Reasoning about Continuous Processes

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Abstract

Overcoming the disadvantages of equidistant discretization of continuous actions, we introduce an approach that separates time into slices of varying length bordered by certain events. Such events are points in time at which the equations describing the system's behavior—that is, the equations which specify the ongoing *processes*—change. Between two events the system's parameters stay continuous. A high-level semantics for drawing logical conclusions about dynamic systems with continuous processes is presented, and we have developed an adequate calculus to automate this reasoning process. In doing this, we have combined deduction and numerical calculus, offering logical reasoning about precise, quantitative system information. The scenario of multiple balls moving in 1-dimensional space interacting with a pendulum serves as demonstration example of our method.

Introduction

In a vast variety of different disciplines it is required to reason logically about physical systems that are described by numerical equations rather than symbolical definitions. The standard approach is to quantify the whole scenario into a finite number of points in time at which all system parameters are represented in variables. If there were infinitely many points at infinitely small distances, this might be sufficient—even though impossible to calculate. But, since discretization is always finite, a problem arises when an action or event takes place in between two of these points. Imagine two balls moving at constant speed into two different directions but their courses crossing each other (Billiard Scenario in (Shoham & McDermott 1988)). Imagine further that these two balls will collide on their courses at a certain point of time. Now, if the discretization does not take into regard this very point of time then the collision is not detected and the balls seem to be moving on into their original directions, which results in entirely wrong final positions of the balls. In (Shoham & McDermott 1988) this problem of not being able to predict the future without an infinite number of discretizations is called the *extended prediction problem*.

While some work has been done to extend specific action calculi in order to deal with continuous change, e.g. (McDermott 1982; Shoham 1988; Shanahan 1990), these ideas have not yet been exploited to define a high-level action semantics serving as basis for a formal justification of such calculi, their comparison, and an assessment of the range of their applicability. Such semantics have recently been developed for the discrete case (Gelfond & Lifschitz 1993; Sandewall 1994; Thielscher 1995) and successfully applied to concrete calculi, e.g. (Kartha 1993; Doherty & Łukaszewicz 1994; Thielscher 1994). However, neither of these formalisms is suitable for calculi dealing with continuous processes. The *Action Description Language* (Gelfond & Lifschitz 1993) is based on the concept of single-step actions and does not include a notion of time. In (Sandewall 1994), the duration of actions is not fixed, but equidistant discretization is assumed and state transitions only occur when actions are executed—otherwise the world description is assumed to remain stable. While in contrast the approach developed in (Thielscher 1995) allows for user-independent events to cause state transitions, again equidistant discretization is assumed.

In this paper, we propose a new semantics for reasoning about continuous change which allows for varying temporal distances between state transitions. The described system may have non-continuous characteristics but must be separable into continuous sections by a finite number of discontinuities. While *fluents* (McCarthy & Hayes 1969) constitute the basic entities for state descriptions in (Gelfond & Lifschitz 1993; Sandewall 1994; Thielscher 1995), we propose the more general notion of *processes* as the underlying concept for constructing state descriptions. In contrast to fluents, whose values are static except in case a state transition occurs, a process may contain parameters whose values change continuously. Such parameters are formalized as functions over time. Much like fluents may change their value during state transitions in the dis-

*on leave from FG Intellektik, TH Darmstadt crete case, in the continuous case a state transition may cause existing processes to disappear and new processes to arise. State transitions are either triggered by the execution of actions (so-called external events, e.g., hitting an idling ball) or by interactions between processes (so-called internal events, e.g., collisions of moving balls). Both external and internal events are specified by *transition laws*.

On this basis, we have developed a formal, model-theoretic semantics for reasoning about domains involving continuous change. An application example will be used to illustrate the formalism. We moreover have developed an adequate (wrt. our semantics) extension of an action calculus using logic programming based on an approach developed in (Hölldobler & Schneeberger 1990). Due to lack of space, a description had to be omitted; it can be found in (Herrmann & Thielscher 1996). At the end of this paper, the interested reader may in addition find an internet address for a PROLOG implementation of our calculus.

