example

Return



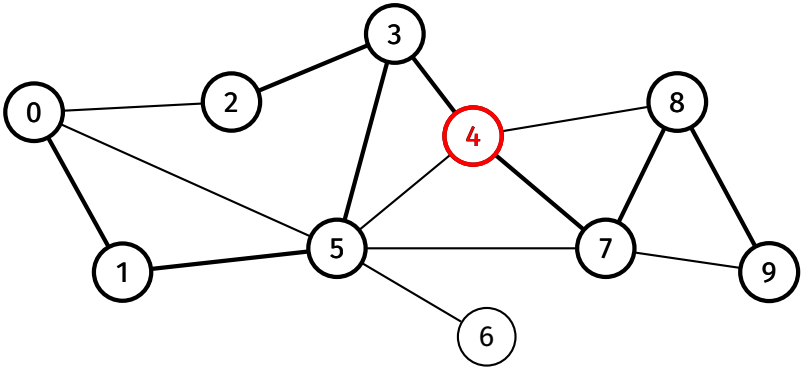
call stack

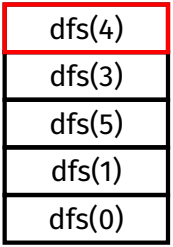
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

DFS Example

Return





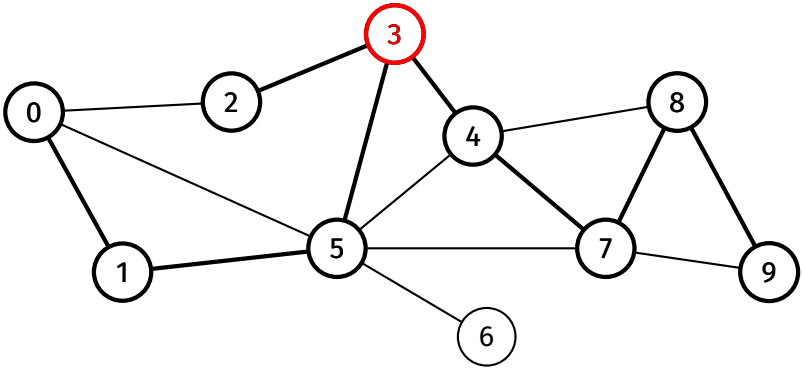
call stack

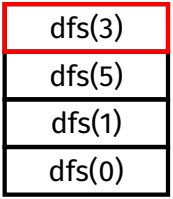
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

DFS Example

Return





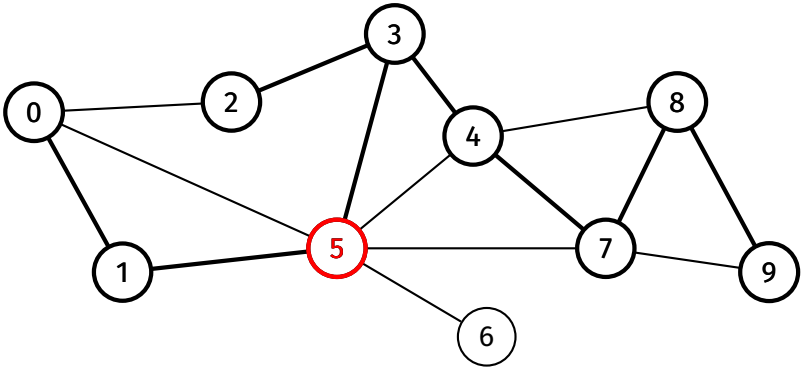
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

DFS Example

Return



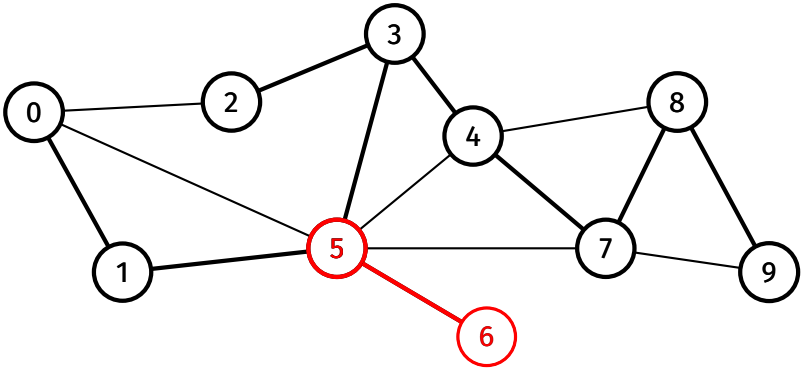
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

DFS Example

6 has not been visited



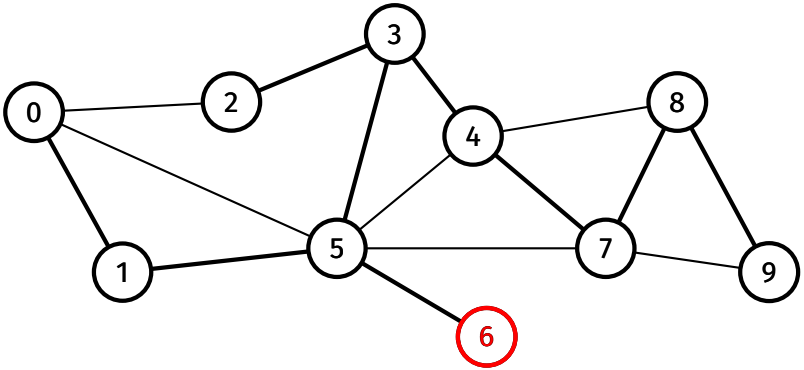
call stack

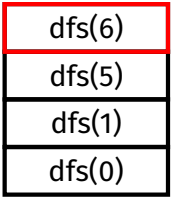
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

