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Letter to the Editor

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There is reason to make the following two comments on the research note "An automatic proof of Gödel's incompleteness theorem" (*Artificial Intelligence* 61 (1993) 291–306) by K. Ammon, which describes the performance of a proof system, SHUNYATA, in an experiment resulting in a proof of Gödel's famous theorem.

The paper fails to relate SHUNYATA's proof techniques to any of the methods known in automated deduction. Thereby the reader is implicitly left with the impression (stated explicitly in other writings of the author) that the reason for this successful experiment lies in the departure from methods like the resolution or the connection method. In order to correct this false impression we report here that the general proof system SETHEO [1], which is based on the connection method easily reproduces the same proof of Gödel's theorem provided with the same information as in Ammon's experiment. The following details illustrate our experiment.

SHUNYATA consists of a number of "heuristics"; most of which (e.g. the negation and implication heuristics) are dealing with Gentzen-like inference rules (handled in SETHEO by its logical machinery). A remarkable exception is the *composition* heuristics which constructs first-order formulas following rather restricted design instructions. This heuristic is essential for proving Gödel's theorem since it enables the system to guess an undecidable formula. In a standard proof system this ability of guessing formulas can be easily achieved by adding a procedure which generates unit clauses following precisely the design instructions used in Ammon's proof. For simplicity, we simulated this feature by adding the appropriate unit clause. In addition, Ammon provided the system with three definitions and five lemmata which

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define and describe various properties concerning the consistency of formal theories, provability, and Gödel numberings. Using the representational technique of reification the same information is presented to SETHEO along with the theorem to be proved. Provided with all this information exactly as in Ammon's experiment SETHEO finds the proof in less than 15 minutes (on a Sparcstation 1). Since Ammon's system needs three unsuccessful, time restricted attempts before guessing the right formula, it seems fair to multiply this time with a factor of four to account for the mentioned simulation. The resulting time of about one hour for a uniform and general prover favorably compares with the specialized SHUNYATA's 9 hours needed in the same experiment.

Our second comment questions the following statement in Ammon's note: "Since the composition heuristic and ... rules are rather elementary and do not contain Cantor's method, it seems justified to say that SHUNYATA implicitly rediscovered Cantor's diagonal method while it was proving Gödel's theorem." (p. 302). Although it is correct to say that the heuristics are elementary and do not contain the diagonal method, we strongly disagree with the author's conclusion. The instructions for designing formulas as described above are indeed simple but on the other hand very restricted. The formula is forced to contain a single, fixed predicate given along with its definition, which states that the predicate expresses a relation between the Gödel number of an arbitrary formula and the Gödel numbers of the proofs of this formula. The task left to the proof system is just to negate the predicate and quantify it appropriately. The intellectual achievement in Gödel's proof is contained in those given definitions and lemmata rather than in the way the system constructed the undecidable formula. The system can therefore not at all be said to having rediscovered Cantor's diagonal method.

Reference

[1] R. Letz, J. Schumann, S. Bayerl and W. Bibel, SETHEO—a high-performance theorem prover for first-order logic, *J. Autom. Reasoning* **8** (2) (1992) 183–212.