A Logic of Processes

In this section, we introduce a formal semantics for reasoning about continuous processes, their interaction in the course of time, and their manipulation by means of executing actions.

Processes

While *fluents* form the basic components of situation descriptions in classical, discrete approaches like situation calculus (McCarthy & Hayes 1969), we use a generalized notion called *processes* as the basic entities of situation descriptions in our model of continuous change. Any concrete process is an instance of a general type of processes, like "continuous movement of a physical object in a 1-dimensional space." The type a process belongs to determines its description components. More precisely, each type is associated with a so-called scheme specifying two kinds of parameters: the *static* parameters, which do not change as long as the process is in progress (like the coordinates of the starting point or the velocity of a continuously moving object), and the *dynamic* parameters, whose actual values are time-dependent and are therefore subject to change in the course of the process (like the actual location of a moving object). Components of the dynamic description part are formally represented as functions whose arguments are the static parameters plus two time-points, namely, the starting time of the process and the actual time:

Definition 1 A *process scheme* is a pair $\langle C, F\rangle$ where C is a finite, ordered set of symbols of size $n \geq 0$ and F is a finite set of functions $f : \mathbb{R}^{n+2} \mapsto \mathbb{R}$.

For example, the two components $\langle C, F\rangle$ of a process scheme describing continuous movement in a 1-dimensional space are as follows: $C = \{x_0, \dot{x}\}$ and $F = \{x(x_0, \dot{x}, t_0, t) = x_0 + \dot{x}(t - t_0)\}$, where x_0 denotes the starting coordinate; $\dot{x}$ the velocity; t_0 and t the starting and the actual time, respectively, of the process; and x denotes the actual location of the moving object at time t.

Any process is an instance of some process scheme and referred to by a (unique) name:

Definition 2 Let $\mathcal{N}$ be a set of symbols (called *names*). A *process* is a 4-tuple $\langle \mathtt{n}, \mathcal{T}, t_0, \vec{p}\,\rangle$ where

- $\mathtt{n} \in \mathcal{N}$;
- $\mathcal{T} = \langle C, F\rangle$ is a process scheme (the *type*), where C is of size n;
- $t_0 \in \mathbb{R}$ (the *starting time*); and
- $\vec{p} = (p_1, \ldots, p_n) \in \mathbb{R}^n$ is an n-dimensional vector over $\mathbb{R}$ (the *parameter vector*)

For example, let $\mathcal{T}_{\text{move}}$ denote the example scheme from above then[1]

$$\begin{array}{l} \langle\, \mathtt{Train\_A}\,,\ \mathcal{T}_{\text{move}}\,,\ \text{1:00pm}\,,\ (\text{0mi}, \text{25mph})\,\rangle \\ \langle\, \mathtt{Train\_B}\,,\ \mathcal{T}_{\text{move}}\,,\ \text{1:30pm}\,,\ (\text{80mi}, -\text{20mph})\,\rangle \end{array} \quad (1)$$

are two processes describing two trains starting at different times and, then, moving towards each other, with different speed.[2]

Based on these notions, we call a set S of processes in conjunction with a particular time-point $t_S \in \mathbb{R}$ a *situation*. Value t_S denotes the time when S arose. We assume that two distinct processes occurring in the same situation have different names, chosen from a given set. If neither an interaction between the given processes nor actions take place then the individual processes are assumed to continue eternally. In this case, S provides a description of the system being modeled at any time $t \geq t_S$.

Events and Transition Laws

However, even without manipulating the ongoing processes by means of executing actions, processes may interact and, by doing this, destroy the harmony. In such cases, a situation $\langle S, t_S\rangle$ is only a time-limited description, whose suitability ends as soon as interaction gives rise to changes within the collection of processes. Such a breakpoint, which causes a discontinuity in the state of affairs, is called *event*. In general, an event causes some running processes to end and some new processes to start at a particular point in time. For instance, an inelastic collision between two moving objects terminates, at the time they meet, both movements and initiates two new processes where both objects move side-by-side, possibly in a new direction and with changed velocity.

