Incentives for Early Arrival in Cooperative Games

(AAMAS24 Best Paper)

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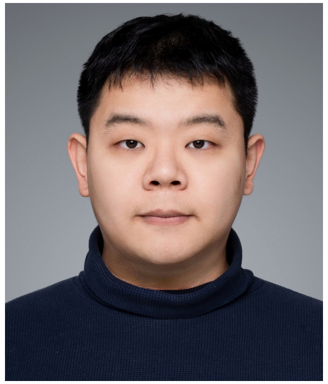






About Me

2002

Macau University of Science and Technology

B.Sc., Computer Science

First Class Honors; Full Scholarship

2006

Technische Universität Dresden, DE

Universidad Politécnica de Madrid, ES

Double M.Sc., Computational Logic

Erasmus Mundus Full Scholarship

IBM

2009

University of Western Sydney, AU

University of Toulouse, FR

Double PhD, Computer Science

ARC Discovery Scholarship

The Best Thesis Award

2013

Kyushu University, JP

Postdoc

Mentor: Prof. Makoto Yokoo

AAAI Fellow (first in Asia)

2014

University of Southampton, UK

Research Fellow

Mentor: Prof. Nick Jennings

IEEE/AAAI Fellow

Rigius Professor of CS

Vice-Provost at Imperial College

2017

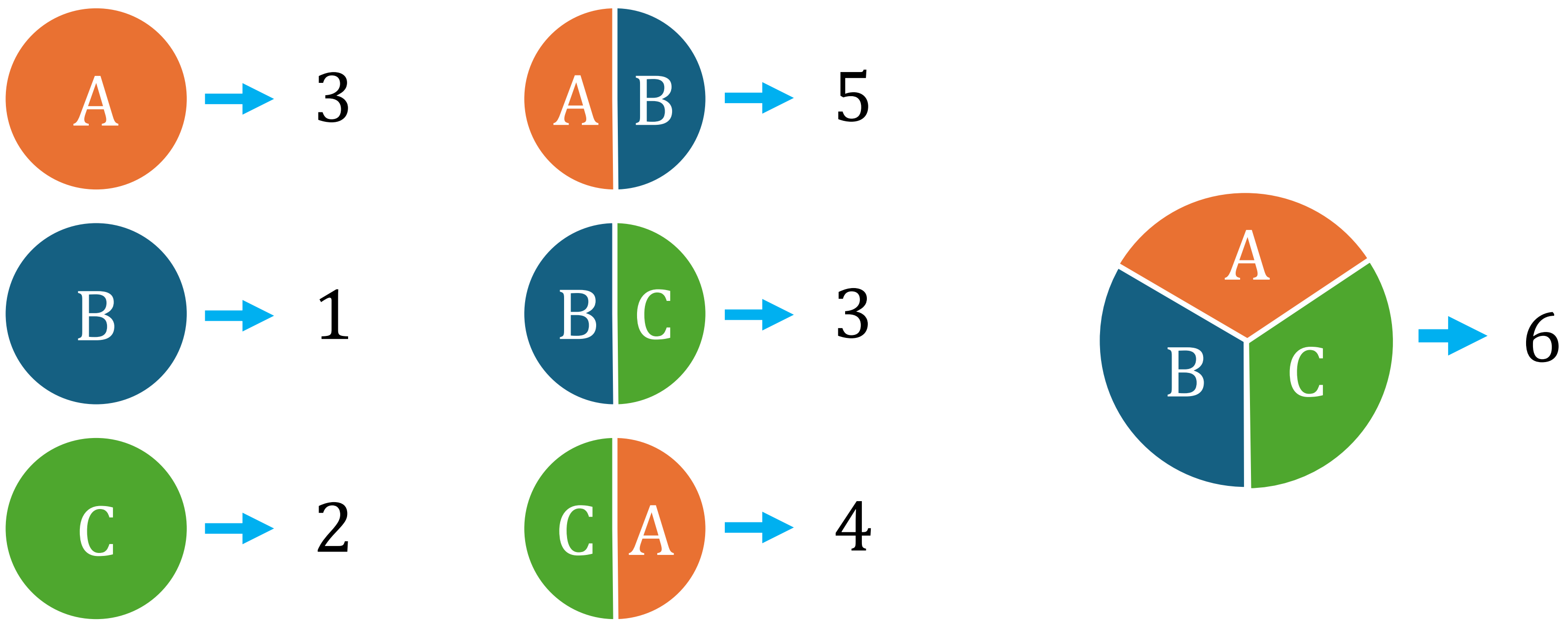
ShanghaiTech University

Assistant Professor

上海青年东方学者

Cooperative Game

- Players cooperate to create different values



- Determine how to share the value:

  Shapley value, core, …

Cooperative Game and Shapley Value

- Players: $N = \{A, B, C\}$
- Valuation Function: $v: 2^N \to \mathbf{R}$
- Marginal Contribution: $\mathrm{MC}(i, v, S) = v(S) - v(S \setminus i), \forall i \in S$

- Shapley Value:

$$\mathrm{SV}_i(v) \coloneqq \frac{1}{|N|!} \sum_{S \subseteq N \setminus \{i\}} |S|!\,(|N| - |S| - 1)\mathrm{MC}(i, v, S \cup \{i\})$$

The averaged marginal contribution on all possible joining orders.

Shapley Value

- SV: Averaged MC on all orders
- E.g.

Coalition	A	B	C	AB	AC	BC	ABC
Value	3	1	2	5	4	3	6

C A B
2 2 2

B C A
1 2 3

A B C
3 2 1

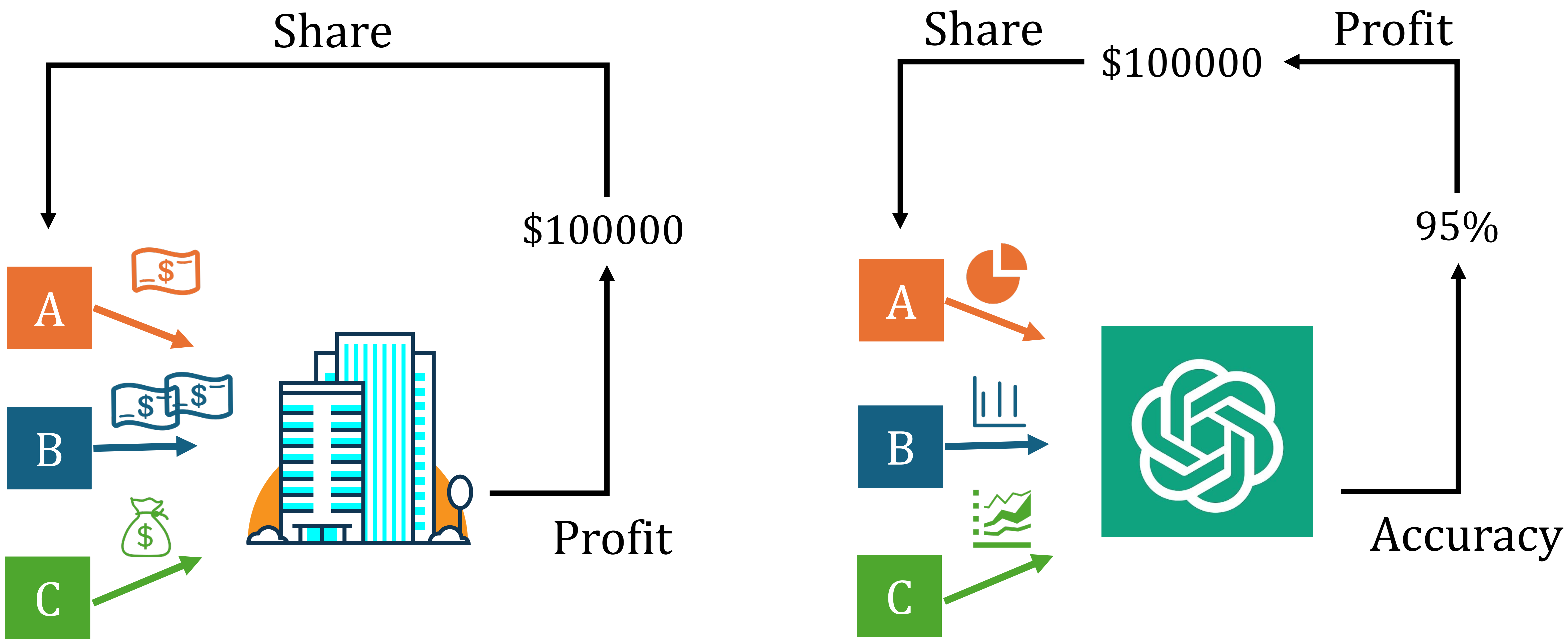
A C B
3 1 2

C B A
2 1 3

B A C
1 4 1

$$SV_A = \frac{2+3+3+3+3+4}{6} = 3$$

Venture Capital / Data Acquisition: Join Order Matters



Online Cooperative Games

Joining Order	A	B	C	Unknown
Value after joining	3	5	6	…
Marginal Contribution (MC)	3	2	1	…
The Value Share			?	