DFS Example

Recurse into 6





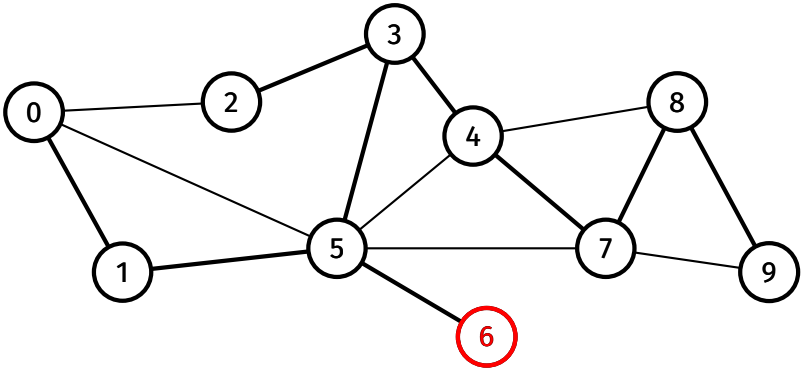
call stack

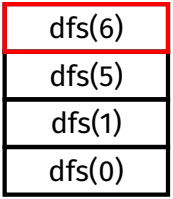
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	1	1

visit order 0 1 5 3 2 4 7 8 9

DFS Example

Mark 6 as visited





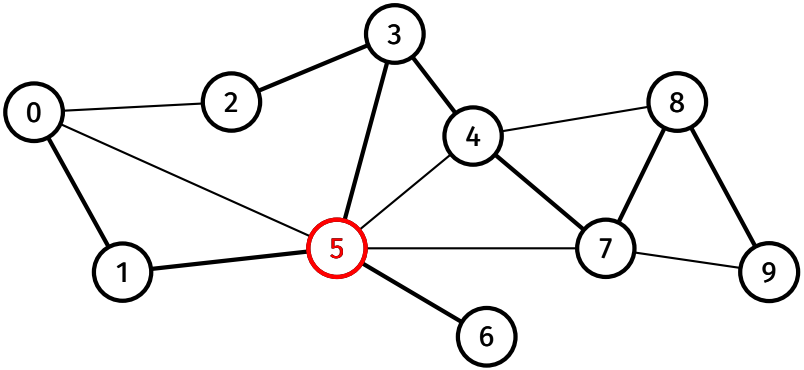
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1

visit order 0 1 5 3 2 4 7 8 9 6

DFS Example

Return



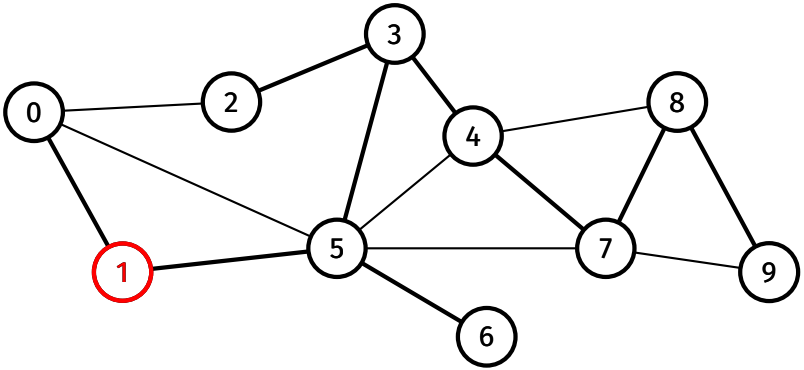
call stack

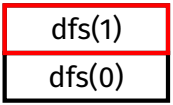
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1

visit order 0 1 5 3 2 4 7 8 9 6

DFS Example

Return



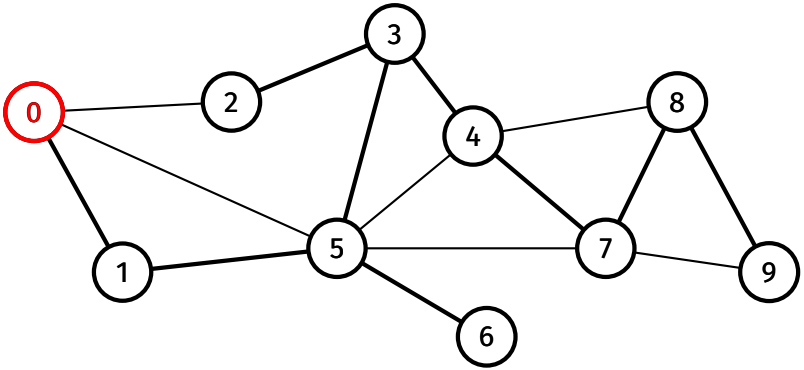


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1

visit order 0 1 5 3 2 4 7 8 9 6

DFS Example

Return



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1

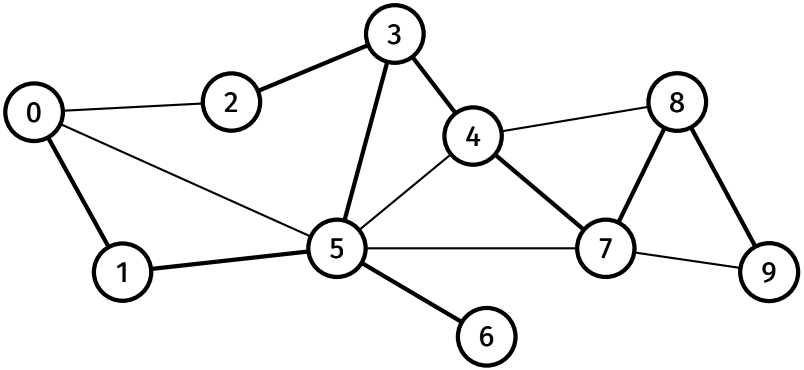
visit order 0 1 5 3 2 4 7 8 9 6

dfs(0)