[1] The following example was inspired by (Shoham & McDermott 1988).

[2] The starting location of the first train, `Train_A`, is taken as reference point of the 1-dimensional coordinate system; hence, the initial distance between the two trains is 80 miles.

Generally, any non-trivial situation $\langle S, t_S \rangle$ gives rise to a variety of potential events at various time-points $t > t_S$. Whether such an event actually occurs depends on whether the situation remains stable until the expected occurrence of the event. It is therefore crucial to find the very *next* event; only this one is guaranteed to occur as expected. To illustrate this point, which we call the *nearest event problem*, consider the situation displayed in Figure 1. While an analysis of the movements of Balls A and B results in the expectation that they collide, an analysis of A and C shows that prior to this we have to check whether these two balls collide first. If this is indeed the case then we should compute the effect of that discontinuity first and see whether Balls A and B still move towards each other.

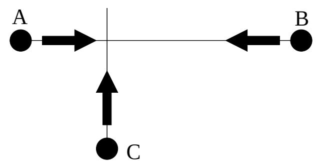

Figure 1: To predict the courses of the balls it is crucial to determine the *nearest event*. In case of any collision, will A and C collide first, or A and B?

This is reflected in the following definition:

Definition 3 An *event* is a triple $\langle C, t, E \rangle$ where C (the *condition*) and E (the *effect*) are (possibly empty) finite sets of processes and $t \in \mathbb{R}$ is the time at which the event is expected to occur.

Let $\langle S, t_S \rangle$ be a situation then an event $\langle C, t, E \rangle$ is *potentially applicable* iff $C \subseteq S$ and $t > t_S$. If $\mathcal{E}$ is a set of events then some $\langle C, t, E \rangle \in \mathcal{E}$ is *applicable* to $\langle S, t_S \rangle$ wrt. $\mathcal{E}$ iff it is potentially applicable and for each potentially applicable $\langle C', t', E' \rangle \in \mathcal{E}$ we have $t \leq t'$.

While in general more than just one actually applicable event may exist, we will restrict ourselves to non-simultaneity in this first approach. As an example, let S denote the two processes defined in (1) and let $t_S = $ 1:30pm then the following event—describing an inelastic collision[3]—is applicable to $\langle S, t_S \rangle$ provided no other event occurs in between:

$$\begin{aligned} \langle\ C &= \{\, \langle \texttt{Train\_A}, \mathcal{T}_{\text{move}}, \text{1:00pm}, (\text{0mi}, \text{25mph}) \rangle\,, \\ &\qquad \langle \texttt{Train\_B}, \mathcal{T}_{\text{move}}, \text{1:30pm}, (\text{80mi}, -\text{20mph}) \rangle \,\}\,, \\ t &= \text{3:00pm}\,, \\ E &= \{\, \langle \texttt{Train\_A}, \mathcal{T}_{\text{move}}, \text{3:00pm}, (\text{50mi}, \text{5mph}) \rangle\,, \\ &\qquad \langle \texttt{Train\_B}, \mathcal{T}_{\text{move}}, \text{3:00pm}, (\text{50mi}, \text{5mph}) \rangle \,\} \end{aligned} \tag{2}$$

Concrete events are instances of general transition laws, which contain variables and, possibly, constraints to guide the process of instantiation. In particular the event's time is usually determined by the instances of other variables. For example, the transition law for inelastic collisions of two continuously moving objects in a 1-dimensional space is as follows (variables are denoted by uppercase letters):

$$\begin{aligned} \langle\ C &= \{\, \langle N_A, \mathcal{T}_{\text{move}}, T_{A0}, (X_{A0}, \dot{X}_A) \rangle\,, \\ &\qquad \langle N_B, \mathcal{T}_{\text{move}}, T_{B0}, (X_{B0}, \dot{X}_B) \rangle \,\}\,, \\ t &= T\,, \\ E &= \{\, \langle N_A, \mathcal{T}_{\text{move}}, T, (X_{new}, \dot{X}_A + \dot{X}_B) \rangle\,, \\ &\qquad \langle N_B, \mathcal{T}_{\text{move}}, T, (X_{new}, \dot{X}_A + \dot{X}_B) \rangle \,\}\ \rangle \end{aligned} \tag{3}$$