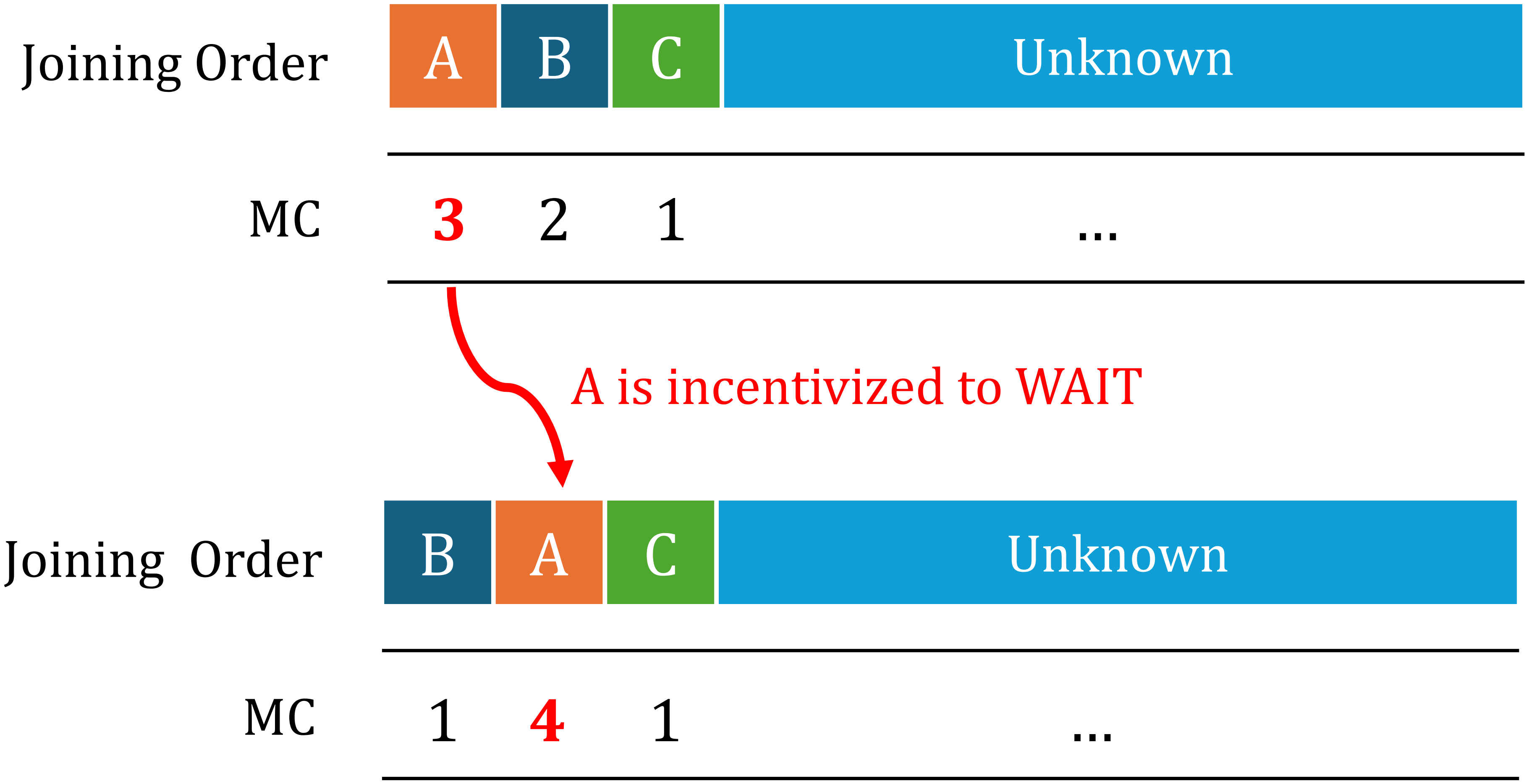
Online Cooperative Game Model

- Players: $N = \{A, B, C\}$
- Valuation Function: $v: 2^N \to \mathbf{R}$
- Joining Order: $\pi \in \Pi(N)$ (a permutation of players)
- Marginal Contribution: $\mathrm{MC}(i, v, S) = v(S) - v(S \setminus i), \forall i \in S$

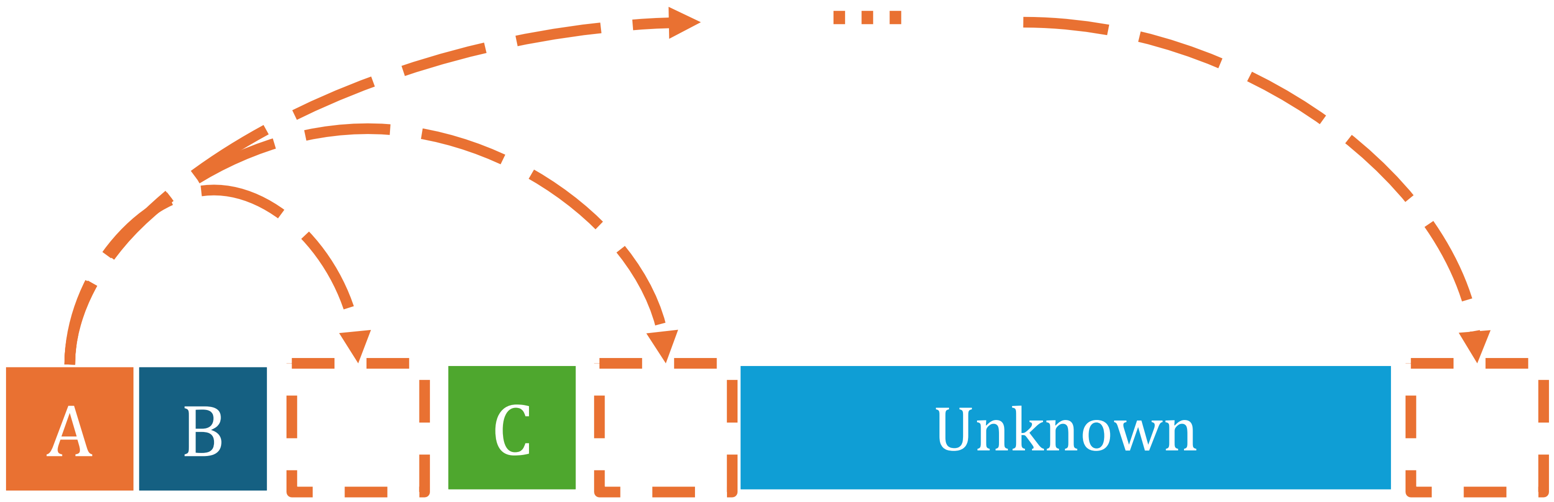
- Shapley Value:

$$\mathrm{SV}_i(v) := \frac{1}{|N|!} \sum_{S \subseteq N \setminus \{i\}} |S|!\,(|N| - |S| - 1)\mathrm{MC}(i, v, S \cup \{i\})$$

How to share the value: *Marginal Contribution*



Incentivizing for Early Arrival (I4EA)



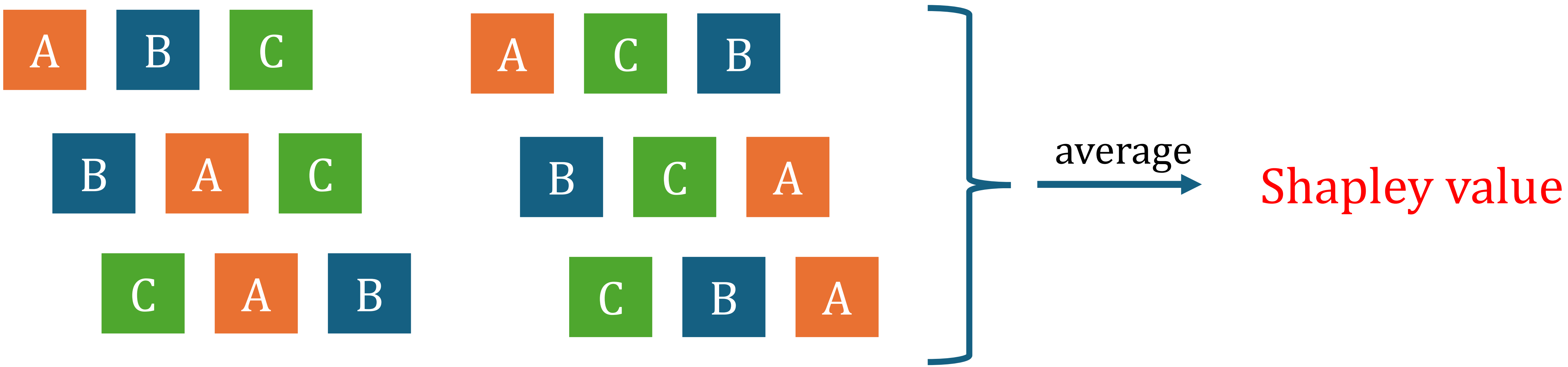
I4EA: When the order of others are fixed, the players are *incentivized to join as soon as possible*.

One Solution: *The First Gets All*

Joining Order	A	B	C	Unknown
MC	3	2	1	...
Value Share	6	0	0	...