call stack

DFS Example

Return



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	1	1	1	1

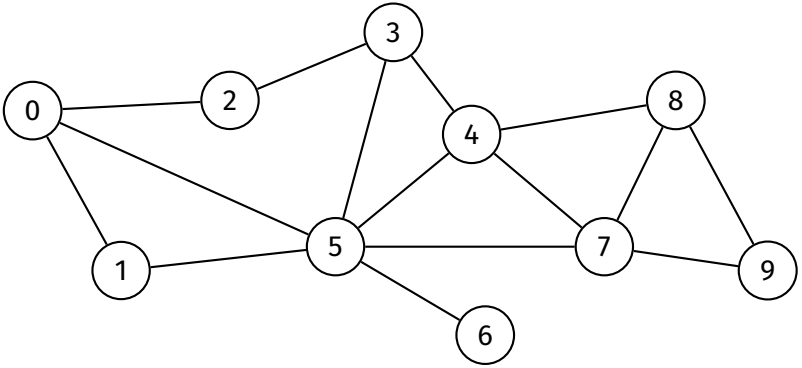
visit order 0 1 5 3 2 4 7 8 9 6

call stack

Path-Checking with Recursive DFS

Example

Is there a path between 0 and 7?

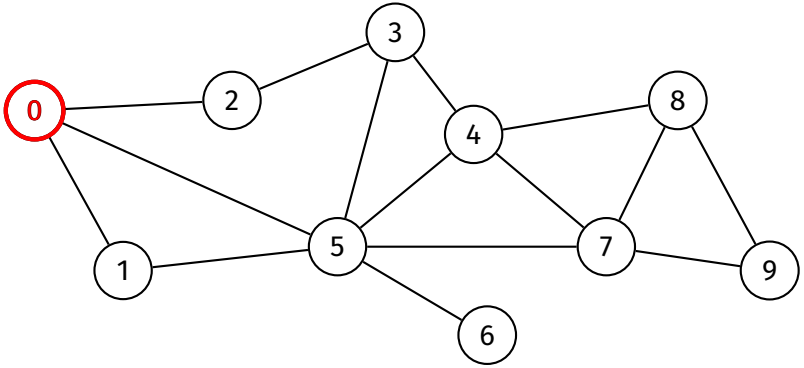


call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	0	0	0	0	0	0	0	0	0	0

Path-Checking with Recursive DFS

Example



path(0, 7)?

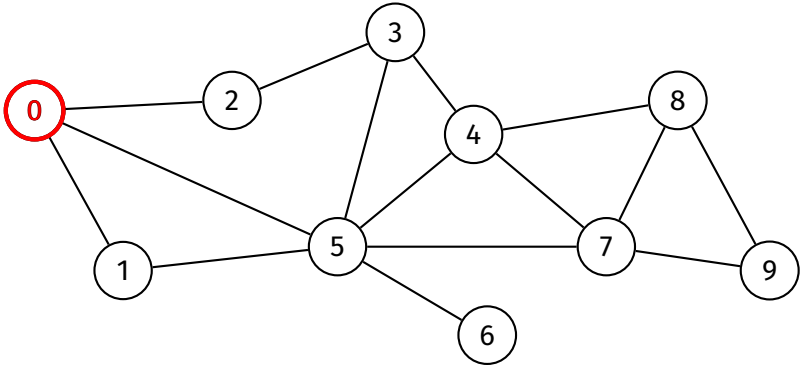
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	0	0	0	0	0	0	0	0	0	0

Path-Checking with Recursive DFS

Example

Mark 0 as visited



path(0, 7)?

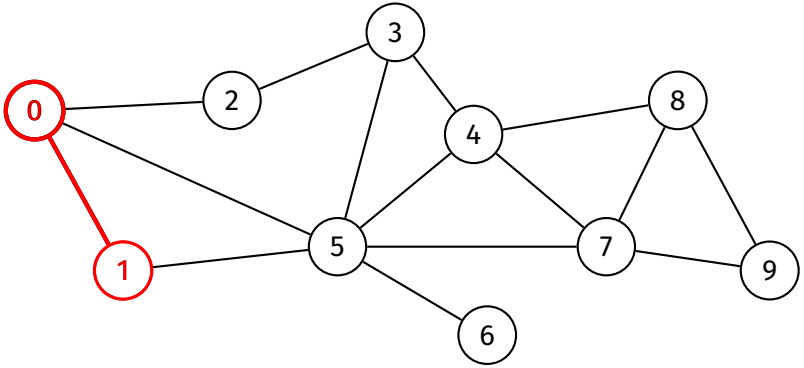
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0

Path-Checking with Recursive DFS

Example

1 has not been visited



path(0, 7)?

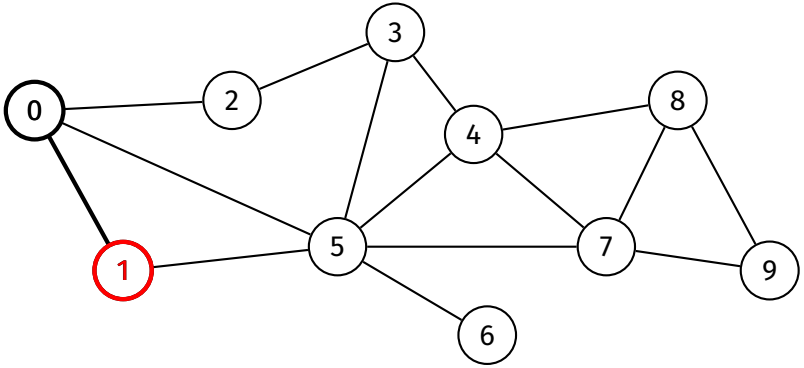
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0