where it is required $N_A \neq N_B$, $\dot{X}_A - \dot{X}_B \neq 0$, and $x_{N_A} = x_{N_B} = X_{new}$ at time T. In our example, where the two movement differentials are $x(X_{A0}, \dot{X}_A, T_{A0}, T) = X_{A0} + \dot{X}_A(T - T_{A0})$ and $x(X_{B0}, \dot{X}_B, T_{B0}, T) = X_{B0} + \dot{X}_B(T - T_{B0})$, this leads to:

$$\begin{aligned} T &= \tfrac{X_{A0} - X_{B0} - \dot{X}_A T_{A0} + \dot{X}_B T_{B0}}{\dot{X}_B - \dot{X}_A} \\ \text{and} \quad X_{new} &= X_{A0} + \dot{X}_A(T - T_{A0}) \end{aligned} \tag{4}$$

Note that in case the two objects do not head towards each other, this equation will result in some $T < T_{A0}, T_{B0}$; that is, the corresponding event can never be (potentially) applicable to a situation with time $T_S \geq T_{A0}, T_{B0}$. The reader is invited to verify that (2) is indeed a valid instance of (3).

On the basis of a set of events (i.e., the collection of all ground instances of given transition laws), the behavior of the model, starting in a particular initial situation, can be described by repeatedly searching for applicable events and, then, calculating their impact. As indicated, we restrict our model to non-simultaneous events, which is reflected in the following definition:

Definition 4 Let $\mathcal{E}$ be a set of events and $\langle S, t_S \rangle$ a situation then the *successor situation* $\Phi(\langle S, t_S \rangle)$ is defined as follows:

1. If no applicable event exists in $\mathcal{E}$ then $\Phi(\langle S, t_S \rangle) = \langle S, \infty \rangle$.
2. If $\langle C, t, E \rangle \in \mathcal{E}$ is the only applicable event then $\Phi(\langle S, t_S \rangle) = \langle S', t_{S'} \rangle$ where
   - $S' = (S \setminus C) \cup E$
   - $t_{S'} = t$
3. Otherwise $\Phi(\langle S, t_S \rangle)$ is undefined.

In words, if no applicable event exists then the system has reached a stable state, which is assumed to hold forever; else the result of an applicable event is obtained by exchanging processes according to the event's description and adjusting the initiating time-point of the new situation accordingly. The former represents the assumption of persistence: Each process which is not affected by the event continues to run in the new situation just like it did in the preceding one. In what follows, we implicitly assume Φ be always defined.

The repeated application of the successor situation function yields an infinite sequence of situations,

[3] This collision is to be interpreted as a coupling of trains rather than a violent crash.

$\langle S_0, t_0 \rangle$, $\Phi(\langle S_0, t_0 \rangle)$, $\Phi^2(\langle S_0, t_0 \rangle)$, ... Then the state of the system at a particular time-point $t \geq t_0$ is correctly described by the collection of processes S where $\langle S, t_S \rangle = \Phi^k(\langle S_0, t_0 \rangle)$ and $\langle S', t_{S'} \rangle = \Phi^{k+1}(\langle S_0, t_0 \rangle)$ such that $t_S \leq t < t_{S'}$, for some $k \geq 0$. For example, starting with $\langle (1), \text{1:30pm} \rangle$, the locations of the two trains, say, are determined by (1) until 3:00pm, while after the collision the new processes, E in (2), have to be used instead.