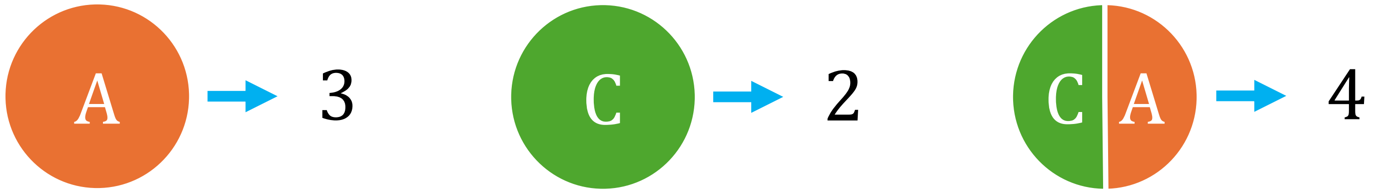
Not Fair !

One Fairness: Shapley Fair (SF)



SF: The expected share to a player equals her *Shapley value.*

Shapley value?



Order: A | Unknown

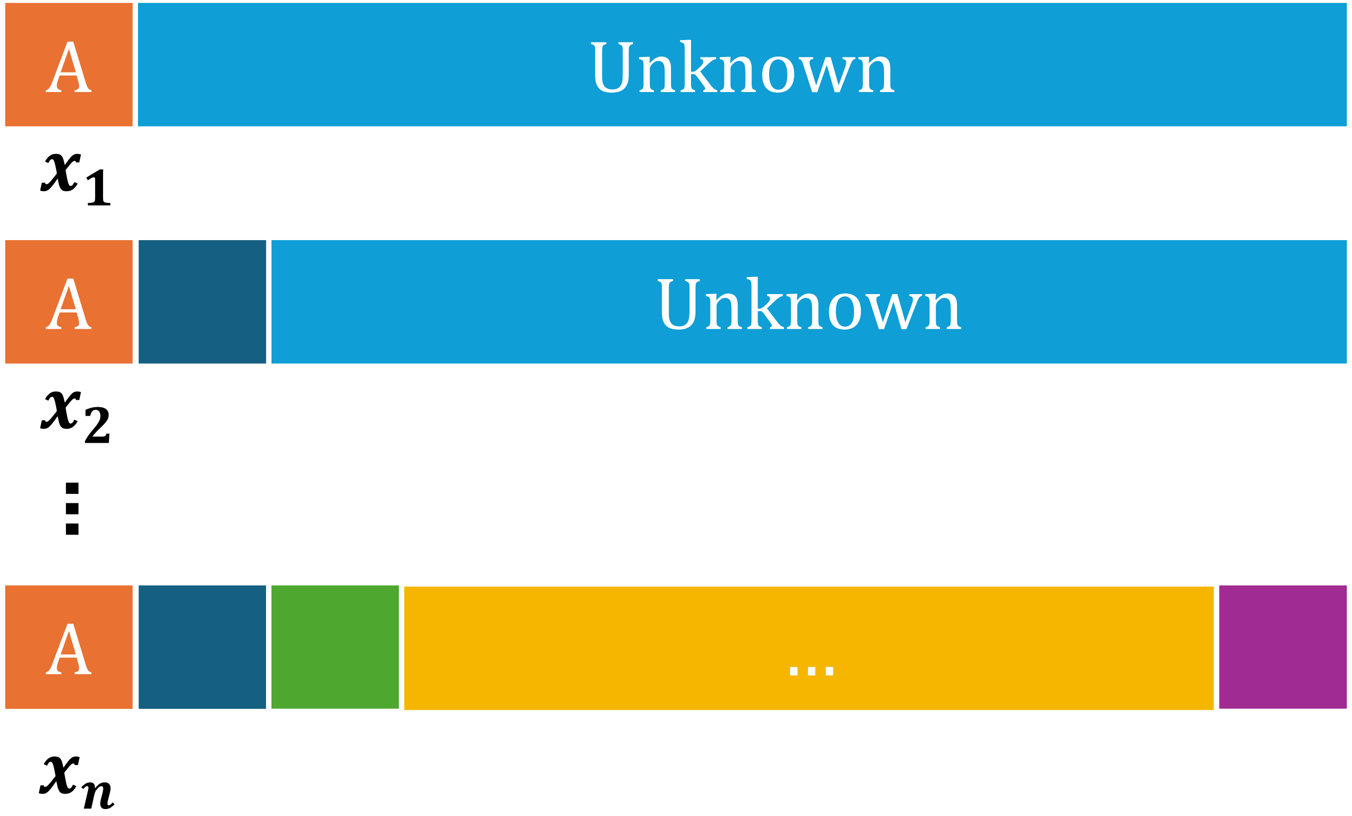
Shapley value: 3 ...

Order: A | C | Unknown

Shapley value: 2.5 1.5 ...

A's share Decreases!

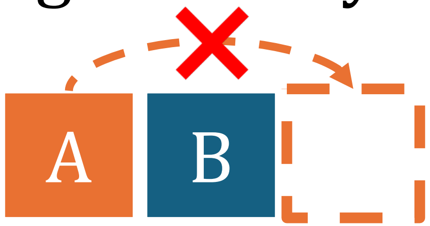
Online Individual Rational (OIR)



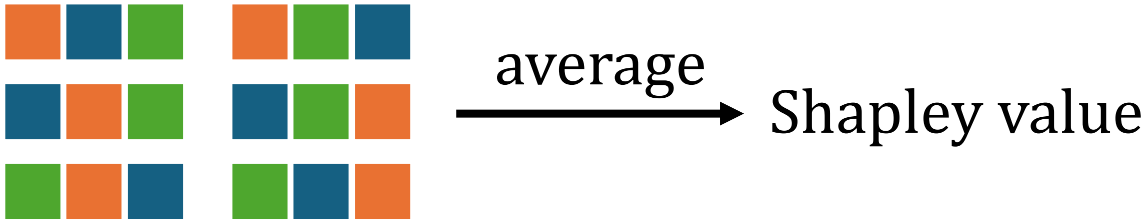
OIR: $x_n \geq \cdots \geq x_2 \geq x_1 \geq 0$ *(non-decreasing, non-negative)*

All the Desire Properties

- Incentivizing For Early Arrival (I4EA)

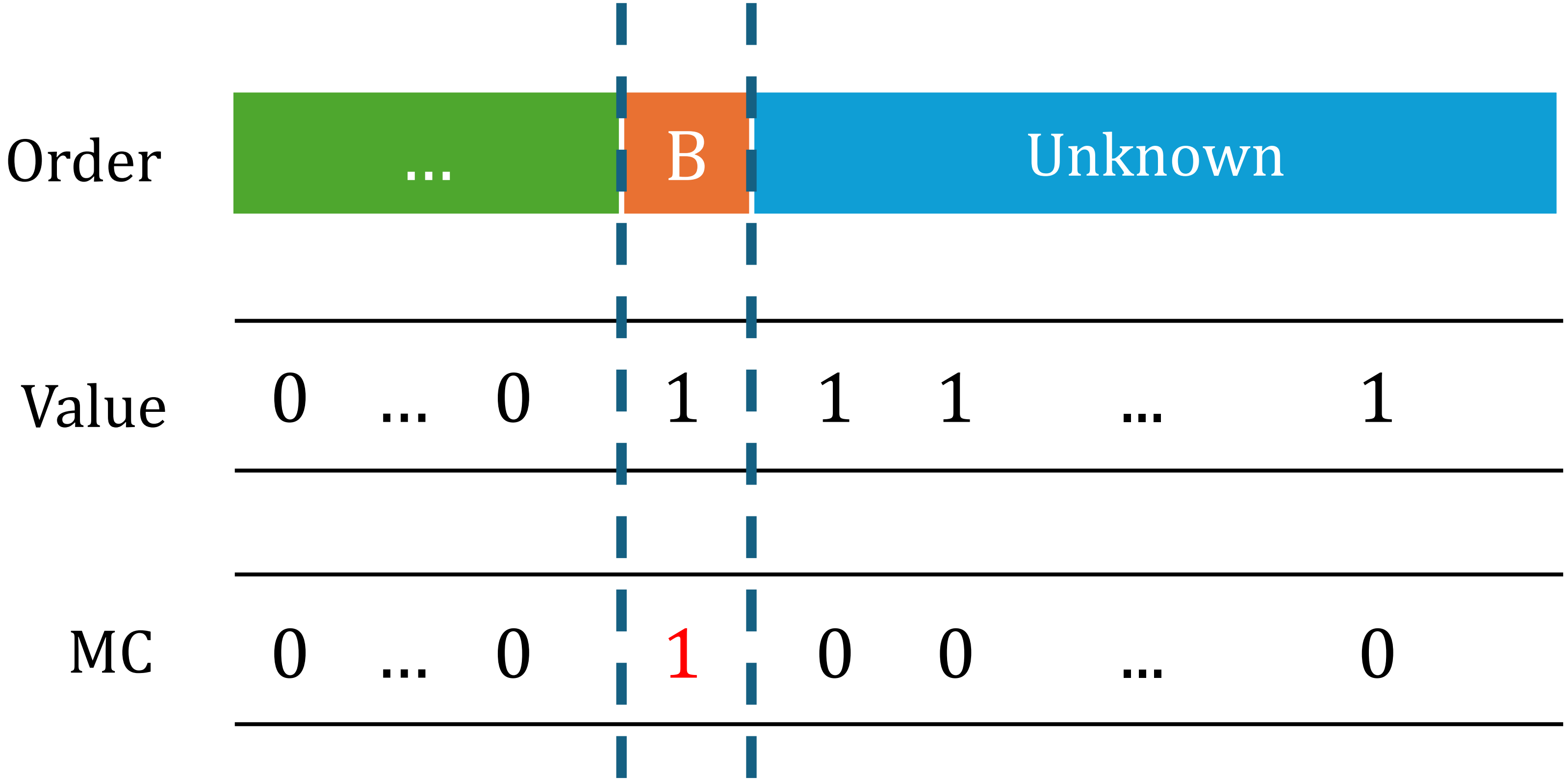


- Shapley-fair (SF)