Path-Checking with Recursive DFS

Example

Recurse into 1



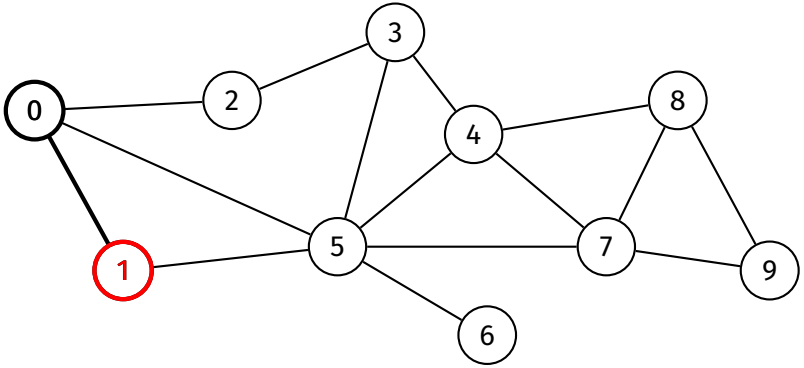
path(1, 7)?
path(0, 7)?
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	0	0	0	0	0	0	0	0	0

Path-Checking with Recursive DFS

Example

Mark 1 as visited



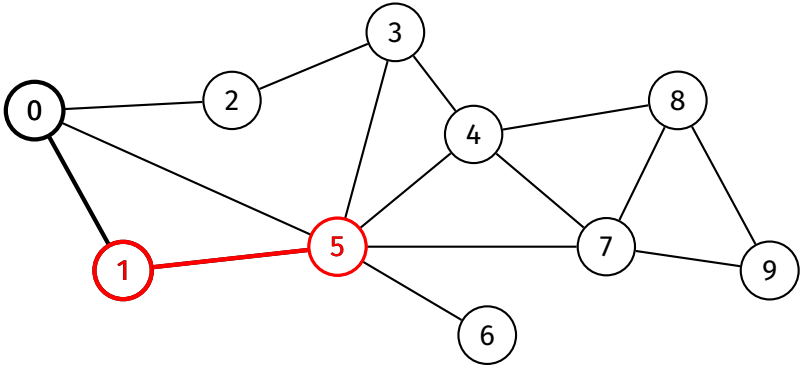
path(1, 7)?
path(0, 7)?
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0

Path-Checking with Recursive DFS

Example

5 has not been visited



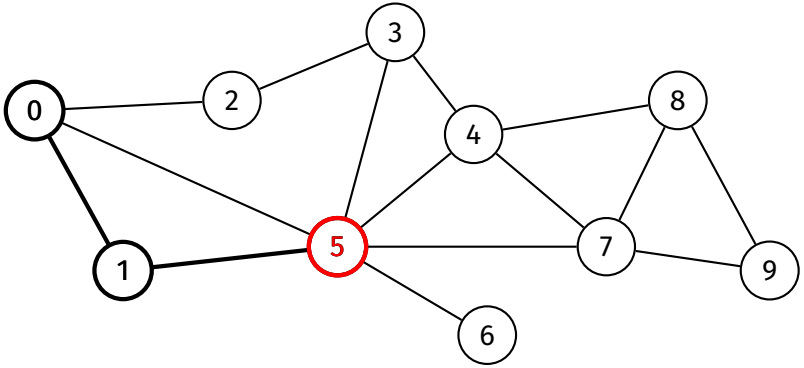
path(1, 7)?
path(0, 7)?
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0

Path-Checking with Recursive DFS

Example

Recurse into 5



path(5, 7)?
path(1, 7)?
path(0, 7)?

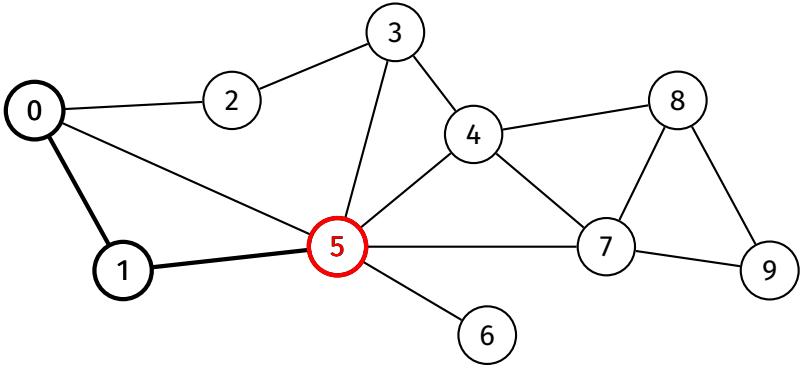
call stack

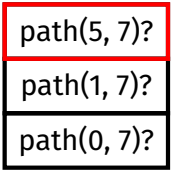
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	0	0	0	0	0

Path-Checking with Recursive DFS

Example

Mark 5 as visited



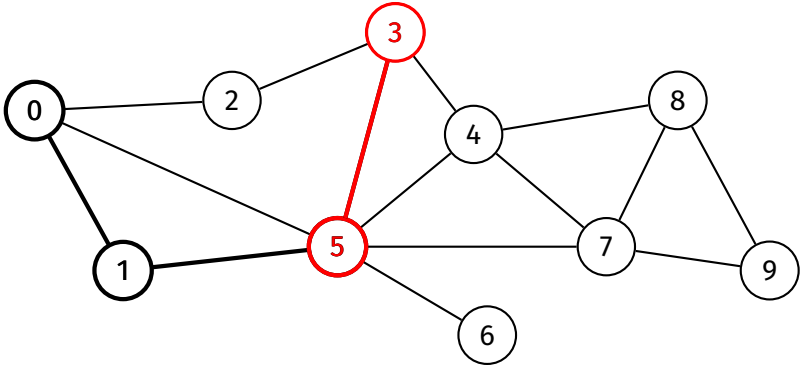


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	1	0	0	0	0

Path-Checking with Recursive DFS

Example

3 has not been visited



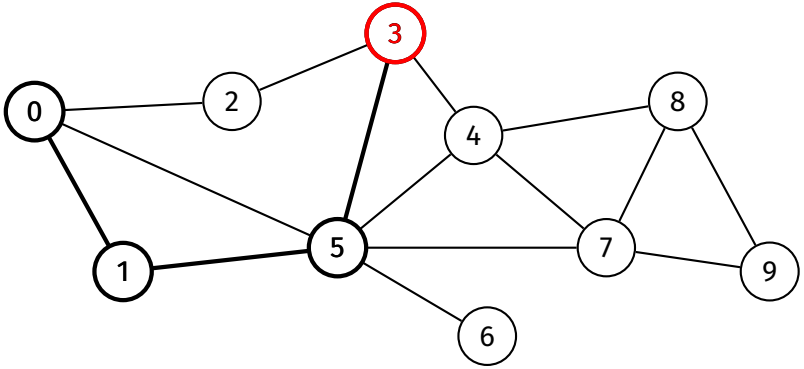
path(5, 7)?
path(1, 7)?
path(0, 7)?
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	1	0	0	0	0

Path-Checking with Recursive DFS

Example

Recurse into 3



path(3, 7)?
path(5, 7)?
path(1, 7)?
path(0, 7)?