The above concept supports the notion of truth and falsity of *observations* made during a development of the system being modeled. Formally, an observation is an expression of the form $[t]\,\alpha(\mathtt{n}) = r$ where

- $t \in \mathbb{R}$ is the time of the observation;
- α is either a symbol in C or the name of a function in F for some process scheme $\langle C, F \rangle$;
- $\mathtt{n}$ is a symbol denoting a process name; and
- $r \in \mathbb{R}$ is the observed value.

Given an initial situation along with a set of events, such an observation is *true* iff the following holds: Let S be the collection of processes describing the system at time t (determined as discussed above) then S contains a process $\langle \mathtt{n}, \langle C, F \rangle, t_0, (r_1, \ldots, r_n) \rangle$ such that

1. either $C = \{c_1, \ldots, c_k = \alpha, \ldots, c_n\}$ and $r_k = r$;
2. or $\alpha \in F$ and $\alpha(r_1, \ldots, r_n, t_0, t) = r$.

E.g., observation $[\text{2:15pm}]x(\mathtt{Train\_B}) = 65\text{mi}$ is true in our example according to (1)[4] while observation $[\text{3:15pm}]x(\mathtt{Train\_B}) = 45\text{mi}$, say, is not since the latter does not take into account the train collision.

On this basis, we can define temporal projection as well as postdiction problems (called chronicle completion in (Sandewall 1994)) in the standard way (Gelfond & Lifschitz 1993; Sandewall 1994; Thielscher 1995) using model-theoretic concepts: A *model* for a set of observations Ψ (under given sets of names $\mathcal{N}$ and events $\mathcal{E}$) is a system development $\langle S_0, t_0 \rangle$, $\Phi(\langle S_0, t_0 \rangle)$, $\Phi^2(\langle S_0, t_0 \rangle)$, ... which satisfies all elements of Ψ. Such a set Ψ *entails* an (additional) observation ψ iff ψ is true in all models of Ψ.

Actions

The concept of agents executing actions in a system of continuous processes can be easily integrated into our model by viewing actions as (artificial) events as well. For example, the following event describes the action of starting Train_B at time 1:30pm:

$$\begin{aligned} \langle\ C &= \{\, \langle \mathtt{Train\_B}, \mathcal{T}_{\text{move}}, \text{1:00pm}, (80\text{mi}, 0\text{mph}) \rangle \,\}, \\ t &= \text{1:30pm}, \\ E &= \{\, \langle \mathtt{Train\_B}, \mathcal{T}_{\text{move}}, \text{1:30pm}, (80\text{mi}, -20\text{mph}) \rangle \,\}\ \rangle \end{aligned} \tag{5}$$

[4]We have $x(x_0 = 80\text{mi}, \dot{x} = -20\text{mph}, t_0 = \text{1:30pm}, t = \text{2:15pm}) = 80\text{mi} - 20\text{mph}(\text{2:15pm} - \text{1:30pm}) = 65\text{mi}$.

In words, the process describing Train_B idle at location $x_0 = 80\text{mi}$ will be replaced by the process describing the train's movement with velocity $\dot{x} = -20\text{mph}$.

As before, such events may be instances of more general transition laws whose applicability is triggered by the intention to execute some action. A most interesting feature of this representation of actions is that the time of their execution may depend on the situation itself. For instance, the specification "Start the idling Train_B to move with velocity -20mph as soon as Train_A passes the 12.5mi mark." can be represented by this transition law:

$$\begin{aligned} \langle\ C &= \{\, \langle \mathtt{Train\_A}, \mathcal{T}_{\text{move}}, T_{A0}, (X_{A0}, \dot{X}_A) \rangle, \\ &\qquad \langle \mathtt{Train\_B}, \mathcal{T}_{\text{move}}, T_B, (X_{B0}, 0\text{mph}) \rangle \,\}, \\ t &= T, \\ E &= \{\, \langle \mathtt{Train\_A}, \mathcal{T}_{\text{move}}, T_A, (X_{A0}, \dot{X}_A) \rangle, \\ &\qquad \langle \mathtt{Train\_B}, \mathcal{T}_{\text{move}}, T, (X_{B0}, -20\text{mph}) \rangle \,\}\ \rangle \end{aligned} \tag{6}$$

where it is required $\dot{X}_A \neq 0$ and $X_{A0} + \dot{X}_A(T - T_{A0}) = 12.5\text{mi}$. E.g., given the first process in (1) along with $\langle \mathtt{Train\_B}, \mathcal{T}_{\text{move}}, \text{1:00pm}, (80\text{mi}, 0\text{mph}) \rangle$, the event described in (5) is essentially an instance of (6)—considering the fact that the process describing Train_A is not changed in (6).