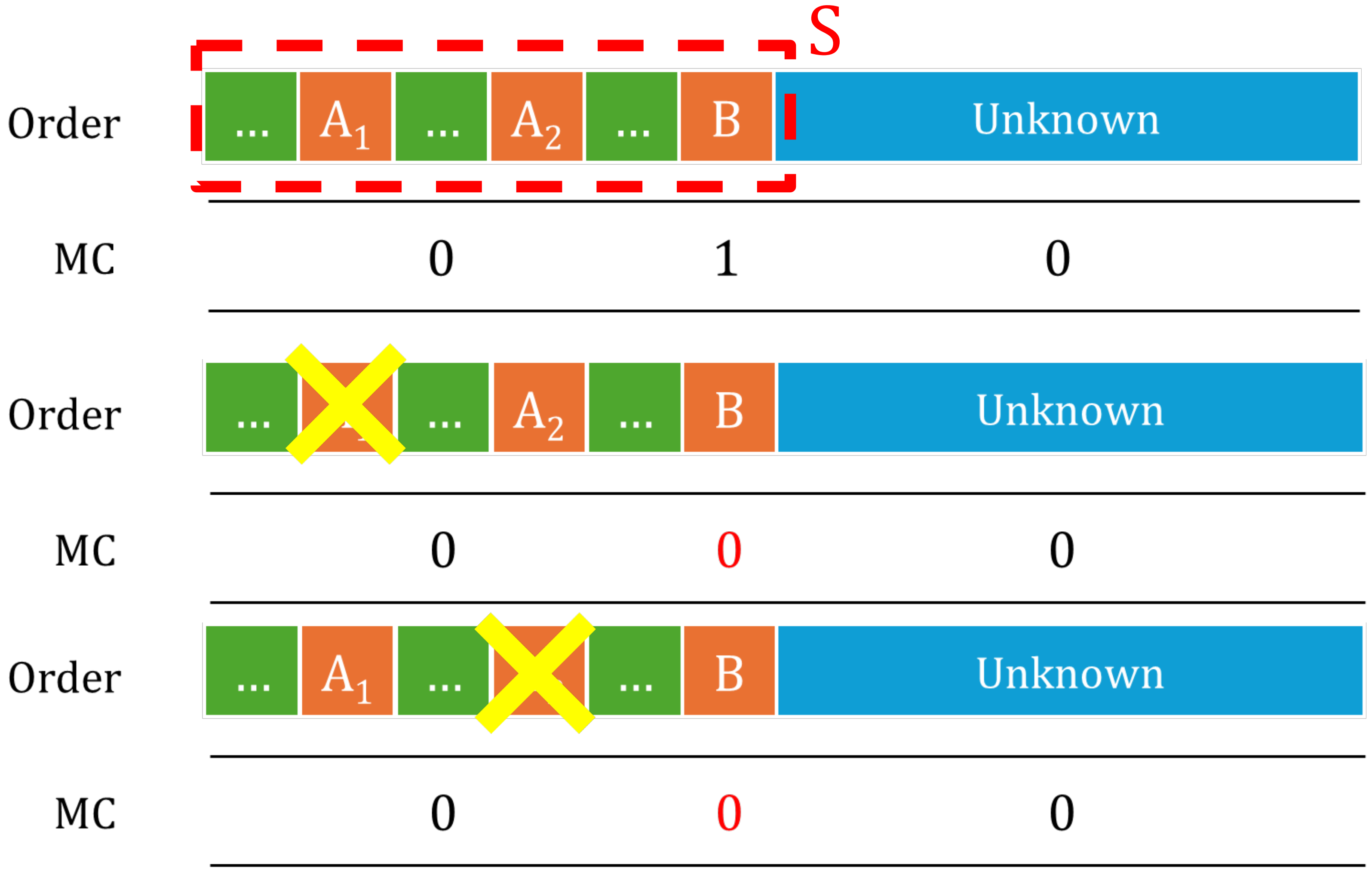
- Online Individual Rational (OIR)

Start with 0-1 Valued Monotone Games: $A_1 \wedge A_2 \wedge B$



Marginal Player: B is the only player who creates a MC of 1.

0-1 Valued Monotone Games: $A_1 \wedge A_2 \wedge B$

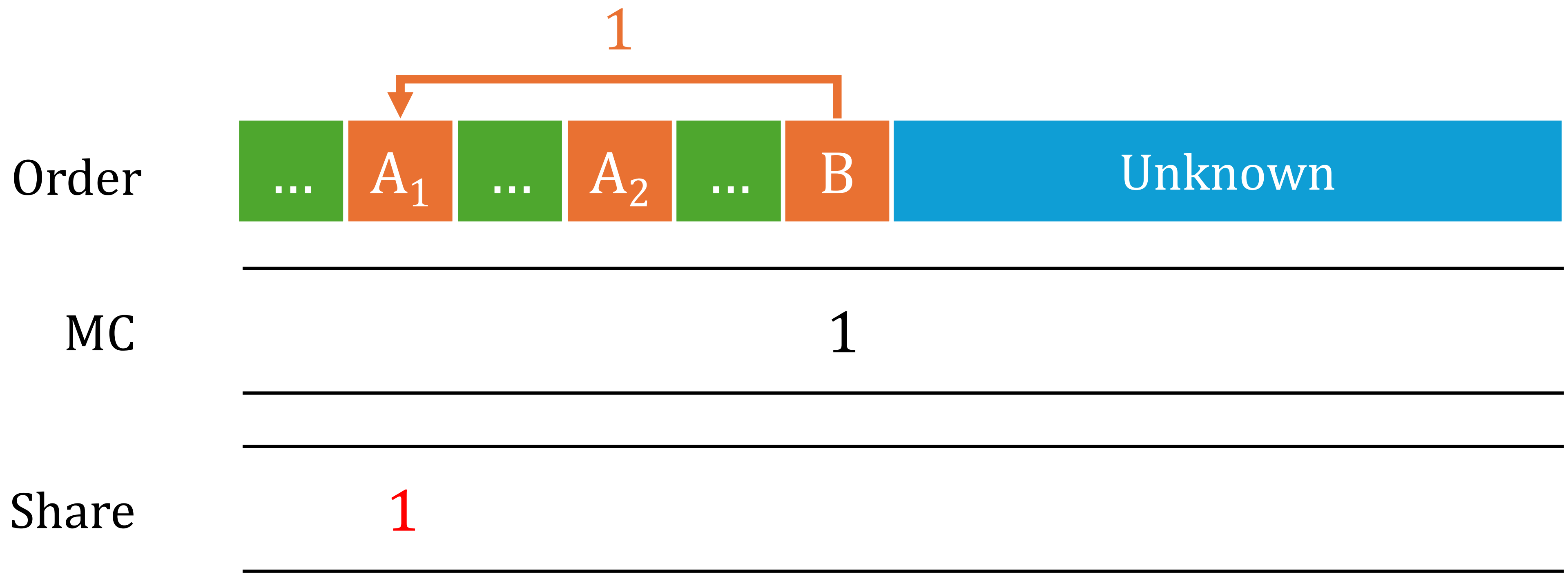


Critical Players: $\{i \mid v(S \setminus \{i\}) = 0\}$

Reward First Critical Player (RFC)

Definition: RFC

Give the MC of the marginal player to the first critical player in S.



Properties Overview

I4EA + SF + OIR —satisfiable→ RFC is one of the solutions

given a game —satisfiable?→ we know

Properties of RFC

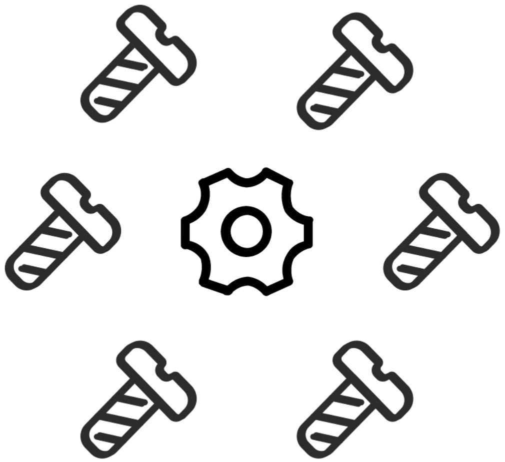
Theorem

RFC is SF and OIR on every 0-1 valued monotone game.