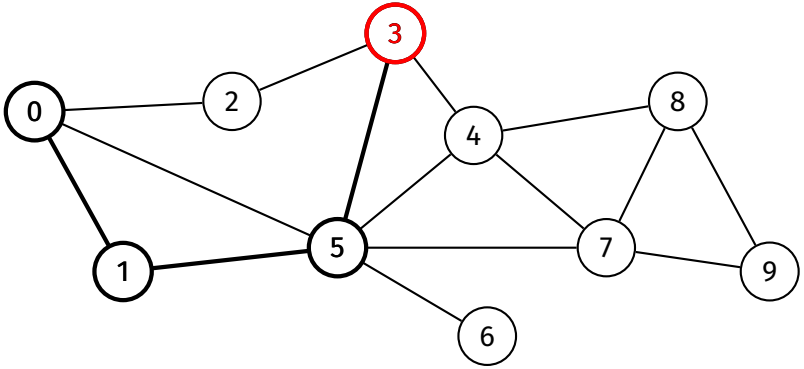
call stack

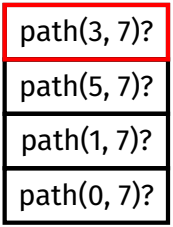
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	0	0	1	0	0	0	0

Path-Checking with Recursive DFS

Example

Mark 3 as visited



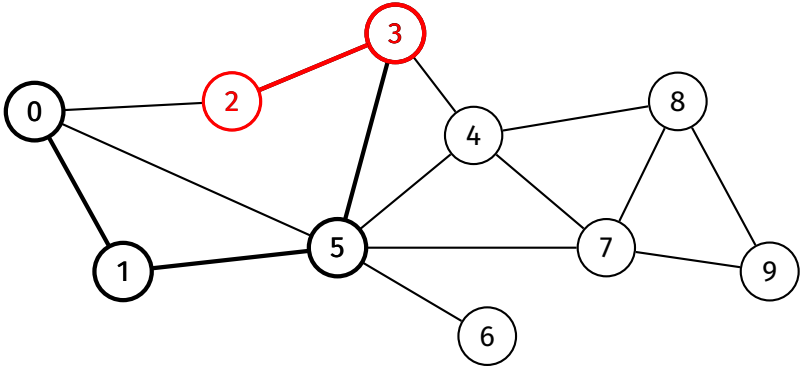


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	1	0	1	0	0	0	0

Path-Checking with Recursive DFS

Example

2 has not been visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	1	0	1	0	0	0	0

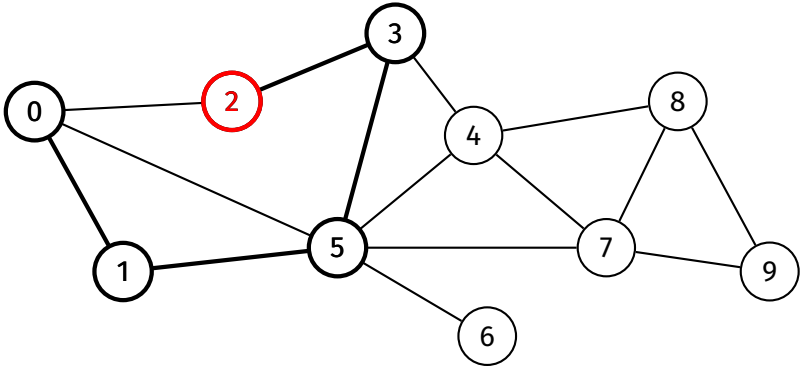
path(3, 7)?
path(5, 7)?
path(1, 7)?
path(0, 7)?

call stack

Path-Checking with Recursive DFS

Example

Recurse into 2



path(2, 7)?
path(3, 7)?
path(5, 7)?
path(1, 7)?
path(0, 7)?

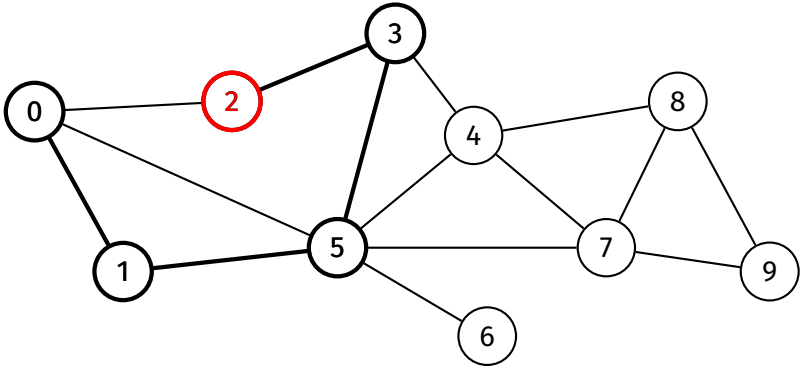
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	0	1	0	1	0	0	0	0