Given a set of events representing actions that are to be executed, these are added to the 'natural' events considered in the previous subsection and, then, all definitions concerning successor situations, developments, and observations remain as they are.

Pendulum and Balls Scenario

In this section, we illustrate how a more complex domain, namely, the interaction between a pendulum and balls that travel along a 1-dimensional space, can be modeled on the basis of our formalism. Figure 2 shows the pendulum which will collide at angle $\varphi = 0$ with a ball being at position $y = y_C$ at the same time. Since the logic part of the formalization is essentially domain-independent, the task left is to find appropriate equations describing the various possible movements (that is, defining process schemes) and the possible interactions (that is, defining transition laws).

For simplicity, we will neglect the damping factor of the motion of the pendulum. The differential equation describing it then is

$$m\, l^2\, \frac{d^2\varphi}{dt^2} \;=\; -mgl\, \sin\varphi \;-\; l^2 \frac{d\varphi}{dt}$$

where l is the length of the pendulum, m is the mass of the pendulum, and g is $9.81 \frac{m}{s^2}$. Solving the differential results in the angle of the pendulum φ, the angular velocity $\dot{\varphi}$ and the angular acceleration $\ddot{\varphi}$.[5]

$$\varphi(\varphi_{\max}, \tau, \mathrm{T}_{\mathrm{P0}}, \mathrm{T}) \;=\; -\varphi_{\max}\ \cos(\frac{2\pi}{\tau}\ (\mathrm{T} - \mathrm{T}_{\mathrm{P0}}))$$

[5]For the sake of simplicity, we will regard the time constant τ of the pendulum be given rather than its length l (we have $\tau = 2\pi\sqrt{\frac{l\ \varphi_{max}}{g\ \sin\varphi_{max}}}$).

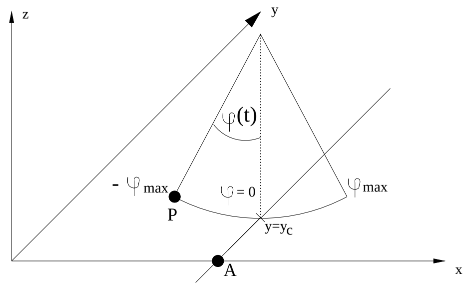

Figure 2: Pendulum P and Ball A in positions $\varphi_P = -\varphi_{max}$ and $Y_A = 0$.

$$\dot{\varphi}(\varphi_{\max}, \tau, \mathrm{T_{P0}}, \mathrm{T}) \;=\; \varphi_{\max}\, \frac{2\pi}{\tau}\, \sin(\frac{2\pi}{\tau}\, (\mathrm{T} - \mathrm{T_{P0}}))$$
$$\ddot{\varphi}(\varphi_{\max}, \tau, \mathrm{T_{P0}}, \mathrm{T}) \;=\; \varphi_{\max}\, \frac{4\pi^2}{\tau^2}\, \cos(\frac{2\pi}{\tau}\, (\mathrm{T} - \mathrm{T_{P0}}))$$

As the process scheme for the pendulum we obtain $\mathcal{T}_{\mathrm{pendulum}} = \langle C, F\rangle$ where $C = \{\varphi_{max},\ \tau,\ y_C\}$ and $F = \{\varphi,\ \dot{\varphi},\ \ddot{\varphi}\}$. E.g., if we start a pendulum with suspension point $y_C = 0.3m$, time constant $\tau = 1sec$ (such a pendulum is often called *second-pendulum*[6]) and starting angle $\varphi_{max} = 10°$ at time $t_{P0} = 0sec$ then the corresponding process becomes

$$\langle \texttt{Pendulum\_P},\ \mathcal{T}_{\mathrm{pendulum}},\ 0,\ (10,\ 1,\ 0.3)\rangle \qquad (7)$$