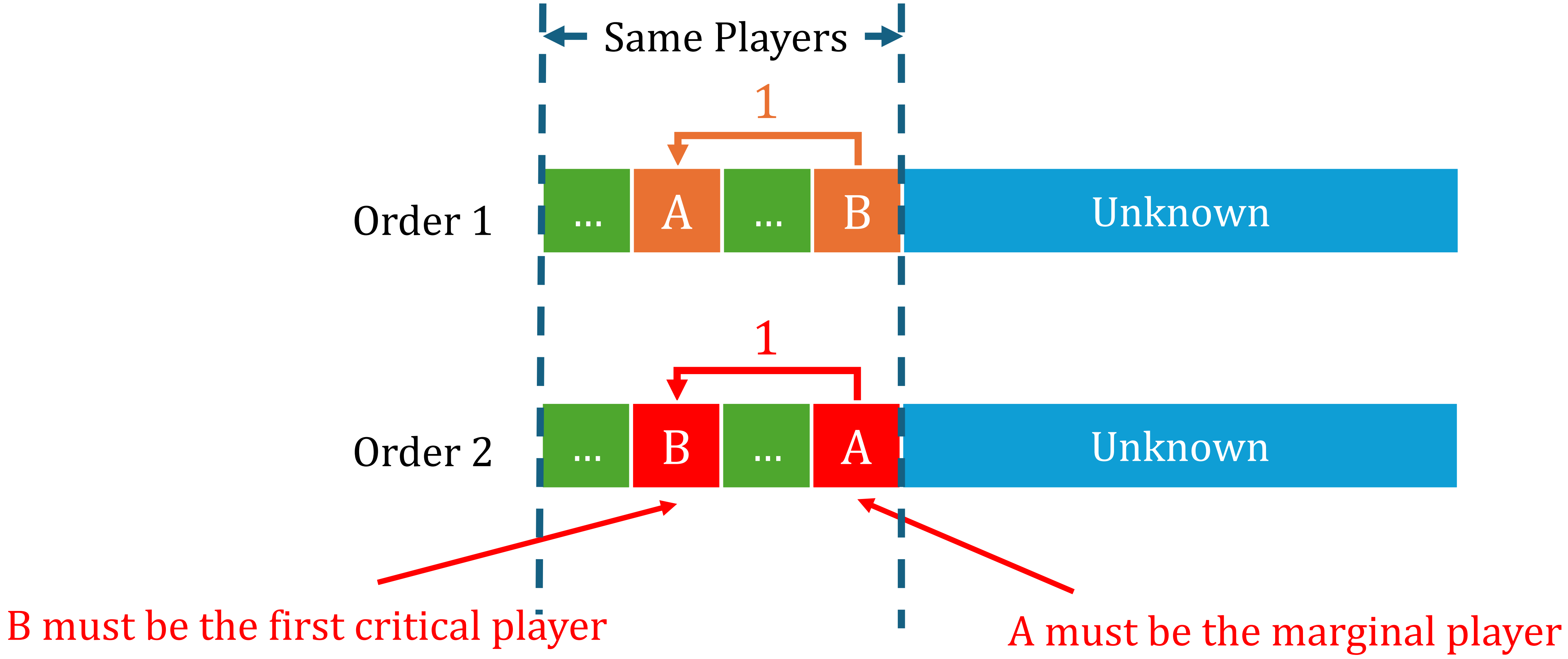
RFC is not I4EA only on games satisfying: $\exists i, v(i) = 0$ and $\exists S \subseteq N, S^* = \{i\}$. Here $S^* := \{i \in S \mid v(S) = 1,\ v(S \setminus \{i\}) = 0\}$.

- SF ✓
- OIR ✓ (obviously)
- I4EA is not satisfied when someone can delay to be the only critical player.

  e.g. $(A \vee B) \wedge C$, $\big((A \wedge B) \vee (E \wedge D)\big) \wedge C$, ...



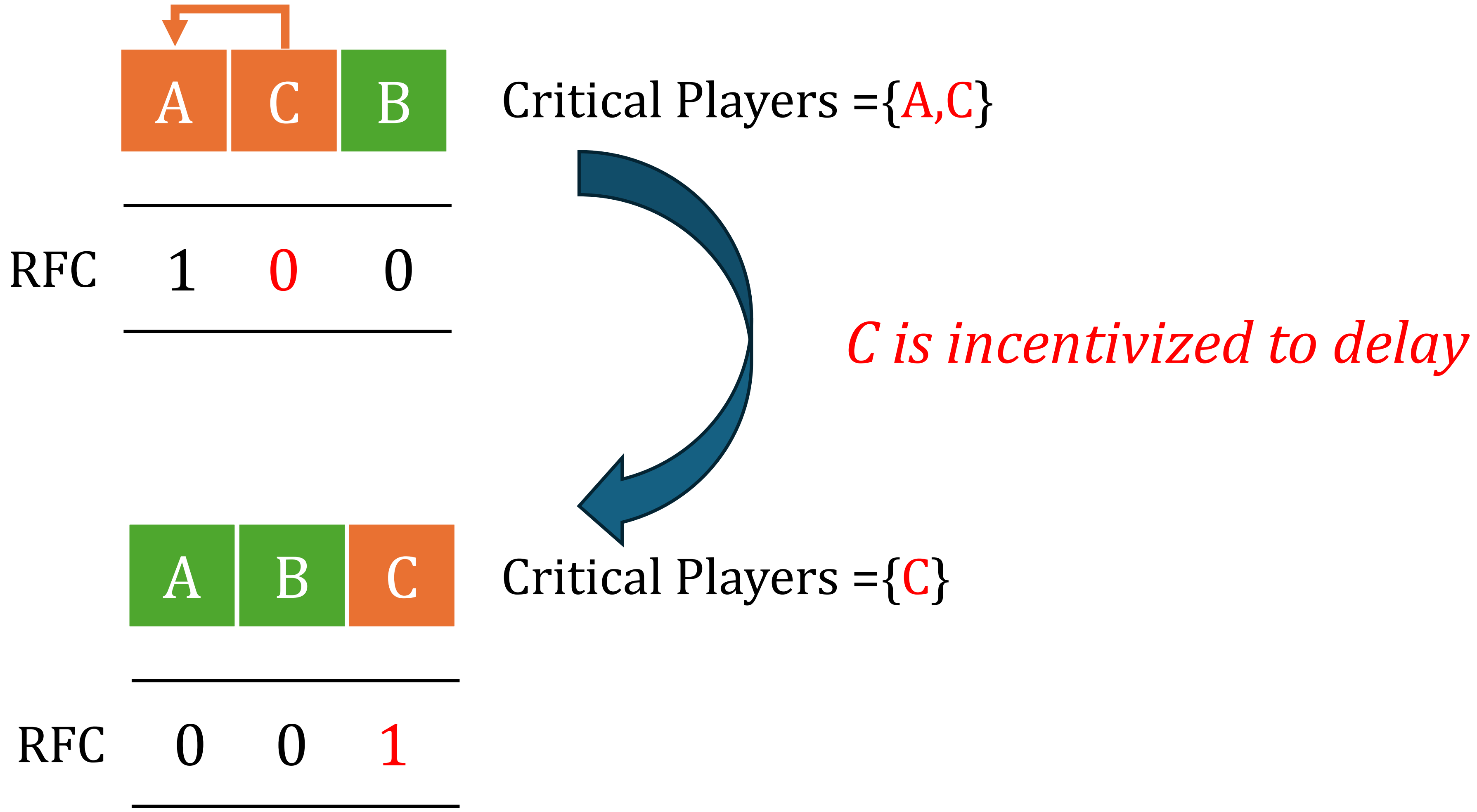
Proof of Shapley-Fair (sketch)



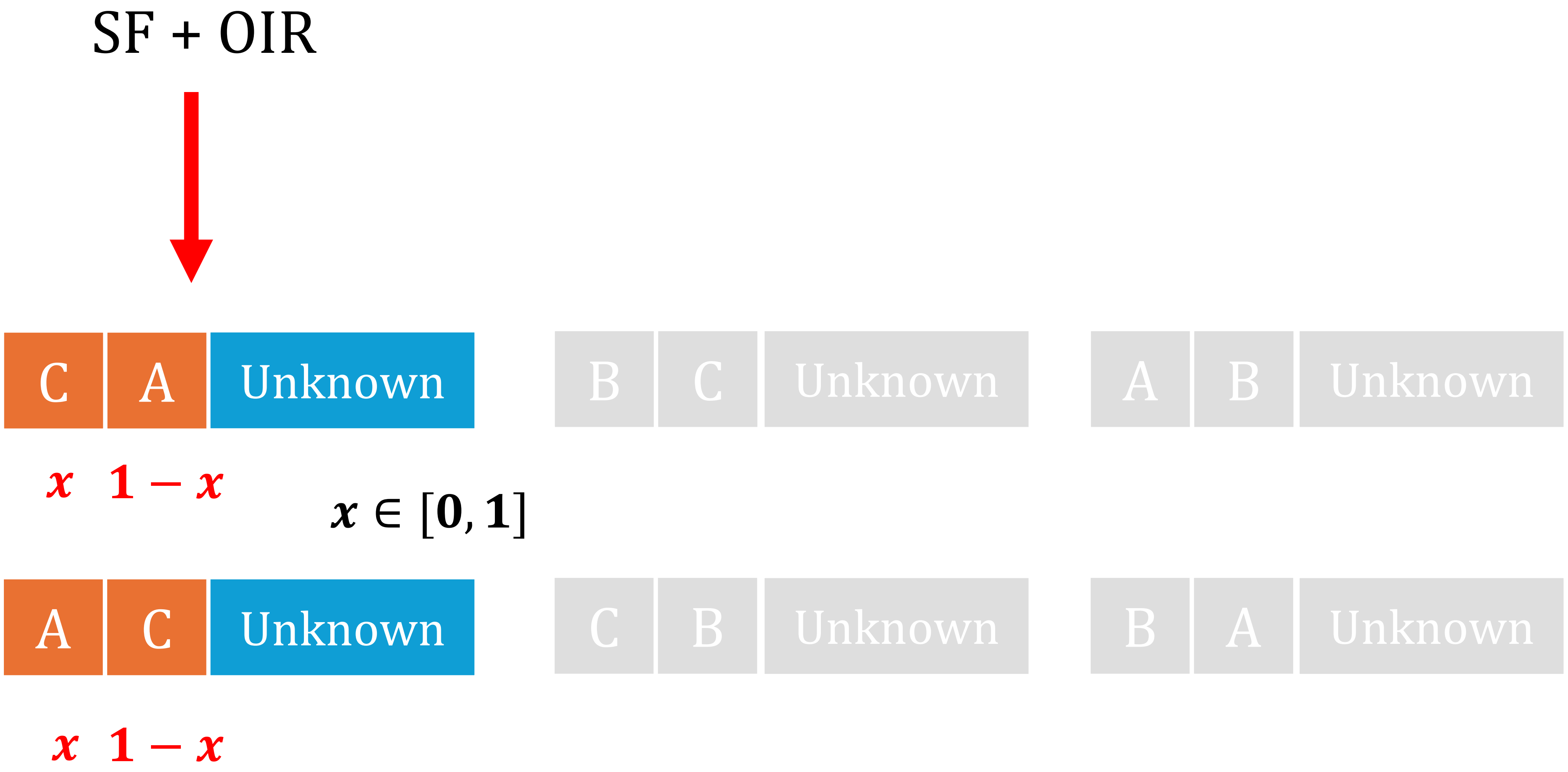
RFC on 3-player 0-1 valued monotone games