Path-Checking with Recursive DFS

Example

Mark 2 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

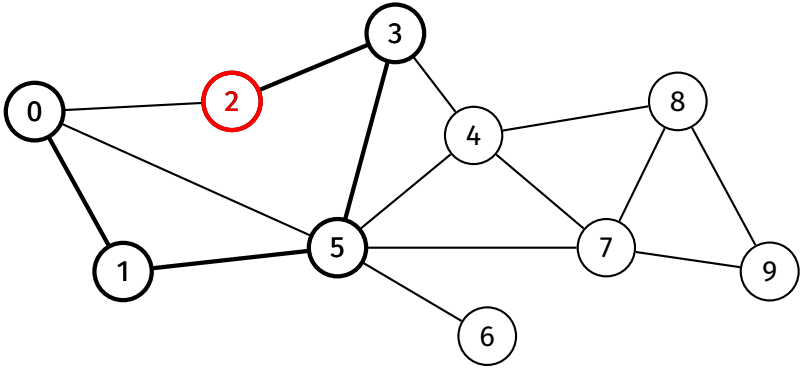
path(2, 7)?
path(3, 7)?
path(5, 7)?
path(1, 7)?
path(0, 7)?

call stack

Path-Checking with Recursive DFS

Example

Return false



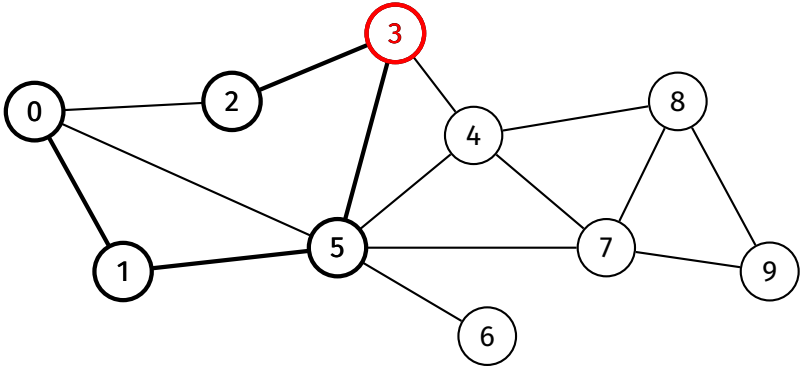
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

path(2, 7)?
path(3, 7)?
path(5, 7)?
path(1, 7)?
path(0, 7)?

call stack

Path-Checking with Recursive DFS

Example



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

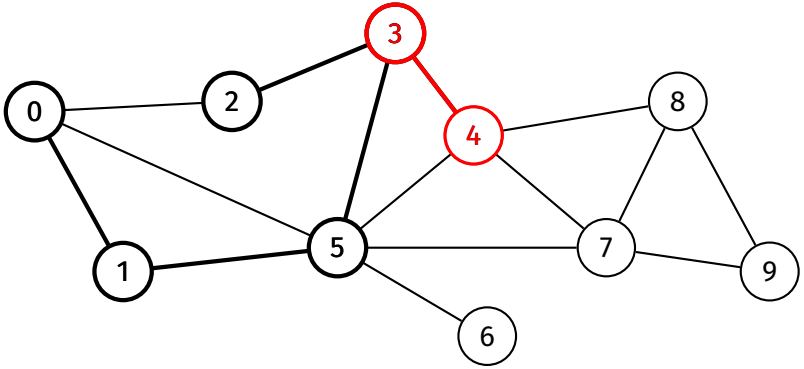
path(3, 7)?
path(5, 7)?
path(1, 7)?
path(0, 7)?

call stack

Path-Checking with Recursive DFS

Example

4 has not been visited



path(3, 7)?
path(5, 7)?
path(1, 7)?
path(0, 7)?

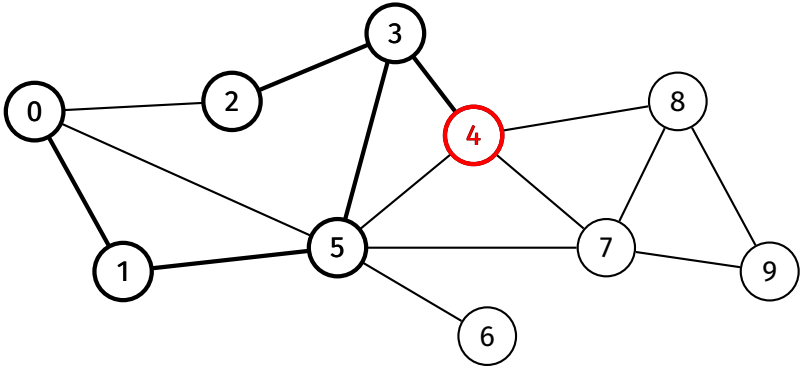
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

Path-Checking with Recursive DFS

Example

Recurse into 4



path(4, 7)?
path(3, 7)?
path(5, 7)?
path(1, 7)?
path(0, 7)?

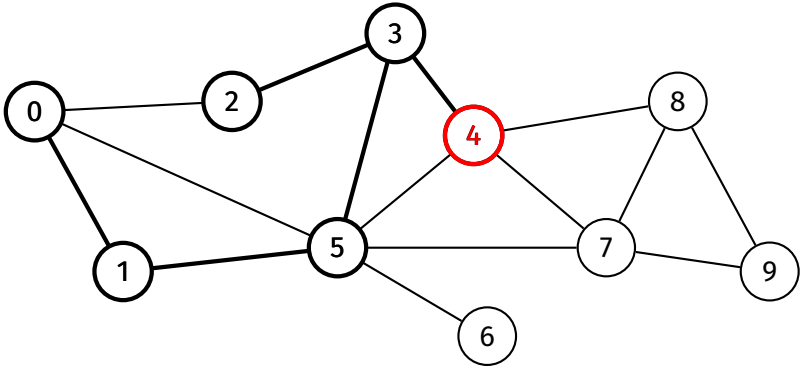
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	0	1	0	0	0	0