For a ball moving along the y-axis, we use a process scheme $\mathcal{T}_{\mathrm{move}} = \langle C, F\rangle$ similar to the one used in the preceding section, viz. $C = \{y_0, \dot{y}\}$ and $F = \{y(y_0, \dot{y}, t_0, t) = y_0 + \dot{y}(t - t_0)\}$.

Based on the two process schemes, we define three different types of events. The first is the collision of two balls, A and B, defined by identical locations at some time t, that is, $y(y_{A0}, \dot{y}_A, t_{A0}, t) = y(y_{B0}, \dot{y}_B, t_{B0}, t)$, similar to equations (4).

The second type of event is the collision between any ball and the pendulum, defined by the angle of the pendulum being zero while the ball's position is at the y-axis position of the pendulum, y_C, at the same time. Since in real physical systems, after such a collision ball and pendulum would move chaotically in 3-dimensional space, we introduce an arbitrary simplification for the sake of a deterministic behavior. The pendulum is assumed to be fixed in its y-axis and to be of much larger mass than the ball, such that the collision will simply be an elastic impact with a still-standing object for the ball (reflection into opposite direction) while the pendulum keeps moving continuously. This results in the following transition law:

$$\begin{aligned} \langle\ C \;&=\; \{\ \langle N_A, \mathcal{T}_{\mathrm{move}}, T_{A0}, (Y_{A0}, \dot{Y}_{A0})\rangle\,, \\ &\qquad \langle N_P, \mathcal{T}_{\mathrm{pendulum}}, T_{P0}, (\varphi_{max}, \tau, Y_C)\rangle\ \}\,, \\ t \;&=\; T\,, \qquad\qquad\qquad\qquad\qquad\qquad (8) \\ E \;&=\; \{\ \langle N_A, \mathcal{T}_{\mathrm{move}}, T, (Y_C, -\dot{Y}_{A0})\rangle\,, \\ &\qquad \langle N_P, \mathcal{T}_{\mathrm{pendulum}}, T_{P0}, (\varphi_{max}, \tau, Y_C)\rangle\ \}\ \rangle \end{aligned}$$

[6] A second-pendulum is in its $\varphi = 0$ position at every point of time $(t - t_{P0}) = \frac{2n-1}{4} sec\ (n \in \mathbb{N})$.

where it is required that $\dot{Y}_{A0} \neq 0$, $\tau \neq 0$,

$$\frac{1}{2}\left(\frac{4(\frac{Y_C - Y_{A0}}{\dot{Y}_A} + T_{A0} - T_{P0})}{\tau} + 1\right) \in \mathbb{N}$$
$$\text{and} \quad T \;=\; \frac{Y_C - Y_{A0}}{\dot{Y}_A} + T_{A0}$$

That is, the pendulum process remains unchanged and new parameters result for the ball. As above, if no collision will occur we obtain a value T smaller than the actual time.

The last type of event is simply user interaction like inserting a new ball into the scenario or starting the pendulum. E.g., the following event formalizes our intention to start a ball from position $y_0 = 0m$ at time $t = 2sec$ to move with speed $\dot{y} = 0.4m/sec$:

$$\langle\ \mathrm{C} = \{\,\}\,,\ \mathrm{t} = 2sec\,,\ \mathrm{E} = \{\langle \texttt{Ball\_A},\ \mathcal{T}_{\mathrm{move}},\ 2,\ (0, 0.4)\rangle\}\ \rangle$$