v	Value Receiver	I4EA
A	to A	Yes
$A \vee B$	1^{st} of $\{A, B\}$	Yes
$A \vee B \vee C$	1^{st}	Yes
$A \wedge B$	1^{st} of $\{A, B\}$	Yes
$A \wedge B \wedge C$	1^{st}	Yes
$(A \wedge B) \vee C$	C is 1^{st} or 2^{nd} $\rightarrow$ C Otherwise $\rightarrow$ 1^{st} of {A,B}	Yes
$(A \vee B) \wedge C$	**C is 1^{st} or 3^{rd} $\rightarrow$ C** **Otherwise $\rightarrow$ 1st of {A,B}**	**No**
$(A \wedge B) \vee (A \wedge C) \vee (B \wedge C)$	1^{st}	Yes

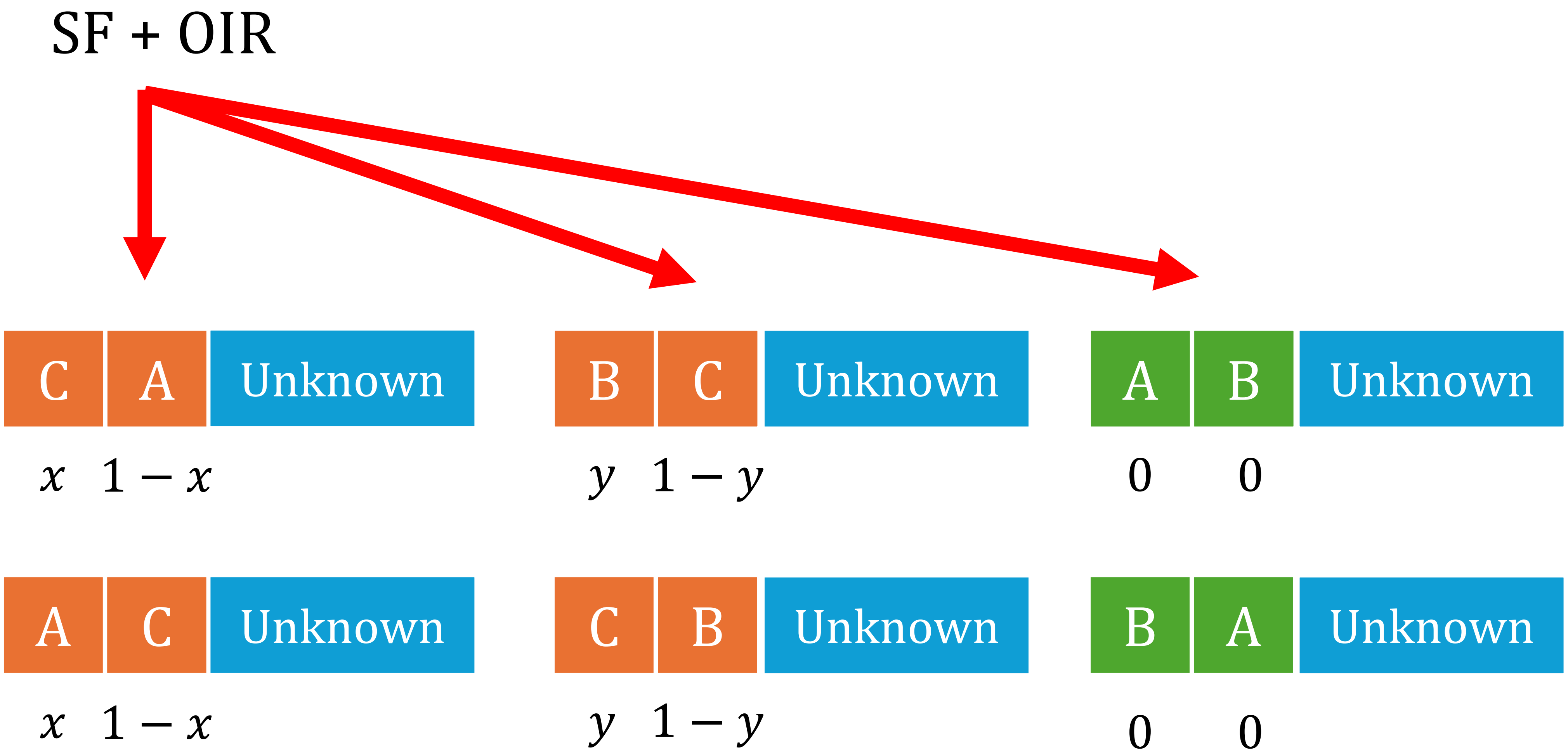
RFC is not I4EA on $(A \vee B) \wedge C$