Path-Checking with Recursive DFS

Example

Mark 4 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0

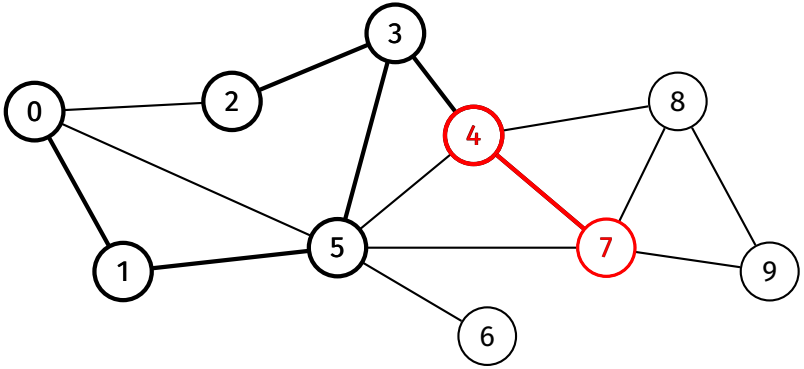
path(4, 7)?
path(3, 7)?
path(5, 7)?
path(1, 7)?
path(0, 7)?

call stack

Path-Checking with Recursive DFS

Example

7 has not been visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0

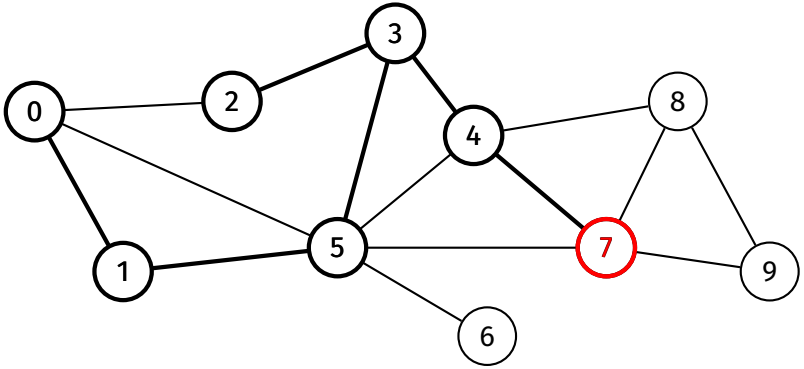
path(4, 7)?
path(3, 7)?
path(5, 7)?
path(1, 7)?
path(0, 7)?

call stack

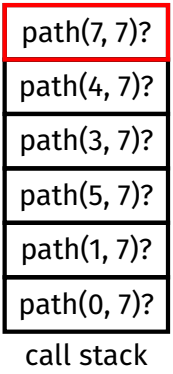
Path-Checking with Recursive DFS

Example

Recurse into 7



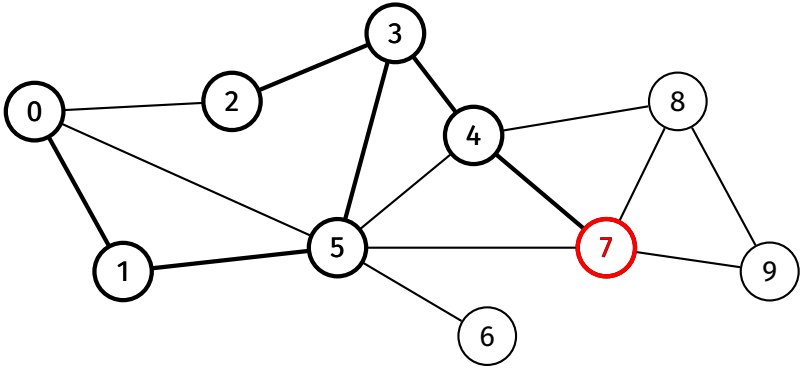
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	0	0	0



Path-Checking with Recursive DFS

Example

Mark 7 as visited



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

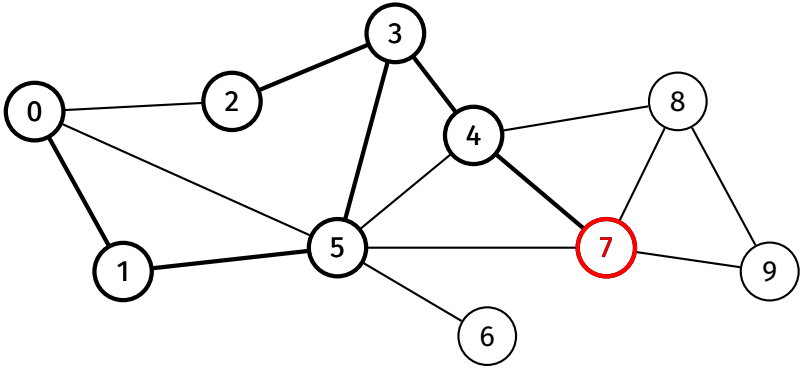
path(7, 7)?
path(4, 7)?
path(3, 7)?
path(5, 7)?
path(1, 7)?
path(0, 7)?

call stack

Path-Checking with Recursive DFS

Example

Return true



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

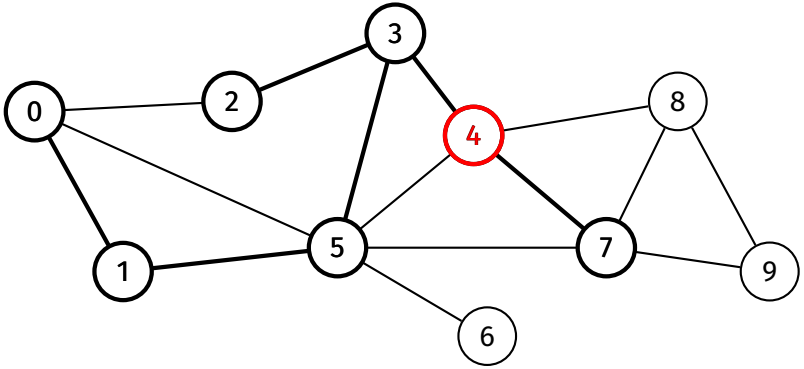
path(7, 7)?
path(4, 7)?
path(3, 7)?
path(5, 7)?
path(1, 7)?
path(0, 7)?

call stack

Path-Checking with Recursive DFS

Example

Return true



path(4, 7)?
path(3, 7)?
path(5, 7)?
path(1, 7)?
path(0, 7)?