Now, given the initial situation $\langle\{(7)\}, 0sec\rangle$, the above event designs a successor state at time $t = 2sec$ by adding the moving ball. Following this state transition, the transition law (8) will be fulfilled for time $t = 2.75sec$ as the *nearest event* (see previous section), since

$$t \;=\; \frac{y_C - y_{A0}}{\dot{y}_A} + t_{A0} \;=\; \tfrac{3}{4}sec + 2sec \quad \text{and}$$
$$\tfrac{1}{2}\left(\frac{4(\frac{0.3m - 0m}{0.4m/sec} + 2sec - 1sec)}{1sec} + 1\right) \;=\; 4\ \in \mathbb{N}$$

This results in the pendulum moving unaltered while the ball now moves into the opposite direction, that is, we obtain $\langle \texttt{Ball\_A},\ \mathcal{T}_{\mathrm{move}},\ 2.75,\ (0.3, -0.4)\rangle$ as a new process. With our approach, we can query for the occurrence of such discontinuities and detect further collisions after this *nearest event*. Also, we can insert new balls at arbitrary time and location or stop and start the pendulum with variable velocity.

Discussion

Generalizing the concept of state descriptions as collections of fluents, our approach of *process separation* introduces a syntax and semantics to formalize and reason about descriptions of continuous physical systems. In addition, in (Herrmann & Thielscher 1996) we offer a suitable calculus that is based on logic programming and allows for logical reasoning in terms of meaningful expressions (e.g. collision event) as well as precise numerical values about the processes (e.g. location at a certain time). Hence, a process separation model of a real world system can serve to evaluate the proper behavior of a system that may be specified by differential equations in our calculus.

While purely numerical approaches do not allow logical reasoning about system phenomena, the so-called *Qualitative Reasoning* uses qualitative descriptions to model a system and offers a logic based on descriptive attributes (Kuipers 1994). Applications of this method model physical systems in *qualitative simulations* (Faltings & Struss 1992). A reproach of Qualitative Reasoning is the absence of quantitative values.

Our process separation model allows to reason logically about predefined events *and* is still based on precise numerical values accessible to the user.

We have been able to solve the *extended prediction problem* by limiting the number of future time slices in concentrating on 'interesting' points in time, that is, where events occur. Since we take all possible events at which the system is assumed to change its parameters into account, this quantization avoids the false predictions mentioned in (Shoham & McDermott 1988). Our approach is not limited to a previously fixed number of objects in our scenarios, since we allow actions that may start new objects to participate in our system model. Also, the occurrence of potential events or their amount need not be known in advance. It is just necessary to specify the conditions under which an event occurs and the new parameters that result from such an event, in form of *transition laws*.

A previous approach to temporal reasoning about continuous systems has introduced *manifested histories* vs. *potential histories* (Shoham 1988). In that approach the potential future courses of two balls may be altered if a collision manifests different courses which only works for discrete points of time as the author admits in his "Technical Limitations." In our calculus, time is real-valued and not affected by this problem.

Another way to reason about physical systems using differential equations and temporal logic was introduced in (Sandewall 1989). Discontinuous systems were divided into piecewise continuous ones separated by discontinuities. As argued in (Allen 1984), such a representation in the form of time-points rather than intervals bears the problem of not being able to correctly model the change of multiple parameters in one time-point. We overcome this problem by separating the system into multiple processes, where all parameters can be adjusted for a new interval of time.

Finally, let us recall the restriction that the current versions of our semantics and calculus do not allow events to occur simultaneously. In case two or more events occur at the same time but without mutual influence, this could be straightforwardly modeled. But if two or more simultaneous events concern identical objects (e.g., three balls all moving towards a single collision) then the overall result might not simply be the combination of the results of the involved events. Rather, such situations require more sophisticated means to construct suitable state transition functions. Recent solutions to this problem for the discrete case, e.g. (Lin & Shoham 1992; Baral & Gelfond 1993; Thielscher 1995), will help to establish an adequate extension of our formalism; yet, this is left as future work.

Internet Availability. For the interested reader, we have provided PROLOG sources on our ftp-server `aida.intellektik.informatik.th-darmstadt.de` in the directory `/pub/AIDA/ContProc`.

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