$(A \vee B) \wedge C$ is unsolvable

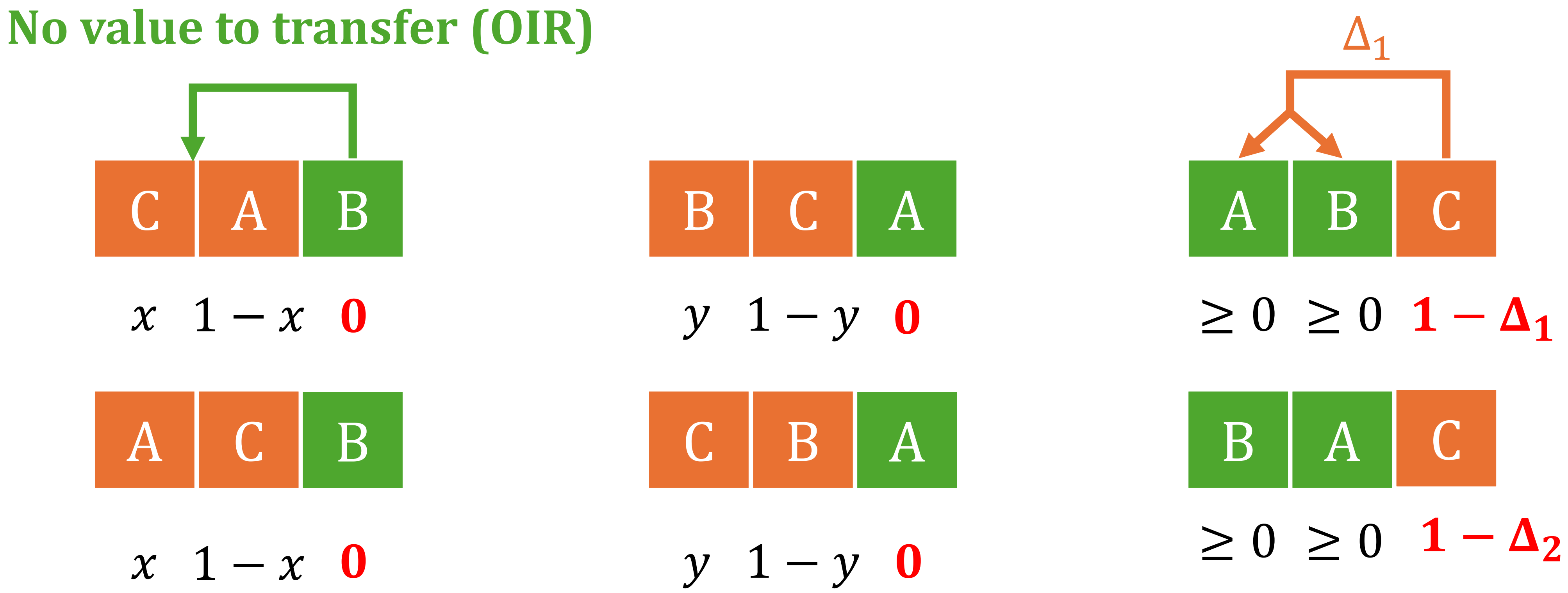


$(A \vee B) \wedge C$ is unsolvable



$(A \vee B) \wedge C$ is unsolvable

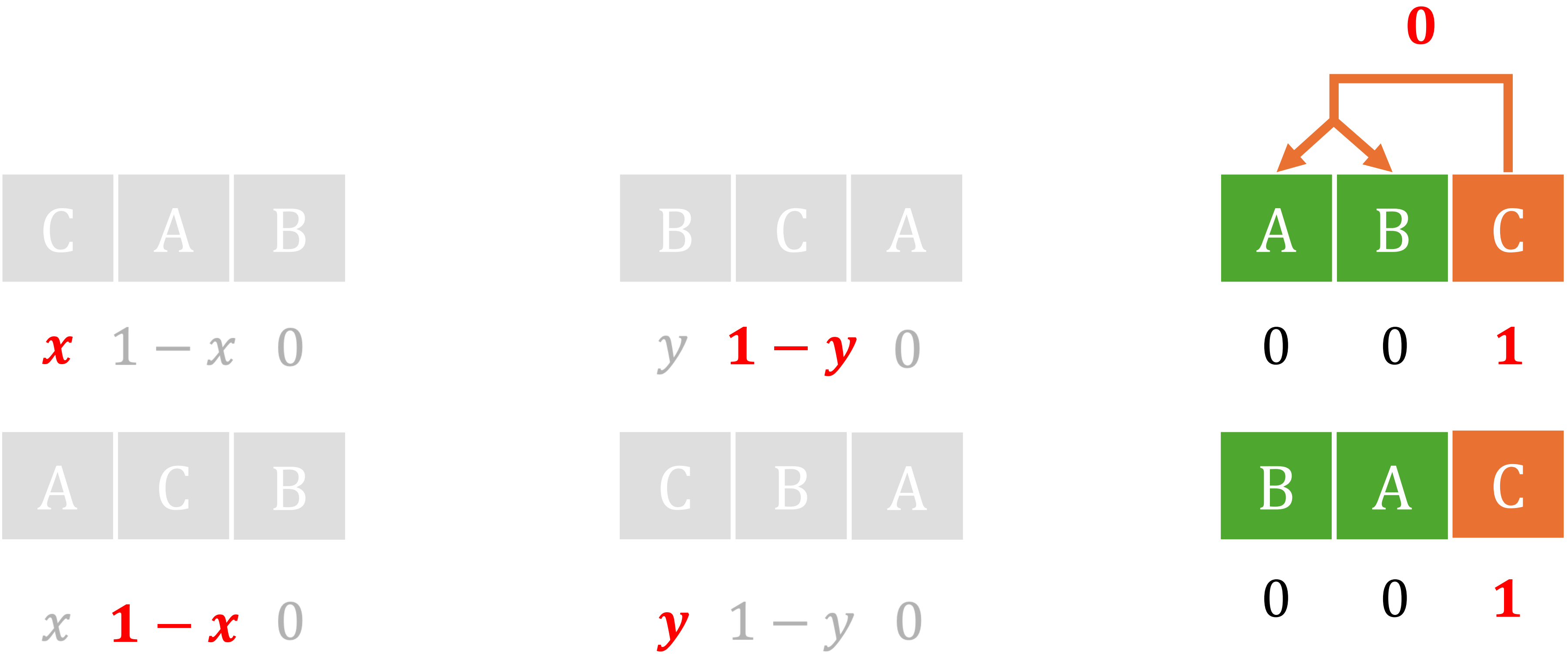
The 3rd player joins…



$(A \lor B) \land C$ is unsolvable

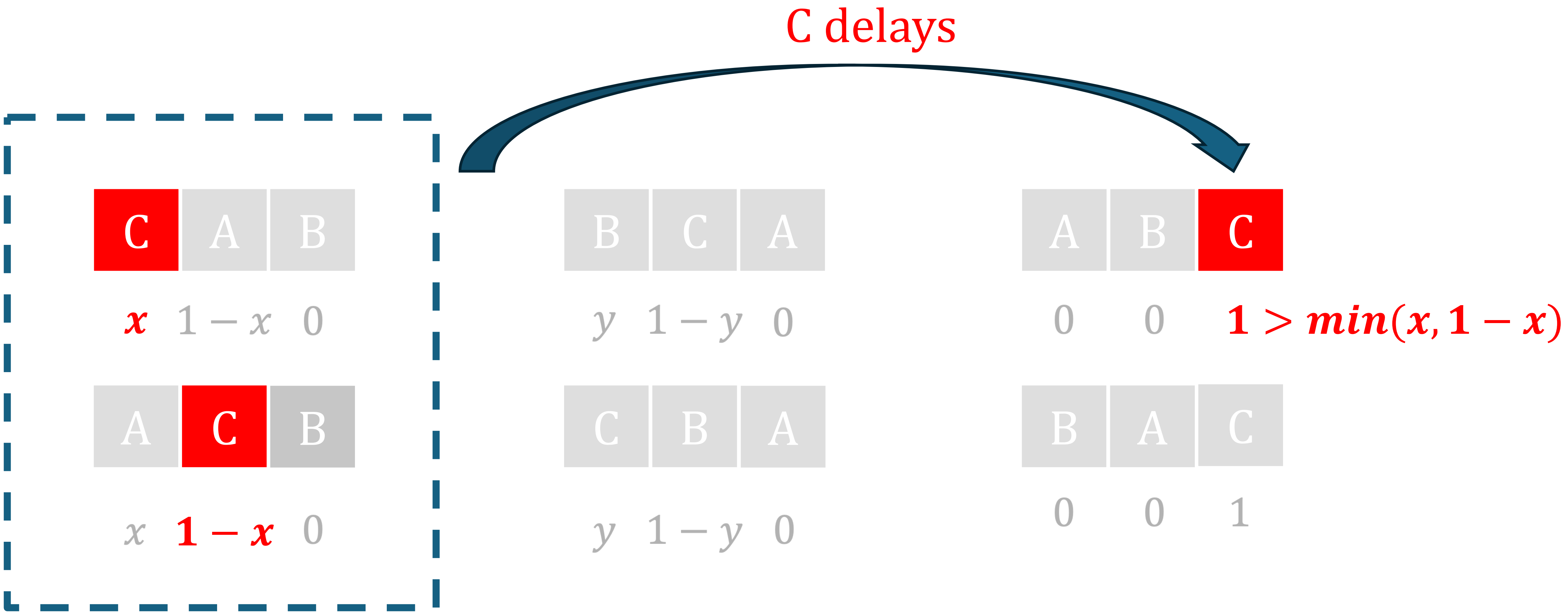
SF $\Rightarrow$ $SV_C = 2/3 = (x + 1 - x + y + 1 - y + 1 - \Delta_1 + 1 - \Delta_2)/6$