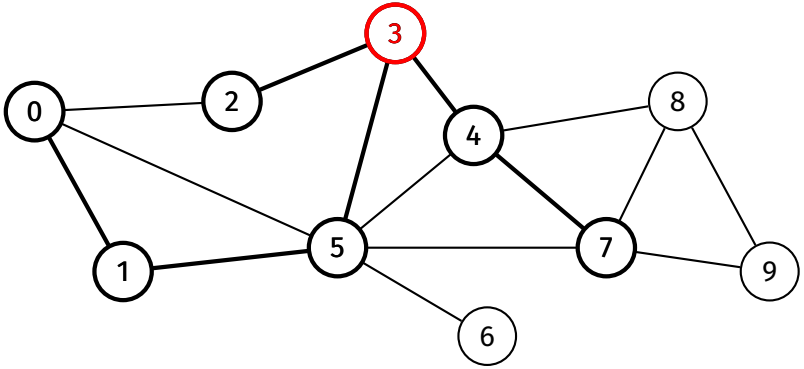
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

Path-Checking with Recursive DFS

Example

Return true



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

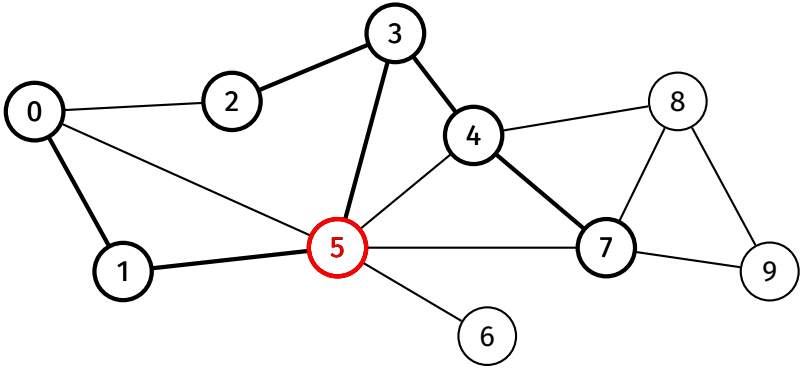
path(3, 7)?
path(5, 7)?
path(1, 7)?
path(0, 7)?

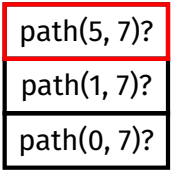
call stack

Path-Checking with Recursive DFS

Example

Return true



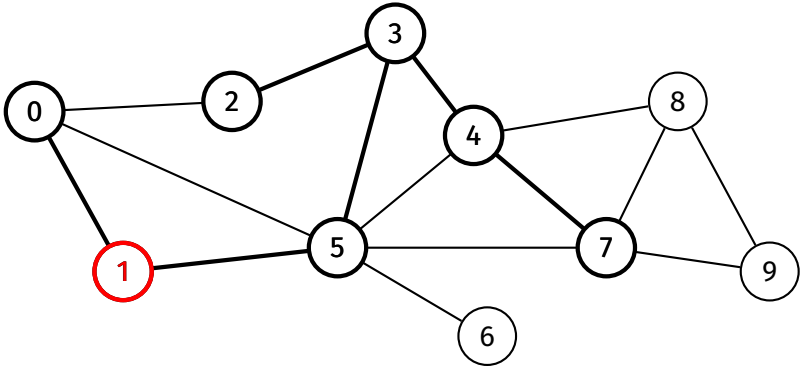


	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

Path-Checking with Recursive DFS

Example

Return true



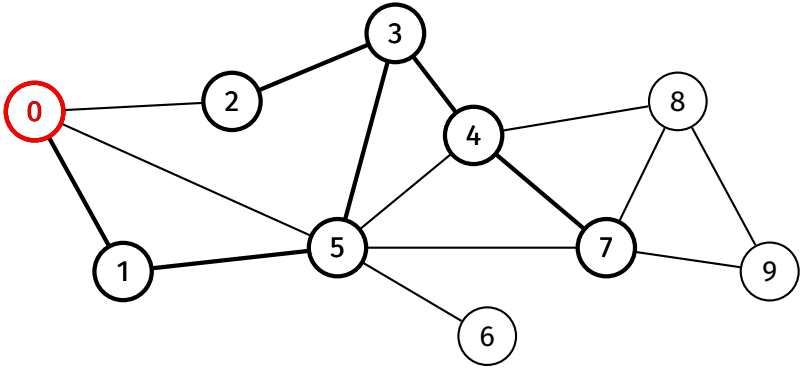
path(1, 7)?
path(0, 7)?
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

Path-Checking with Recursive DFS

Example

Return true



path(0, 7)?

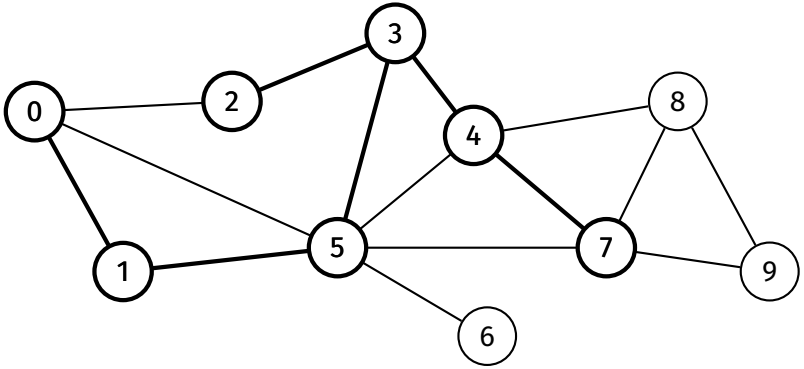
call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0

Path-Checking with Recursive DFS

Example

Answer: Yes



call stack

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
visited	1	1	1	1	1	1	0	1	0	0