$\Rightarrow$ $\Delta_1 = \Delta_2 = 0$

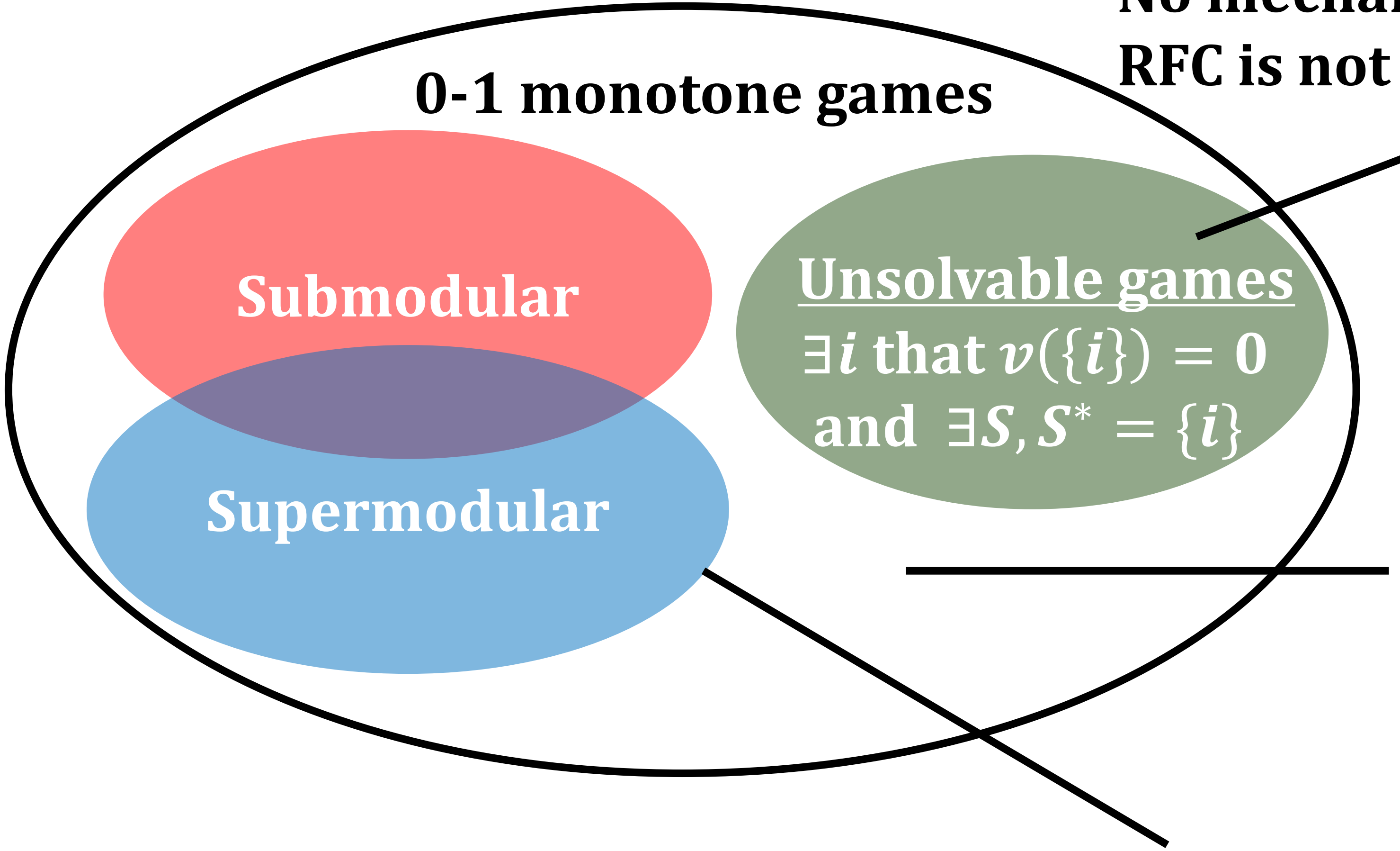


$(A \vee B) \wedge C$ is unsolvable

SF + OIR → not I4EA



Summary of Properties



Characterization of Unsolvable Games:
No mechanism satisfying OIR SF and I4EA.
RFC is not I4EA.

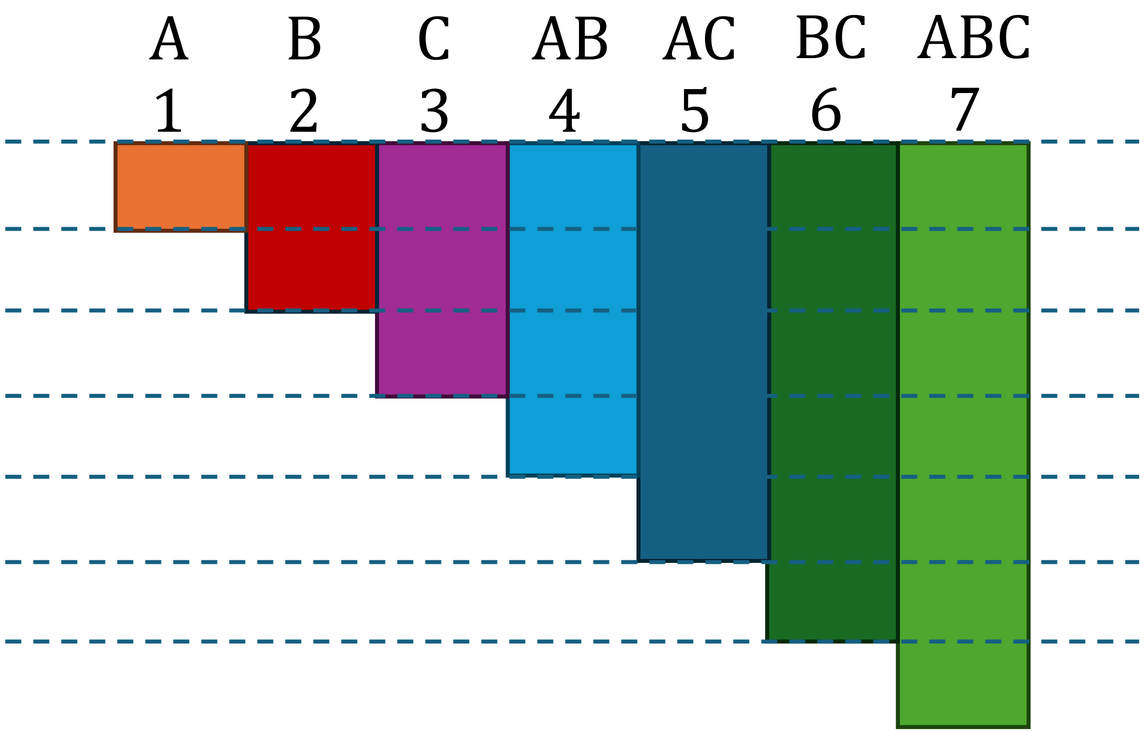
Completeness of RFC:
RFC is I4EA, SF and OIR.

submodular and supermodular games are solvable

Extend RFC to General Valued Monotone Games

Definition: eRFC

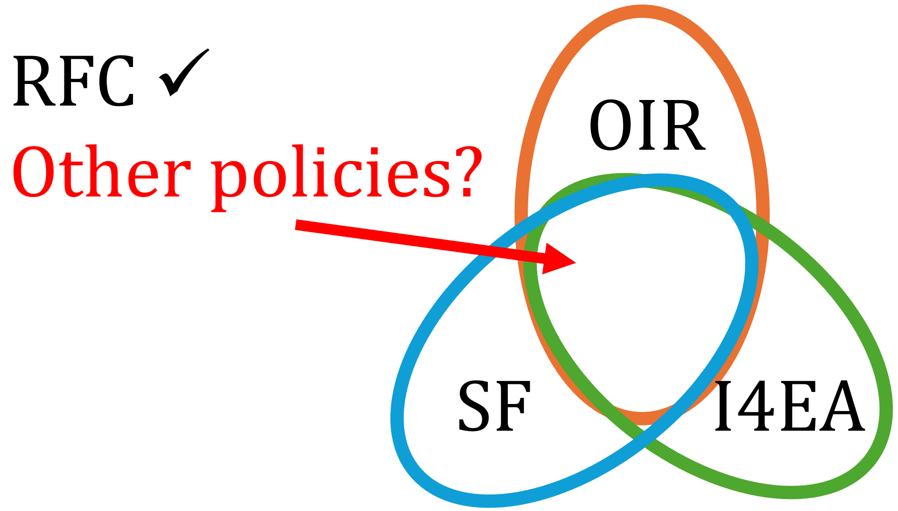
(1) decompose game online (2) accumulate the share



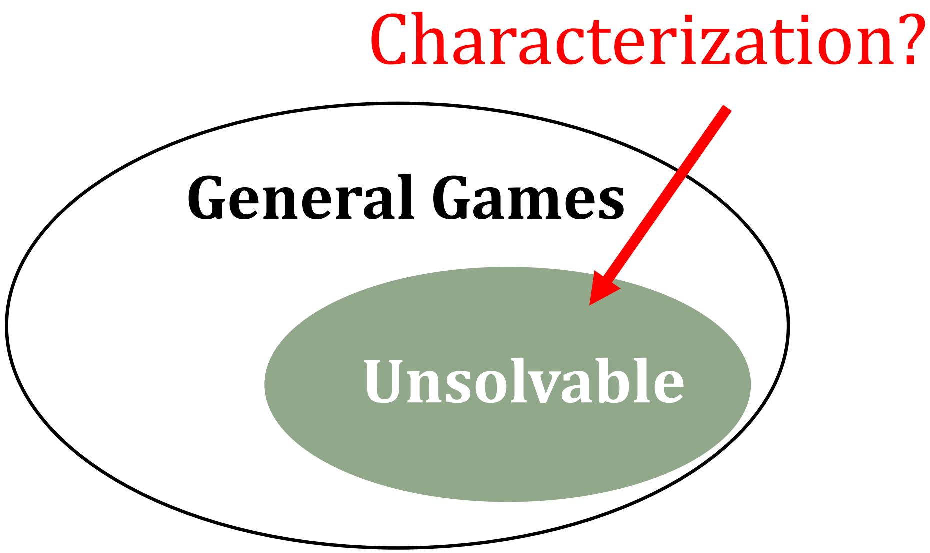
A	B	C	AB	AC	BC	ABC
1	2	3	4	5	6	7
1	1	1	1	1	1	1
	1	1	1	1	1	1
		1	1	1	1	1
			1	1	1	1
				1	1	1
					1	1
						1

0-1 valued components

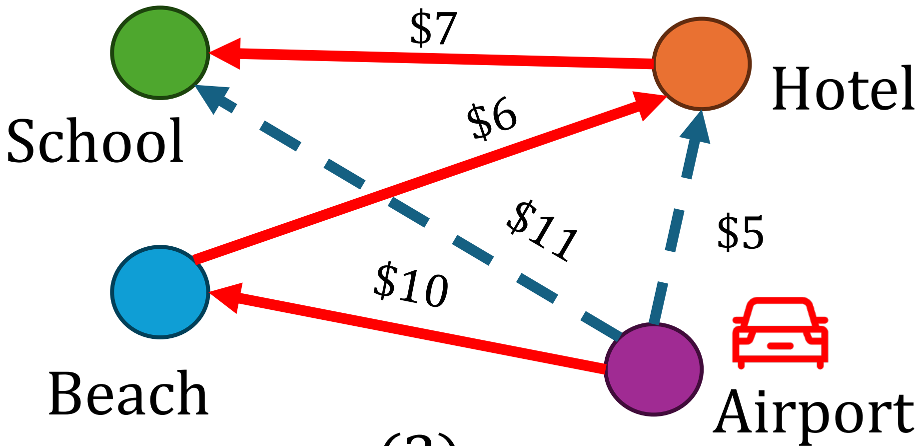
Future Work



(1)



(2)



(3)